Complete Mathematics, Part-2

SSC
Higher Mathematics

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Raj

Lucent's

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English Edition

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01 Algebraic Identities

1. Formulae for two index, two term.

1.1. $(a + b)^2 = a^2 + 2ab + b^2$

$= (a - b)^2 + 4ab$

1.2. $(a - b)^2 = a^2 - 2ab + b^2$

$= (a + b)^2 - 4ab$

1.3. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

1.4. $(a + b)^2 - (a - b)^2 = 4ab$

2. Formulae for three index, two term.

2.1. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= a^3 + b^3 + 3ab(a + b)$

2.2. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= a^3 - b^3 - 3ab(a - b)$

2.3. $(a + b)^3 + (a - b)^3 = 2(a^3 + 3ab^2) = 2a(a^2 + 3b^2)$

2.4. $(a + b)^3 - (a - b)^3 = 3a^2b + 2b^3 = 2b(3a^2 + b^2)$

3. Formulae for four and five index, two term.

3.1. $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

3.2. $(a - b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

3.3. $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

3.4. $(a - b)^5 = a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$

4. Ordinary factors.

4.1. $a^2 - b^2 = (a - b)(a + b)$

4.2. $a^3 + b^3 = (a + b)(a^2 - ab + b^2) = (a + b)^3 - 3ab(a + b)$

4.3. $a^3 - b^3 = (a - b)(a^2 + ab + b^2) = (a - b)^3 + 3ab(a - b)$

4.4 .$a^4 - b^4 = (a^2 - b^2)(a^2 + b^2) = (a - b)(a + b)(a^2 + b^2)$

$= (a - b)(a^3 + a^2b + ab^2 + b^3)$

4.5. $x^2 + (a + b)x + ab = (x + a)(x + b)$

4.6. $x^2 - (a + b)x + ab = (x - a)(x - b)$

5. Factor for $a^n - b^n$.

5.1. $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots\ldots + ab^{n-2} + b^{n-1})$

How to Remember: *First factor of RHS is $(a-b)$. In the second factor, power of a in first term is $n-1$. In consecutive terms power of a is decreasing by 1 where as power of b is increasing by 1. Last term does not contain a and power of b (in last term) is $n-1$.*

If $n = 2$ then, $a^2 - b^2 = (a-b)(a+b)$

$n = 3$ then, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$n = 4$ then, $a^4 - b^4 = (a-b)(a^3 + a^2b + ab^2 + b^3)$

$n = 5$ then, $a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$ etc.

5.2. If $a = 1, b = x$ then,

$$(1 - x^n) = (1-x)(1 + x + x^2 + x^3 + \ldots\ldots + x^{n-1})$$

at, $n = 2$ $\quad 1 - x^2 = (1-x)(1+x)$

at, $n = 3$ $\quad 1 - x^3 = (1-x)(1 + x + x^2)$

at, $n = 4$ $\quad 1 - x^4 = (1-x)(1 + x + x^2 + x^3)$

at, $n = 5$ $\quad 1 - x^5 = (1-x)(1 + x + x^2 + x^4)$ etc.

6. Special factors :

6.1. $a^4 + a^2b^2 + b^4 = (a^2 + b^2)^2 - a^2b^2$

$$= (a^2 - ab + b^2)(a^2 + ab + b^2)$$

6.2. $(a + b + c)(bc + ca + ab) - abc = (b + c)(c + a)(a + b)$

6.3. $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(b + c)(c + a)(a + b)$

7. 7.1. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

7.2. If $a + b + c = 0$, then, $a^3 + b^3 + c^3 = 3abc$

7.3. $\because \quad a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\}$

$\therefore \quad a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)\{(a-b)^2 + (b-c)^2 + (c-a)^2\}$

But, if $\{(a-b)^2 + (b-c)^2 + (c-a)^2\} = 0$

then, $a - b = 0, b - c = 0$, and $c - a = 0$

or, $a = b,\ b = c,\ c = a$

or, $a = b = c$

Hence, $a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow a + b + c = 0 \quad$ or, $\quad a = b = c$

7.4. $(a-b)^2 + (b-c)^2 + (c-a)^2 \geq 0$

$\Rightarrow \quad 2(a^2 + b^2 + c^2 - ab - bc - ca) \geq 0$

$\Rightarrow \quad a^2 + b^2 + c^2 \geq ab + bc + ca$

7.5. $(a + b + c)^2 \geq 0$

$\Rightarrow \quad a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 0$

$\Rightarrow \quad ab + bc + ca \geq \frac{-1}{2}(a^2 + b^2 + c^2)$

8. Cyclic factor

8.1. $a^2(b - c) + b^2(c - a) + c^2(a - b) = -(a - b)(b - c)(c - a)$

8.2. $bc(b - c) + ca(c - a) + ab(a - b) = -(a - b)(b - c)(c - a)$

8.3. $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) = (a - b)(b - c)(c - a)$

9. Formulae for two index, three and four terms.

9.1. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

9.2. $(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd)$

Note : $(a - b - c)^2 = a^2 + b^2 + c^2 + 2a(-b) + 2a(-c) + 2(-b)(-c)$

$= a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$ etc.

9.3. $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(b + c)(c + a)(a + b)$

10. Special formula :

10.1. If $x + \frac{1}{x} = a$ then $x^2 + \frac{1}{x^2} = a^2 - 2$

10.2. If $x + \frac{1}{x} = a$ then $x^3 + \frac{1}{x^3} = a^3 - 3a$

10.3. If $x + \frac{1}{x} = a$ then $x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$

10.4. If $x + \frac{1}{x} = a$ then $x^5 + \frac{1}{x^5} = a^5 - 5a^3 + 5a$

10.5. If $x + \frac{1}{x} = a$ then $x^6 + \frac{1}{x^6} = a^6 - 6a^4 + 9a^2 - 2$

11. 11.1. If $x - \frac{1}{x} = a$ then $x^2 + \frac{1}{x^2} = a^2 + 2$

11.2. If $x - \frac{1}{x} = a$ then $x^3 - \frac{1}{x^3} = a^3 + 3a$

11.3. If $x - \frac{1}{x} = a$ then $x^4 + \frac{1}{x^4} = a^4 + 4a^2 + 2$

11.4. If $x - \frac{1}{x} = a$ then $x^5 - \frac{1}{x^5} = a^5 + 5a^3 + 5a$

11.5. If $x - \frac{1}{x} = a$ then $x^6 + \frac{1}{x^6} = a^6 + 6a^4 + 9a^2 + 2$

Proof of some of the above identities are given in solved examples.

12. 12.1. If $x^2 + \frac{1}{x^2} = 1$ then $x^6 = -1$

12.2. If $x^2+\frac{1}{x^2}=-1$ then $x^6=1$

Explanation : $x^2+\frac{1}{x^2}=1$

$\Rightarrow\ x^4-x^2+1=0]$

Multiplying both sides by (x^2+1), $(x^2+1)(x^4-x^2+1)=0$

or, $\left(x^2\right)^3+1=0$

$\therefore\ x^6=-1$ etc.

Note : *Here x is an imaginary (complex) Number.*

13. 13.1. Componendo-dividendo

If $\frac{a}{b}=\frac{c}{d}$ then, $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ or, $\frac{a-b}{a+b}=\frac{c-d}{c+d}$

13.2. If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{b}=\frac{c+d}{d}$ *(by componendo)*

13.3. If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a-b}{b}=\frac{c-d}{d}$ *(by dividendo)*

4. Ratio-proportion

14.1. $\frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}=\frac{a-c}{b-d}=\frac{\sqrt{ac}}{\sqrt{bd}}=\frac{\sqrt{a^2+c^2}}{\sqrt{b^2+d^2}}$

14.2. $\frac{a}{b}=\frac{c}{d}=\frac{ka+mb}{kb+md}=\frac{ka-mb}{kb-md}$

14.3. $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{a+c+e}{b+d+f}=\frac{a+c-e}{b+d-f}=\frac{\sqrt[3]{ace}}{\sqrt[3]{bdf}}=\frac{\sqrt{a^2+c^2+e^2}}{\sqrt{b^2+d^2+f^2}}$ etc.

15. Remainder Theorem and Factor Theorem :

15.1. Remainder theorem

When a polynomial $p(x)$ of one or more than one degree divided by $x-a$, the remainder is $p(a)$.

e.g., suppose $p(x)=2x^3+3x^2-x-1$ is a polynomial.

When it is divided by $x-3$,

Remainder $p(3)=2\cdot3^3+3\cdot3^2-3-1=54+27-4=77$.

15.2. Factor theorem

When a polynomial $p(x)$ of one or more than one degree is divided by $(x-a)$ and remainder is zero. Then '$x-a$' is a factor of $p(x)$. e.g.,

In, $p(x)=x^3-x^2+x-1$, $p(1)=1-1+1-1=0$

Hence, $(x-1)$ is a factor of x^3-x^2+x-1.

16. Special cases for Remainder and Factor theorem.

16.1. When a polynomial $p(x)$ of one or more than one degree is divided by $(ax + b)$, the remainder is $p\left(\frac{-b}{a}\right)$.

16.2. When a polynomial $p(x)$ of one or more than one degree is divided by $(ax + b)$ and the remainder is zero then $ax + b$ is a factor of $p(x)$.

16.3. Suppose a polynomial $p(x)$ is divided by $x - a$ and $x - b$ and remainder are respectively m and n. When the polynomial is divided by $(x - a)(x - b)$, the remainder will be of the form $Ax + B$, where $p(a) = Aa + B = m$ and $p(b) = Ab + B = n$. Solve these two equations to get values of A and B. *(See solved example 22)*

Solved Example

1. If $a^2 + ab + b^2 = b^2 + bc + c^2$ where $a \neq b \neq c$ then find the value of $a + b + c$.

Solution : $a^2 + ab + b^2 = b^2 + bc + c^2$

$\Rightarrow \quad a^2 + ab = bc + c^2$

$\Rightarrow \quad a^2 - c^2 + ab - bc = 0$

$\Rightarrow \quad (a - c)(a + c) + b(a - c) = 0$

$\Rightarrow \quad (a - c)(a + c + b) = 0$

$\because \quad a \neq c \quad \therefore \quad a + c + b = 0$

2. If $a + b = z$, $c + a = y$, $b + c = x$, $x^2 + y^2 + z^2 = 50$ and $xy + yz + zx = 47$ then find the value of $a^2 + b^2 + c^2 - ab - bc - ca$.

Solution : Here $x - y = b - a$, $y - z = c - b$ and $z - x = a - c$

Now, $a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\{(a - b)^2 + (b - c)^2 + (c - a)^2\}$

$= \frac{1}{2}\{(x - y)^2 + (y - z)^2 + (z - x)^2\} \qquad (\because (a - b)^2 = (b - a)^2)$

$= \frac{1}{2}\{2(x^2 + y^2 + z^2 - xy - yz - zx)\}$

$= x^2 + y^2 + z^2 - (xy + yz + zx) = 50 - 47 = 3$

3. If $ab + bc + ca = 0$ then prove that $a^2b^2 + b^2c^2 + c^2a^2 = -2abc(a + b + c)$

Solution : $(ab + bc + ca)^2 = (ab)^2 + (bc)^2 + (ca)^2 + 2(abbc + abca + bcca)$

or, $0 = a^2b^2 + b^2c^2 + c^2a^2 + 2abc(b + a + c)$

$\therefore \quad a^2b^2 + b^2c^2 + c^2a^2 = -2abc(b + a + c)$

4. If $x^3 - \frac{1}{x^3} = k^3 + 3k$ then $x - \frac{1}{x} = k$ Prove it.

Solution : $\left(x - \frac{1}{x}\right)^3 = x^3 - 3x + 3\frac{1}{x} - \frac{1}{x^3}$

$\Rightarrow \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$

$\Rightarrow \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right) = x^3 - \frac{1}{x^3}$ ...(i)

But it is given that, $k^3 + 3k = x^3 - \frac{1}{x^3}$ ...(ii)

Comparing (1) & (2), $x - \frac{1}{x} = k$

5. If $x + \frac{1}{x} = 4$ then find the value of $x^3 + \frac{1}{x^3}$.

Solution : $\left(x + \frac{1}{x}\right)^3 = x^3 + 3x + 3\frac{1}{x} + \frac{1}{x^3}$

$\Rightarrow 4^3 = x^3 + 3\left(x + \frac{1}{x}\right) + \frac{1}{x^3}$

$\Rightarrow 64 = x^3 + 3 \times 4 + \frac{1}{x^3}$

$\therefore x^3 + \frac{1}{x^3} = 64 - 12 = 52$

6. If $p^3 - \frac{1}{p^3} = 140$ then find the value of $p - \frac{1}{p}$

Solution : $\left(p - \frac{1}{p}\right)^3 = p^3 - 3p + \frac{3}{p} - \frac{1}{p^3}$

$\Rightarrow \left(p - \frac{1}{p}\right)^3 = p^3 - \frac{1}{p^3} - 3\left(p - \frac{1}{p}\right)$

$\Rightarrow \left(p - \frac{1}{p}\right)^3 + 3\left(p - \frac{1}{p}\right) = 140 = 125 + 15$ ...(i)

$\therefore \left(p - \frac{1}{p}\right)^3 + 3\left(p - \frac{1}{p}\right) = 5^3 + 3 \times 5$

Comparing $p - \frac{1}{p} = 5$

Second Method : Let $p - \frac{1}{p} = t$

$\therefore$ from (i) of above method,

$\Rightarrow t^3 + 3t - 140 = 0$

$\Rightarrow \quad t^3 - 125 + 3t - 15 = 0$

$\Rightarrow \quad (t-5)(t^2 + 5t + 25) + 3(t-5) = 0$

$\Rightarrow \quad (t-5)(t^2 + 5t + 25 + 3) = 0$

$\Rightarrow \quad t = 5, t^2 + 5t + 28 = 0$

$\Rightarrow \quad t = 5, t = \frac{-5 \pm \sqrt{25-112}}{2}$, which is imaginary.

7. If $x + \frac{1}{x} = a$ then $x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$ prove it.

Solution : $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow \quad a^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = a^2 - 2$

Squaring both sides, $x^4 + \frac{1}{x^4} + 2 = a^4 - 4a^2 + 4$

$\therefore \quad x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$

8. If $t + \frac{1}{t} = c$ then $t^5 + \frac{1}{t^5} = c^5 - 5c^3 + 5c$ prove it. *[SSC Tier-I 2014]*

Solution : $t + \frac{1}{t} = c \Rightarrow t^2 + \frac{1}{t^2} + 2 = c^2 \Rightarrow t^2 + \frac{1}{t^2} = c^2 - 2$

and $t + \frac{1}{t} = c$

$\Rightarrow \quad t^3 + 3t + \frac{3}{t} + \frac{1}{t^3} = c^3$

$\Rightarrow \quad t^3 + \frac{1}{t^3} + 3\left(t + \frac{1}{t}\right) = c^3$

$\Rightarrow \quad t^3 + \frac{1}{t^3} + 3c = c^3$

$\Rightarrow \quad t^3 + \frac{1}{t^3} = c^3 - 3c$

Now, $\left(t^2 + \frac{1}{t^2}\right)\left(t^3 + \frac{1}{t^3}\right) = t^5 + \frac{1}{t} + t + \frac{1}{t^5}$

$\Rightarrow \quad (c^2 - 2)(c^3 - 3c) = t^5 + c + \frac{1}{t^5}$

$\Rightarrow \quad c^5 - 3c^3 - 2c^3 + 6c = t^5 + \frac{1}{t^5} + c$

$\therefore \quad t^5 + \frac{1}{t^5} = c^5 - 5c^3 + 5c$

9. If $a^4 + \frac{1}{a^4} = 47$ then find the value of $a + \frac{1}{a}$.

Solution : We know that $a + \frac{1}{a} = t$ *(As in question no. 7)*

$\Rightarrow \quad a^4 + \frac{1}{a^4} = t^4 - 4t^2 + 2$

$t^4 - 4t^2 + 2 = 47$

$t^4 - 4t^2 - 45 = 0$

$(t^2 - 9)(t^2 + 5) = 0$

$\Rightarrow \quad t^2 = 9$

$\Rightarrow \quad t = \pm 3$

10. If $x - \frac{1}{x} = a$ then prove that $x^4 + \frac{1}{x^4} = a^4 + 4a^2 + 2$.

Solution : $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

$\Rightarrow \quad a^2 + 2 = x^2 + \frac{1}{x^2}$

Squaring, $a^4 + 4a^2 + 4 = x^4 + \frac{1}{x^4} + 2$

$\therefore \quad x^4 + \frac{1}{x^4} = a^4 + 4a^2 + 2$

11. If $x^4 + \frac{1}{x^4} = 14$ then find the value of $x - \frac{1}{x}$.

Solution : We know that if $x - \frac{1}{x} = a$ then $x^4 + \frac{1}{x^4} = a^4 + 4a^2 + 2$

According to question if $x - \frac{1}{x} = a$ then $a^4 + 4a^2 + 2 = 14$

$\Rightarrow \quad a^4 + 4a^2 - 12 = 0$

$\Rightarrow \quad (a^2 + 6)(a^2 - 2) = 0$

$\Rightarrow \quad a^2 = 2$

$\therefore \quad a = \pm\sqrt{2}$

12. If $x + \frac{1}{x} = a$ then prove that $x^6 + \frac{1}{x^6} = a^6 - 6a^4 + 9a^2 - 2$. *[SSC Tier-I 2014]*

Solution : $x^6 + \frac{1}{x^6} = (x^2)^3 + \left(\frac{1}{x^2}\right)^3$

$$= \left(x^2 + \frac{1}{x^2}\right)\left(x^4 - x^2 \cdot \frac{1}{x^2} + \frac{1}{x^4}\right)$$

$$= \left\{\left(x+\frac{1}{x}\right)^2-2\right\}\left\{\left(x^2+\frac{1}{x^2}\right)^2-2-1\right\}$$

$$= \left\{\left(x+\frac{1}{x}\right)^2-2\right\}\left\{\left(\left(x+\frac{1}{x}\right)^2-2\right)^2-3\right\}$$

$$= \left\{\left(a^2-2\right)\right\}\left\{\left(a^2-2\right)^2-3\right\} = (a^2-2)(a^4-4a^2+1)$$

$$= a^6-4a^4+a^2-2a^4+8a^2-2 = a^6-6a^4+9a^2-2$$

13. If $x = b - c + a$, $y = c - a + b$, $z = a - b + c$, then prove that

$(b-a)x + (c-b)y + (a-c)z = 0$

Solution : 1^{st} term $= (b-a)x = (b-a)(b-c+a) = (b-a)\{(b+a)-c\}$

$= (b-a)(b+a) - (b-a)c = b^2 - a^2 - bc + ac$ ... (i)

2^{nd} term $= (c-b)y = (c-b)(c-a+b) = (c-b)\{(c+b)-a\}$

$= (c-b)(c+b) - (c-b)a = c^2 - b^2 - ca + ab$ ... (ii)

3^{rd} term $= (a-c)z = (a-c)(a-b+c) = (a-c)\{(a+c)-b\}$

$= (a-c)(a+c) - (a-c)b = a^2 - c^2 - ab + bc$ ... (iii)

$\therefore$ from (i), (ii) and (iii), $(b-a)x + (c-b)y + (a-c)z$

$= b^2 - a^2 + c^2 - b^2 + a^2 - c^2 - bc + ac - ca + ab - ab + bc = 0$

14. If $2s = a + b + c$,

then prove that, $(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2 = a^2 + b^2 + c^2$.

Solution : LHS$= (s-a)^2 + (s-b)^2 + (s-c)^2 + s^2$

$= (s^2 - 2as + a^2) + (s^2 - 2bs + b^2) + (s^2 - 2cs + c^2) + s^2$

$= 4s^2 - 2s(a+b+c) + a^2 + b^2 + c^2$

$= 4s^2 - 2s \times 2s + a^2 + b^2 + c^2$ $(\because\ a+b+c = 2s)$

$= 4s^2 - 4s^2 + a^2 + b^2 + c^2 = a^2 + b^2 + c^2$

15. Prove that $2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4$

$= (a+b+c)(b+c-a)(c+a-b)(a+b-c)$

Solution : Given expression $= 4b^2c^2 - (a^4 + b^4 + c^4 + 2b^2c^2 - 2c^2a^2 - 2a^2b^2)$

$= (2bc)^2 - (a^2 - b^2 - c^2)^2$

$= \{2bc + (a^2 - b^2 - c^2)\}\{2bc - (a^2 - b^2 - c^2)\}$

$(\because\ x^2 - y^2 = (x+y)(x-y))$

$= \{a^2 - (b^2 - 2bc + c^2)\}\{(b^2 + 2bc + c^2) - a^2\}$

$= \{a^2 - (b-c)^2\}\{(b+c)^2 - a^2\}$

$= [a + (b - c)]\,[a - (b - c)]\,[(b + c) + a]\,[(b + c) - a]$

$= (a + b - c)\,(a - b + c)\,(b + c + a)\,(b + c - a)$

$= (a + b + c)\,(b + c - a)\,(c + a - b)\,(a + b - c)$

16. Prove that $(a + b + c)^3 = a^3 + b^3 + c^3 + 3\,(b + c)\,(c + a)\,(a + b)$

Solution : $(a + b + c)^3 = [(a + b) + c]^3$

$= (a + b)^3 + c^3 + 3(a + b)\,c\,[(a + b) + c]$

$\because \quad (x + y)^3 = x^3 + y^3 + 3xy\,(x + y)$

Here, $x = a + b,\ y = c)$

$= [a^3 + b^3 + 3ab\,(a + b)] + c^3 + 3(a + b)\,c\,(a + b + c)$

$= a^3 + b^3 + c^3 + [3ab\,(a + b) + 3\,(a + b)\,c\,(a + b + c)]$

$= a^3 + b^3 + c^3 + 3(a + b)\,[ab + c\,(a + b + c)]$

$= a^3 + b^3 + c^3 + 3(a + b)\,[c^2 + c\,(a + b) + ab]$

$= a^3 + b^3 + c^3 + 3\,(a + b)\,(c + b)\,(c + a)$

$= a^3 + b^3 + c^3 + 3\,(b + c)\,(c + a)\,(a + b)$

17. If $a + b + c = 0$ then prove the following.

(i) $a^2 + b^2 + c^2 = -2\,(bc + ca + ab)$

(ii) $a^3 + b^3 + c^3 = 3abc$

(iii) $(bc + ca + ab)^2 = b^2c^2 + c^2a^2 + a^2b^2 = \frac{1}{4}\,(a^2 + b^2 + c^2)^2$

(iv) $a^4 + b^4 + c^4 = 2\,(b^2c^2 + c^2a^2 + a^2b^2) = \frac{1}{2}\,(a^2 + b^2 + c^2)^2$

(v) $a^5 + b^5 + c^5 = -5abc\,(bc + ca + ab)$

$= \frac{5}{2}\,abc\,(a^2 + b^2 + c^2)$

$= \frac{5}{6}\,(a^2 + b^2 + c^2)\,(a^3 + b^3 + c^3)$

Solution : (i) We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2bc + 2ca + 2ab$

$\therefore \quad 0^2 = a^2 + b^2 + c^2 + 2\,(bc + ca + ab)$

or, $a^2 + b^2 + c^2 = -2\,(bc + ca + ab)$

(ii) We know that $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)\,(a^2 + b^2 + c^2 - bc - ca - ab)$

$= 0 \times (a^2 + b^2 + c^2 - bc - ca - ab) = 0$

or, $a^3 + b^3 + c^3 = 3abc$

(iii) $(bc + ca + ab)^2 = b^2c^2 + c^2a^2 + a^2b^2 = \frac{1}{4}\,(a^2 + b^2 + c^2)^2$

$\because \quad (bc + ca + ab)^2 = b^2c^2 + c^2a^2 + a^2b^2 + 2abc\,(a + b + c)$

$\therefore \quad (bc + ca + ab)^2 = b^2c^2 + c^2a^2 + a^2b^2 + 2abc \times 0 = b^2c^2 + c^2a^2 + a^2b^2$

But from (i), $bc + ca + ab = -\frac{1}{2}(a^2 + b^2 + c^2)$

$\therefore \quad (bc + ca + ab)^2 = \frac{1}{4}(a^2 + b^2 + c^2)^2$

Hence, $(bc + ca + ab)^2 = b^2c^2 + c^2a^2 + a^2b^2 = \frac{1}{4}(a^2 + b^2 + c^2)^2$

(iv) We know from question no. (15) that,

$2b^2c^2 + 2c^2a^2 + 2a^2b^2 - a^4 - b^4 - c^4$

$= (a + b + c)(b + c - a)(c + a - b)(a + b - c)$

$= 0 \times (b + c - a)(c + a - b)(a + b - c) = 0$

$\therefore \quad a^4 + b^4 + c^4 = 2b^2c^2 + 2c^2a^2 + 2a^2b^2 = 2(b^2c^2 + c^2a^2 + a^2b^2)$

$= \frac{1}{2}(a^2 + b^2 + c^2)^2$ (from (iii)

(v) $\because \quad a + b + c = 0 \Rightarrow a + b = -c$

$\therefore \quad (a + b)^5 = (-c)^5$

or, $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 = -c^5$

or, $a^5 + b^5 + c^5 = -5a^4b - 10a^3b^2 - 10a^2b^3 - 5ab^4$

$= -5ab(a^3 + 2a^2b + 2ab^2 + b^3)$

$= -5ab(a + b)(a^2 + ab + b^2)$

$= -5ab(-c)\{(a + b)^2 - ab\} = 5abc\{(a + b)(-c) - ab\}$

$= 5abc(-ac - bc - ab) = -5abc(bc + ca + ab)$

$= \frac{5abc}{2}(a^2 + b^2 + c^2) = \frac{5}{6}(a^2 + b^2 + c^2)\cdot 3abc$

$= \frac{5}{6}(a^2 + b^2 + c^2)(a^3 + b^3 + c^3)$ $[\because a^3 + b^3 + c^3 = 3abc]$

18. If $x^2 + y^2 + z^2 = a$,

then prove that value of $xy + yz + zx$ lies between $\frac{-a}{2}$ and a.

Solution : From, $(x + y + z)^2 \geq 0$

$x^2 + y^2 + z^2 + 2(xy + yz + zx) \geq 0$

or, $a + 2(xy + yz + zx) \geq 0$

or, $xy + yz + zx \geq \frac{-a}{2}$

or, $\frac{-a}{2} \leq xy + yz + zx$ ... (i)

Again, $(x - y)^2 + (y - z)^2 + (z - x)^2 \geq 0$

$x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + z^2 + x^2 - 2zx \geq 0$

or, $2(x^2 + y^2 + z^2 - (xy + yz + zx)) \geq 0$

or, $x^2 + y^2 + z^2 \geq xy + yz + zx$

or, $a \geq xy + yz + zx$

or, $xy + yz + zx \leq a$

from (i) & (ii), $\frac{-a}{2} \leq xy + yz + zx \leq a$ ... (ii)

19. If $\frac{(m+n)x-(a-b)}{(m-n)x-(a+b)} = \frac{(m+n)x+a+c}{(m-n)x+a-c}$ then find the value of x.

Solution : We know that,

$$\frac{A}{B} = \frac{C}{D} \Leftrightarrow \frac{A+B}{A-B} = \frac{C+D}{C-D}$$ *(by componendo and dividendo)*

Given relation is,

$$\frac{mx+nx-a+b}{mx-nx-a-b} = \frac{mx+nx+a+c}{mx-nx+a-c}$$

or, $\frac{(mx-a)+(nx+b)}{(mx-a)-(nx+b)} = \frac{(mx+a)+(nx+c)}{(mx+a)-(nx+c)}$

or, $\frac{mx-a}{nx+b} = \frac{mx+a}{nx+c}$ *(by componendo and dividendo)*

or, $(mx - a)(nx + c) = (mx + a)(nx + b)$

or, $mnx^2 + cmx - anx - ac = mnx^2 + mbx + anx + ab$

or, $cmx - anx - mbx - anx = ab + ac$

or, $x(cm - 2an - mb) = a(b + c)$

or, $x = \frac{a(b+c)}{cm-2an-bm}$

20. If $\frac{2y+2z-x}{a} = \frac{2z+2x-y}{b} = \frac{2x+2y-z}{c}$ then prove that $\frac{x}{2b+2c-a} = \frac{y}{2c+2a-b} = \frac{z}{2a+2b-c}$

Solution : We know that $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{ka+mc+ne}{kb+md+nf}$

Multiplying first ratio by –1, second by 2, and third by 2,

Each ratio $= \frac{-(2y+2z-x)+2(2z+2x-y)+2(2x+2y-z)}{-a+2b+2c}$

$= \frac{-2y-2z+x+4z+4x-2y+4x+4y-2z}{-a+2b+2c}$

$= \frac{9x}{-a+2b+2c}$ ... (i)

Similarly, Multiplying first ratio by 2, second by – 1 and third by 2,

Each ratio $= \frac{9y}{2a-b+2c}$ *(do yourself)* ... (ii)

Ag... by 2, second by 2 and third by −1,

Each ratio $= \dfrac{9z}{2a+2b-c}$ ... (iii)

From (i), (ii) and (iii), $\dfrac{9x}{-a+2b+2c} = \dfrac{9y}{2a-b+2c} = \dfrac{9z}{2a+b-2c}$

$\therefore \quad \dfrac{x}{2b+2c-a} = \dfrac{y}{2c+2a-b} = \dfrac{z}{2a+2b-c}$

21. If $(x-1)$ and $(x-2)$ are two factors of $x^3 - ax^2 + 14x + b$ then find the value of a and b.

Solution : $\because$ $x-1$ is a factor of $x^3 - ax^2 + 14x + b$.

$\therefore$ putting $x = 1$,

$1 - a + 14 + b = 0$

or, $a - b = 15$ ... (i)

$\because$ $x-2$ is also a factor of $x^3 - ax^2 + 14x + b$

$\therefore$ putting $x = 2$,

$8 - 4a + 28 + b = 0$

or, $4a - b = 36$ ... (ii)

Solving (i) & (ii) $a = 7,\ b = -8$

22. If a polynomial is divided by $x-2$ the remainder is 1 and when it is divided by $x-3$ the remainder is 2. What will be the remainder when the polynomial is divided by $x^2 - 5x + 6$.

Solution : Let polynomial be $p(x)$ then by remainder theorem $p(2) = 1$ and $p(3) = 2$

$\because$ $x^2 - 5x + 6 = (x-2)(x-3)$

Let $p(x) = h(x)(x-2)(x-3) + ax + b$

$\therefore$ $p(2) = 0 + 2a + b$ ... (i)

or, $1 = 2a + b$

Again, $p(3) = 0 + 3a + b$ ... (ii)

or, $2 = 3a + b$

Subtracting (i) from (ii),

$$\begin{array}{r} 3a + b = 2 \\ 2a + b = 1 \\ \hline a = 1 \end{array}$$

From (i), $2 \times 1 + b = 1$ or, $b = -1$

Hence, Required remainder $ax + b = x - 1$.

Exercise–1A

1. If $x + y + z = 0$, then $(x + y)(y + z)(z + x)$ is equal to which of the followin
   (a) $-xyz$
   (b) $x^2 + y^2 + z^3$
   (c) $x^3 + y^3 + z^3 + 3xyz$
   (d) xyz
2. What is the LCM of $(6x^3 + 60x^2 + 150x)$ and $(3x^4 + 12x^3 - 15x^2)$?
   (a) $6x^2(x+5)^2(x-1)$
   (b) $3x^2(x+5)^2(x-1)$
   (c) $6x^2(x+5)^2(x-1)^2$
   (d) $3x^2(x+5)(x-1)^2$
3. If HCF of $(x^2 + x - 12)$ and $(2x^2 - kx - 9)$ is $(x - k)$, then value of k is
   (a) -3 (b) 3 (c) -4 (d) 4
4. What is the HCF of $(x^2 + bx - x - b)$ and $[x^2 + x(a-1) - a]$?
   (a) $x + b$ (b) $x + a$ (c) $x + 1$ (d) $x - 1$
5. If $(3x^3 - 2x^2y - 13xy^2 + 10y^3)$ is divided by $(x - 2y)$, then what is the remainder?
   (a) 0 (b) $y + 5$ (c) $y + 1$ (d) $y^2 + 3$
6. If $(x^3 + 5x^2 + 10k)$ is divided by $(x^2 + 2)$ then remainder is $-2x$, value of k is
   (a) -2 (b) -1 (c) 1 (d) 2
7. If $(5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2$ is divided by $x^2 + x + 1$, then what is the remainder ?
   (a) -1 (b) 0 (c) 1 (d) 2
8. What are the components of $(x^{29} - x^{24} + x^{13} - 1)$?
   (a) $(x - 1)$ only
   (b) $(x + 1)$ only
   (c) both $(x - 1)$ and $(x + 1)$
   (d) Neither $(x - 1)$ nor $(x + 1)$
9. If $x^2 - 4x + 1 = 0$, then what is the value of $x^3 + \frac{1}{x^3}$?
   (a) 44 (b) 48 (c) 52 (d) 64
10. If $x + y + z = 6$ and $xy + yz + zx = 11$ then what is the value of $x^3 + y^3 + z^3 - 3xyz$?
    (a) 18 (b) 36 (c) 54 (d) 66
11. If a is a rational number such that $(x - a)$ is a factor of $x^3 - 3x^2 - 3x + 9$, then
    (a) a may be any integer
    (b) a is an integer divisible by 9
    (c) a cannot be integer
    (d) a can have three values
12. If $x^2 - 11x + a$ and $x^2 - 14x + 2a$ have a common factor then what are the values of a ?
    (a) 0, 7 (b) 5, 20 (c) 0, 24 (d) 1, 3

13. For what value of k, HCF of $2x^2 + kx - 12$ and $x^2 + x - 2k - 2$ is $(x + 4)$.
(a) 5 (b) 7 (c) 10 (d) -4

14. $x(y - z)(y + z) + y(z - x)(z + x) + z(x - y)(x + y)$ equals
(a) $(x + y)(y + z)(z + x)$ (b) $(x - y)(x - z)(z - y)$
(c) $(x + y)(z - y)(x - z)$ (d) $(y - x)(z - y)(x - z)$

15. If $\left(\frac{x}{y}\right) = \left(\frac{z}{w}\right)$ then $(xy + zw)^2$ equals
(a) $(x^2 + z^2)(y^2 + w^2)$ (b) $x^2y^2 + z^2w^2$
(c) $x^2w^2 + y^2z^2$ (d) $(x^2 + w^2)(y^2 + z^2)$

16. If $\frac{1}{x+1} + \frac{2}{y+2} + \frac{1009}{z+1009} = 1$, then what is the value of $\frac{x}{x+1} + \frac{y}{y+2} + \frac{z}{z+1009}$?
(a) 0 (b) 2 (c) 3 (d) 4

17. If $a = 258$, $b = 260$ and $c = 262$ then value of $a^3 + b^3 + c^3 - 3abc$ is
(a) 9360 (b) 6240 (c) 7040 (d) 10560

18. If HCF of $x^3 - 27$ and $x^3 + 4x^2 + 12x + k$ is a quadratic polynomial then, the value of k is
(a) 27 (b) 9 (c) 3 (d) -3

19. When $x^{40} + 2$ is divided by $x^4 + 1$, then what is the remainder ?
(a) 1 (b) 2 (c) 3 (d) 4

20. If $a^x = b^y = c^z$ and $abc = 1$ then $xy + yz + zx$ is equal to which of the following ?
(a) xyz (b) $x + y + z$ (c) 0 (d) 1

21. If $a = \frac{1+x}{2-x}$ then $\frac{1}{a+1} + \frac{2a+1}{a^2-1}$ equals
(a) $\frac{(1+x)(2+x)}{2x-1}$ (b) $\frac{(1-x)(2-x)}{x-1}$ (c) $\frac{(1+x)(2-x)}{2x-1}$ (d) $\frac{(1-x)(2-x)}{2x+1}$

22. If $pq + qr + rp = 0$ then what is the value of $\frac{p^2}{p^2 - qr} + \frac{q^2}{q^2 - rp} + \frac{r^2}{r^2 - pq}$?
(a) 0 (b) 1 (c) -1 (d) 3

23. If $x + y + z = 0$ then what is the value of

$$\frac{1}{x^2 + y^2 - z^2} + \frac{1}{y^2 + z^2 - x^2} + \frac{1}{z^2 + x^2 - y^2} \text{ ?}$$

(a) $\frac{1}{x^2 + y^2 + z^2}$ (b) 1 (c) -1 (d) 0

24. $\frac{(x-y)^3 + (y-z)^3 + (z-x)^3}{4(x-y)(y-z)(z-x)}$ equals
(a) $-\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) 0

25. If $a + b + c = 6$, $a^2 + b^2 + c^2 = 14$ and $a^3 + b^3 + c^3 = 36$ then value of abc is equal to

(a) 0 (b) 4 (c) 1 (d) 6

26. If $x(x + y + z) = 9$, $y(x + y + z) = 16$ and $z(x + y + z) = 144$ then what is the value of x ?

(a) $\frac{9}{5}$ (b) $\frac{9}{7}$ (c) $\frac{9}{13}$ (d) $\frac{16}{3}$

27. If u, v, w are real numbers such that $u^3 - 8v^3 - 27w^3 = 18uvw$, which one of the following is true ?

(a) $u - v + w = 0$ (b) $u = -v = -w$ (c) $u - 2v = 3w$ (d) $u + 2v = -3w$

28. If $a + b + c = 0$ then what is the value of $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$?

(a) -3 (b) 0 (c) 1 (d) 3

29. If $(x^4 + x^{-4}) = 322$ then which one of the following is the value of $x - x^{-1}$?

(a) 18 (b) 16 (c) 8 (d) 4

30. If x varies as m^{th} power of y, y varies as n^{th} power of z and x varies as p^{th} power of z then which one of the following is true ?

(a) $p = m + n$ (b) $p = m - n$

(c) $p = mn$ (d) None of these

31. If $x = (b - c)(a - d)$, $y = (c - a)(b - d)$, $z = (a - b)(c - d)$ then which one is equal to $x^3 + y^3 + z^3$?

(a) xyz (b) $2xyz$ (c) $3xyz$ (d) $-3xyz$

32. If $a + b + c = 6$, $a^2 + b^2 + c^2 = 26$, then $ab + bc + ca$ is equal to

(a) 0 (b) 2 (c) 4 (d) 5

33. If $3x^3 - 2x^2y - 13xy^2 + 10y^3$ is divided by $x - 2y$, then what will be the remainder ?

(a) 0 (b) x (c) $y + 5$ (d) $x - 3$

34. If $\left(a + \frac{1}{a}\right)^2 = 3$ then what is the value of

$$1 + a^6 + a^{12} + a^{18} + a^{84} + a^{90} + a^{200} + a^{206} ?$$

(a) 1 (b) 0 (c) 8 (d) $2a^2$

35. If $x + y + z = 0$, then $\frac{xyz}{(x+y)(y+z)(z+x)}$ equals $[x \neq -y, y \neq -z, z \neq -x]$

(a) -1 (b) 1

(c) $xy + yz + zx$ (d) None of these

36. If $\frac{5x - 7y + 10}{1} = \frac{3x + 2y + 1}{8} = \frac{11x + 4y - 10}{9}$, then $x + y$ equals

(a) 1 (b) 2 (c) 3 (d) -3

37. If $\frac{2x-3y+1}{2}=\frac{x+4y+8}{3}=\frac{4x-7y+2}{5}$, then $x+y$ is equal to which of the following ?
(a) 3 (b) 2 (c) 0 (d) -2

38. If $x=a(b-c)$, $y=b(c-a)$ and $z=c(a-b)$ then $\left(\frac{x}{a}\right)^3+\left(\frac{y}{b}\right)^3+\left(\frac{z}{c}\right)^3=?$
(a) $\frac{xyz}{3abc}$ (b) $3xyzabc$ (c) $\frac{3xyz}{abc}$ (d) $\frac{xyz}{abc}$

39. If $x^2+2=2x$, then value of $x^4-x^3+x^2+2$ is
(a) 0 (b) 1 (c) -1 (d) $\sqrt{2}$

40. If $2^x=3^y=6^{-z}$, then $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ is equal to
(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

41. If $\frac{1}{x+y}=\frac{1}{x}+\frac{1}{y}$ $(x\neq 0, y\neq 0, x\neq y)$ then value of x^3-y^3 is
(a) 0 (b) 1 (c) -1 (d) 2

42. For real a, b, c if $a^2+b^2+c^2=ab+bc+ca$ then value of $\frac{a+c}{b}$ is
(a) 1 (b) 2 (c) 3 (d) 0

43. If $x+\frac{1}{x}=5$ then $\frac{2x}{3x^2-5x+3}$ is
(a) 5 (b) $\frac{1}{5}$ (c) 3 (d) $\frac{1}{3}$

44. If $x^4+\frac{1}{x^4}=119$ and $x>1$ then what is the value of $x^3-\frac{1}{x^3}$?
(a) 54 (b) 18 (c) 72 (d) 36

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45. If $(x+y-z)^2+(y+z-x)^2+(z+x-y)^2=0$ then what is the value of $x+y+z$?
(a) $\sqrt{3}$ (b) $3\sqrt{3}$ (c) 3 (d) 0

46. If $x-y=\frac{x+y}{7}=\frac{xy}{4}$ then what is the value of xy ?
(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

47. If $x+y+z=0$ then what is the value of $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}$?
(a) $(xyz)^2$ (b) $x^2+y^2+z^2$ (c) 9 (d) 3

48. If $x+y=a$ and $xy=b^2$, then the value of $x^3-x^2y-y^3$ in terms of a, b is
(a) $(a^2+4b^2)a$ (b) a^3-3b^2
(c) a^3-4b^2a (d) a^3+3b^2

49. If $(3a+1)^2+(b-1)^2+(2c-3)^2=0$, then value of $(3a+b+2c)$ is
(a) 3 (b) -1 (c) 2 (d) 5

50. If $xy(x+y)=1$, then value of $\frac{1}{x^3y^3}-x^3-y^3$ is
(a) 0 (b) 1 (c) 3 (d) -2

51. If $\frac{x}{2x^2+5x+2}=\frac{1}{6}$ then value of $\left(x+\frac{1}{x}\right)$ is
(a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) -2

52. Value of the expression $\frac{(a-b)^2}{(b-c)(c-a)}+\frac{(b-c)^2}{(a-b)(c-a)}+\frac{(c-a)^2}{(a-b)(b-c)}$ is
(a) 0 (b) 3 (c) $\frac{1}{3}$ (d) 2

53. If $3x+\frac{1}{2x}=5$ then what is the value of $8x^3+\frac{1}{27x^3}$?
(a) $118\frac{1}{2}$ (b) $30\frac{10}{27}$ (c) 0 (d) 1

54. If $(a-3)^2+(b-4)^2+(c-9)^2=0$ then what is the value of $\sqrt{a+b+c}$?
(a) -4 (b) 4 (c) ± 4 (d) ± 2

55. If average of x and $\frac{1}{x}$ $(x \neq 0)$ is M then what is the average of x^2 and $\frac{1}{x^2}$?
(a) $1-M^2$ (b) $1-2M$ (c) $2M^2-1$ (d) $2M^2+1$

56. If a, b, c are real and $a^2+b^2+c^2=2(a-b-c)-3$, then what is the value of $2a-3b+4c$?
(a) -1 (b) 0 (c) 1 (d) 2

57. If $a+b+c=0$ then what is the value of $\frac{1}{(a+b)(b+c)}+\frac{1}{(a+c)(b+a)}+\frac{1}{(c+a)(c+b)}$?
(a) 1 (b) 0 (c) -1 (d) -2

58. If $5x^2-4xy+y^2-2x+1=0$ then value of $x+y$ is
(a) 1 (b) 0 (c) -3 (d) 3

59. If $x=b+c-2a, y=c+a-2b, z=a+b-2c$ then the value of $x^2+y^2-z^2+2xy$ is
(a) 0 (b) $a+b+c$ (c) $a-b+c$ (d) $a+b-c$

60. $(y-z)^3+(z-x)^3+(x-y)^3$ is equal to ?
(a) $3(y-z)(z+x)(y-x)$
(b) $(x-y)(y+z)(x-z)$
(c) $3(y-z)(z-x)(x-y)$
(d) $(y-z)(z-x)(x-y)$

61. If $x=2-2^{1/3}+2^{2/3}$, then value of $x^3-6x^2+18x+18$ is
(a) 22 (b) 33 (c) 40 (d) 45

62. If $a^3 - b^3 - c^3 - 3abc = 0$, then
(a) $a = b = c$ (b) $a + b + c = 0$ (c) $a + c = b$ (d) $a = b + c$

63. If $x^2 - 3x + 1 = 0$ then what is the value of $x^3 + \frac{1}{x^3}$?
(a) 9 (b) 18 (c) 27 (d) 1

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64. If $2x + \frac{1}{3x} = 5$ then find the value of $\frac{5x}{6x^2+20x+1}$
(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{1}{7}$

65. If $m + \frac{1}{m-2} = 4$ then find the value of $(m-2)^2 + \frac{1}{(m-2)^2}$ is
(a) -2 (b) 0 (c) 2 (d) 4

66. If $a^2 = b + c$, $b^2 = c + a$ and $c^2 = a + b$ then what is the value of $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c}$?
(a) abc (b) $a^2b^2c^2$ (c) 1 (d) 0

67. If $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$ then what is the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$?
(a) 1 (b) 2 (c) 3 (d) 4

68. If $a^2 + b^2 + 2b + 4a + 5 = 0$ then what is the value of $\frac{a-b}{a+c}$?
(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

69. If $\frac{x}{(b-c)(b+c-2a)} = \frac{y}{(c-a)(c+a-2b)} = \frac{z}{(a-b)(a+b-2c)}$ then the value of $x + y + z$ is
(a) $a + b + c$ (b) $a^2 + b^2 + c^2$
(c) 0 (d) $(a + b + c)^2$

70. If $\frac{x^3+3x}{3x^2+1} = \frac{189}{61}$ then value of x is
(a) 9 (b) 11 (c) 7 (d) 13

71. If $(x^2 + y^2)(p^2 + q^2) = (xp + yq)^2$ then
(a) $xy = pq$ (b) $px = yq$ (c) $xq = yp$ (d) None of these

72. If $\frac{x}{2x+y+z} = \frac{y}{x+2y+z} = \frac{z}{x+y+2z}$ then each terms is equal to
(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) None of these

$a^2 + b^2 + c^2$ is

74. If $x^2 + 8y^2 + 9z^2 - 4xy$ ...

(a) $x = y = z$

(b) $3x = 2y = z$

(c) $x = 2y = 3z$

(d) $x + 2\sqrt{2}\,y + 3z = 0$

75. If $p^2(a^2 + b^2 + c^2) - 2p(ab + bc + cd) + (b^2 + c^2 + d^2) \leq 0$ then which of the following statement is true ?

(a) $p = \frac{b}{a} + \frac{c}{a} + \frac{d}{c}$

(b) $p(a + b + c) = 0$

(c) $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

(d) $ab = bc = cd$

76. If $b + c = 10$, $c + a = 20$, $a + b = 30$ then value of $a^3 + b^3 + c^3$ is

(a) 9000 (b) 12000 (c) 18000 (d) 27000

77. If $b + c = 2x$, $c + a = 2y$ and $a + b = 2z$ then value of $a^3 + b^3 + c^3$ is

(a) $(x + y + z)^3$

(b) $(x + y + z)^3 + 24xyz$

(c) $(x + y + z)^3 - 24xyz$

(d) $24xyz$

78. If $a + b + c = 0$ then value of $a^3 + b^3 + c^3$ is

(a) $3a(a + b)(b + c)$

(b) $3a(a + b)(c + a)$

(c) $3a(b + c)(c + a)$

(d) $3(a + b)(b + c)(c + a)$

79. Value of $(x - y)^3 + (y - z)^3 + (z - x)^3$ is

(a) $3xyz$

(b) $3(x + y)(y + z)(z + x)$

(c) $3(x - y)(y - z)(z - x)$

(d) 0

80. If $a + b + c = p$, $abc = q$ and $ab + bc + ca = 0$ then what is the value of $a^2b^2 + b^2c^2 + c^2a^2$?

(a) $2pq$ (b) $-2pq$ (c) $3pq$ (d) $-3pq$

81. If $a = 89$, $b = -69$, $c = 8$ then the value of $9(a + b)^2 + 49c^2 - 42(a + b)c$ is

(a) 2 (b) 4 (c) 16 (d) 0

82. If $x = q + r + s$, $y = r + s - p$ and $z = p + q + r$ then the value of $x^2 + y^2 + z^2 - 2xy - 2xz + 2yz$ is

(a) p^2 (b) q^2 (c) r^2 (d) s^2

83. If $x + y + z = 10$, $x^2 + y^2 + z^2 = 60$ then value of $xy + yz + zx$ is

(a) 40 (b) 80 (c) 160 (d) 20

84. If $a^2 + b^2 = 1$, $c^2 + d^2 = 2$ then $(ac - bd)^2 + (ad + bc)^2$ is

(a) 2 (b) 0 (c) 1 (d) 4

85. $(x - a)(x - b)(a - b) + (x - b)(x - c)(b - c) + (x - c)(x - a)(c - a)$ is equal to which of the following ?

(a) $(a - b)(b - c)(c - a)$

(b) $(x - a)(x - b)(x - c)$

(c) $-(a - b)(b - c)(c - a)$

(d) $-(x - a)(x - b)(x - c)$

86. If $x + \frac{1}{x}$ … $x = 2b$ and $y - \frac{1}{y} = 2d$ then the value of $xy + \frac{1}{xy}$ is

(a) $ac + bd$ (b) $ac - bd$ (c) $2(ac - bd)$ (d) $2(ac + bd)$

87. Which one of the following is not a factor of $x^8 + x^4 + 1$?

(a) $x^2 - \sqrt{x} + 1$ (b) $x^2 - x + 1$ (c) $x^4 - x^2 + 1$ (d) $x^2 - 2x + 1$

88. A factor of $a^4 - 11a^2b^2 + b^4$ is

(a) $(a^2 - b^2 - 3ab)$ (b) $a^2 + b^2 - 3ab$

(c) $(a^2 + b^2 + 3ab)$ (d) $(a^2 - b^2 + 4ab)$

89. If $a^2 + b^2 = x$, $ab = y$ then the value of $\frac{a^4 + b^4}{a^2 - ab\sqrt{2} + b^2}$ is

(a) $x + 2y$ (b) $x + \sqrt{2}y$ (c) $\sqrt{2}x + y$ (d) $2x + y$

90. If $(a + b)x = a$ and $(a + b)y = b$ then the value of $\frac{x^2 + y^2}{x^2 - y^2}$ is

(a) $\frac{a^2 - b^2}{a^2 + b^2}$ (b) $\frac{a^2}{a^2 + b^2}$ (c) $\frac{b^2}{a^2 + b^2}$ (d) $\frac{a^2 + b^2}{a^2 - b^2}$

91. If $x = \frac{p+q}{p-q}$ and $y = \frac{p-q}{p+q}$ then the value of $\frac{x-y}{x+y}$ is

(a) $\frac{p^2 + q^2}{2pq}$ (b) $\frac{2pq}{p^2 + q^2}$ (c) $\frac{2pq}{p^2 - q^2}$ (d) $\frac{2(p^2 - q^2)}{pq}$

92. If $x + \frac{a}{x} = 1$ then the value of $\frac{x^3 - x^2}{x^2 + x + a}$ in terms of a is

(a) $\frac{a}{2}$ (b) $\frac{-a}{2}$ (c) $2a$ (d) a

93. If $a = \frac{xy}{x+y}$, $b = \frac{xz}{x+z}$ and $c = \frac{yz}{y+z}$ where a, b, c are non zero then x is

(a) $\frac{2abc}{ac + bc - ab}$ (b) $\frac{2abc}{ab - ac + bc}$ (c) $\frac{2abc}{ab + bc + ac}$ (d) $\frac{2abc}{ab + ac - bc}$

94. HCF and LCM of two algebraic expressions are respectively $(a + 1)$ and $(a^3 + a^2 - a - 1)$. If one of the expression is $a^2 - 1$, then what is the second expression ?

(a) $(a + 1)$ (b) $(a - 1)^2$ (c) $(a + 1)^2$ (d) $(a + 1)(a - 1)$

95. If $\left(x^2 + \frac{1}{x^2}\right) = p$, then what is the value of $\left(x^3 + \frac{1}{x^3}\right)$?

(a) $p^{3/2}$ (b) $(p + 1)\sqrt{p+2}$ (c) $(p - 1)\sqrt{p+2}$ (d) $(p + 1)\sqrt{p-2}$

96. If $a + b + c = 0$, then what is the value of $\frac{a^2 + b^2 + c^2}{(a-b)^2 + (b-c)^2 + (c-a)^2}$?

(a) 1 (b) 3 (c) $\frac{1}{3}$ (d) 0

97. If $y = \left(x + \frac{1}{x}\right)$, then the expression $x^4 + x^3 - 4x^2 + x + 1 = 0$ can be simplified in terms of y as
(a) $y^2 + y - 2 = 0$ (b) $y^2 + y - 4 = 0$ (c) $y^2 + y - 6 = 0$ (d) $y^2 + y + 6 = 0$

98. Value of $(2+1)(2^2+1)(2^4+1)(2^8+1)(2^{16}+1)(2^{32}+1)(2^{64}+1)$ is
(a) $2^{256} - 1$ (b) $2^{256} + 1$ (c) $2^{128} - 1$ (d) $2^{128} + 1$

99. HCF of polynomials $x^3 + 3x^2y + 2xy^2$ and $x^4 + 6x^3y + 8x^2y^2$ is
(a) $x(x + 2y)$
(b) $x(x + 3y)$
(c) $x + 2y$
(d) None of these

100. If $pqr = 1$, then what is value of the expression $\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$?
(a) 1 (b) -1 (c) 0 (d) $\frac{1}{3}$

101. If $x + y + z = 2s$, then $(s-x)^3 + (s-y)^3 + 3(s-x)(s-y)z$ equals
(a) z^3 (b) $-z^3$ (c) x^3 (d) y^3

102. If $x^2 = y + z, y^2 = z + x, z^2 = x + y$, then what is the value of $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$?
(a) 1 (b) 0 (c) -1 (d) 2

103. Suppose p, q, r are such that $p + q = r$ and $pqr = 30$, then what is the value of $p^3 + q^3 - r^3$?
(a) 0 (b) 90 (c) -90
(d) cannot be determined with given data

104. What is the square root of $\left(\frac{x^5 - 1}{x - 1}\right) + (x^3 + 2x^2 + x)$?
(a) $x^2 + x + 1$ (b) $x^2 - x + 1$ (c) $x^2 - x - 1$ (d) $x^2 + x - 1$

105. $\frac{x^8 + 4}{x^4 + 2x^2 + 2}$ on simplification, equals
(a) $x^4 + 2x^2 - 2$
(b) $x^4 - 2x^2 + 2$
(c) $x^4 - 2x^2 - 2$
(d) cannot be simplified

106. If $x + \left(\frac{1}{x}\right) = p$, then $x^6 + \left(\frac{1}{x^6}\right)$ equals
(a) $p^6 + 6p$
(b) $p^6 - 6p$
(c) $p^6 + 6p^4 + 9p^2 + 2$
(d) $p^6 - 6p^4 + 9p^2 - 2$ [SSC Tier-I 2014]

107. If $x + y + z = 0$, then $[(y - z - x)/2]^3 + [(z - x - y)/2]^3 + [(x - y - z)/2]^3$ equals
(a) $24\,xyz$ (b) $-24\,xyz$ (c) $3\,xyz$ (d) xyz

Answer-1A

1. (a)	2. (a)	3. (b)	4. (d)	5. (a)	6. (c)	7. (b)	8. (a)
9. (c)	10. (a)	11. (d)	12. (c)	13. (a)	14. (b)	15. (a)	16. (b)
17. (a)	18. (b)	19. (c)	20. (c)	21. (c)	22. (b)	23. (d)	24. (c)
25. (d)	26. (c)	27. (c)	28. (d)	29. (d)	30. (c)	31. (c)	32. (d)
33. (a)	34. (b)	35. (a)	36. (c)	37. (d)	38. (c)	39. (a)	40. (a)
41. (a)	42. (b)	43. (b)	44. (d)	45. (d)	46. (a)	47. (d)	48. (c)
49. (a)	50. (c)	51. (b)	52. (b)	53. (b)	54. (b)	55. (c)	56. (c)
57. (b)	58. (d)	59. (a)	60. (c)	61. (c)	62. (d)	63. (b)	64. (d)
65. (c)	66. (c)	67. (d)	68. (c)	69. (c)	70. (a)	71. (c)	72. (a)
73. (d)	74. (c)	75. (c)	76. (a)	77. (c)	78. (b)	79. (c)	80. (b)
81. (c)	82. (c)	83. (d)	84. (a)	85. (c)	86. (d)	87. (d)	88. (a)
89. (b)	90. (d)	91. (b)	92. (a)	93. (a)	94. (c)	95. (c)	96. (c)
97. (c)	98. (c)	99. (a)	100. (a)	101. (a)	102. (a)	103. (b)	104. (a)
105. (b)	106. (d)	107. (c)					

Explanation

1. (a) $\because$ $x + y + z = 0 \Rightarrow x + y = -z, y + z = -x, x + z = -y$

$\therefore$ $(x + y)(y + z)(z + x) = (-z)(-x)(-y) = -xyz$

2. (a) First polynomial $= 6x^3 + 60x^2 + 150x$

$= 6x(x^2 + 10x + 25) = 3 \times 2 \times x \times (x+5)^2$

Second polynomial $= 3x^4 + 12x^3 - 15x^2$

$= 3x^2(x^2 + 4x - 5) = 3x^2(x^2 + 5x - x - 5) = 3x^2(x+5)(x-1)$

$\therefore$ Required LCM $= 3 \times 2 \times x^2 \times (x+5)^2(x-1) = 6x^2(x+5)^2(x-1)$

3. (b) $\because x - k$ is a factor of $2x^2 - kx - 9$

$\therefore$ $2k^2 - k^2 - 9 = 0$

$\Rightarrow$ $k^2 - 9 = 0$

$\therefore$ $k = \pm 3$

But factor of $x^2 + x - 12$ are $(x+4), (x-3)$

Hence value of k is 3.

4. (d) First polynomial $= x^2 + bx - x - b = x(x+b) - 1(x+b)$

$= (x-1)(x+b)$

Second polynomial $= x^2 + xa - x - a = x(x+a) - 1(x+a)$

$= (x+a)(x-1)$

$\therefore$ Required LCM $= x - 1$

5. (a) Required remainder $= 3(2y)^3 - 2(2y)^2 y - 13(2y)y^2 + 10y^3$

(using factor theorem)

$$= 24y^3 - 8y^3 - 26y^3 + 10y^3 = 34y^3 - 34y^3 = 0$$

6. (c) $x^2 + 2)\ x^3 + 5x^2 + 10k\ (x + 5$

$$\underline{\underset{-}{x^3} \underset{-}{+} 2x}$$

$$5x^2 - 2x + 10k$$

$$\underline{\underset{-}{5x^2} \underset{-}{+} 10}$$

$$-2x + 10k - 10$$

But remainder is $-2x$,

$$\Rightarrow -2x = -2x + 10k - 10$$

$$\Rightarrow 10k - 10 = 0$$

$$\therefore \quad k = 1$$

7. (b) $\because (5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2$

$$= (5x^2 + 14x + 2 - 4x^2 + 5x - 7) \times (5x^2 + 14x + 2 + 4x^2 - 5x + 7)$$

$$= (x^2 + 19x - 5)(9x^2 + 9x + 9) = 9(x^2 + 19x - 5)(x^2 + x + 1)$$

Clearly, $(x^2 + x + 1)$is a factor of $\{(5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2\}$.

Hence remainder is zero.

8. (a) At $x = 1$

Polynomial $= (1)^{29} - (1)^{24} + (1)^{13} - 1 = 1 - 1 + 1 - 1 = 0$

$\therefore$ $(x - 1)$ is a factor of $x^{29} - x^{24} + x^{13} - 1$

at, $x = -1$

Polynomial $= (-1)^{29} - (-1)^{24} + (-1)^{13} - 1 = -1 - 1 - 1 - 1 = -4$

$\therefore$ $(x + 1)$ is not a factor of $x^{29} - x^{24} + x^{13} - 1$

Hence option (a) is correct.

9. (c) $x^2 - 4x + 1 = 0$

$\Rightarrow$ $x^2 + 1 = 4x$

Dividing by x, $x + \frac{1}{x} = 4$

$$\therefore \quad x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)\left(x \cdot \frac{1}{x}\right) = 4^3 - 3 \times 4 \times 1 = 64 - 12 = 52$$

10. (a) Given $x + y + z = 6$ and $xy + yz + zx = 11$

$$\therefore \quad x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= (x + y + z)[(x + y + z)^2 - 3(xy + yz + zx)]$$

$$= 6[6^2 - 3(11)] = 6 \times 3 = 18$$

Here factors are $\sqrt{3}, -\sqrt{3}, 3$. Hence option (d) is correct.

12. (c) Let $x - \alpha$ is a common factor.

$\therefore \quad \alpha^2 - 11\alpha + a = 0$ and $\alpha^2 - 14\alpha + 2a = 0$

Solving, $\alpha^2 - 11\alpha + a = \alpha^2 - 14\alpha + 2a$

$\Rightarrow \quad 3\alpha = a$

$\Rightarrow \quad \alpha = \dfrac{a}{3}$

Putting $\alpha = \frac{a}{3}$, in the first equation (i) $\left(\frac{a}{3}\right)^2 - 11\frac{a}{3} + a = 0$

$\Rightarrow \quad a^2 - 33a + 9a = 0$

$\quad a^2 - 24a = 0$

$\Rightarrow \quad a = 0, 24$

13. (a) Since $x + 4$ is HCF, therefore it divides both the expression and each expression becomes zero at $x = -4$.

$\therefore \quad 2(-4)^2 + k(-4) - 12 = 0$ and $(-4)^2 + (-4) - 2k - 2 = 0$

$\Rightarrow \quad 32 - 12 = 4k$ and $16 - 6 = 2k$

$\Rightarrow \quad k = 5$ and $k = 5$

14. (b) $x(y - z)(y + z) + y(z - x)(z + x) + z(x - y)(x + y)$

$= x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$

$= x(y^2 - z^2) + yz^2 - yx^2 + zx^2 - zy^2$

$= x(y - z)(y + z) + x^2(z - y) + yz(z - y)$

$= (y - z)[xy + xz - x^2 - yz] = (y - z)[y(x - z) + x(z - x)]$

$= (y - z)(x - z)(y - x) = (x - y)(x - z)(z - y)$

15. (a) Given, $\dfrac{x}{y} = \dfrac{z}{w} \Rightarrow xw = yz$

Now, $(xy + zw)^2 = x^2y^2 + z^2w^2 + 2(xyzw) = x^2y^2 + z^2w^2 + 2[yz \cdot yz]$

$= x^2y^2 + y^2z^2 + z^2w^2 + y^2z^2 = y^2(x^2 + z^2) + z^2w^2 + x^2w^2$

$= y^2(x^2 + z^2) + w^2(x^2 + z^2) = (x^2 + z^2)(y^2 + w^2)$

16. (b) Given, $\dfrac{1}{x+1} + \dfrac{2}{y+2} + \dfrac{1009}{z+1009} = 1$

$\Rightarrow \quad \dfrac{1}{x+1} - 1 + \dfrac{2}{y+2} - 1 + \dfrac{1009}{z+1009} - 1 = 1 - 3$

$\Rightarrow \quad -\dfrac{x}{x+1} - \dfrac{y}{y+2} - \dfrac{z}{z+1009} = -2$

17. (a) $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$$=(a+b+c)\frac{1}{2}\left((a-b)^2+(b-c)^2+(c-a)^2\right)$$

$$=(258+260+262)\frac{1}{2}\left\{(-2)^2+(-2)^2+4^2\right\}$$

$$=(780)\frac{1}{2}\times 24=780\times 12=9360$$

18. (b) We have, $x^3-27=(x-3)(x^2+9+3x)$

Now, $x^2+9+3x\,)\,x^3+4x^2+12x+k\,(\,x+1$

$$\begin{array}{l} x^3+3x^2+9x \\ \underline{-\quad -\quad -} \\ x^2+3x+k \\ x^2+3x+9 \\ \underline{-\quad -\quad -} \\ k-9 \end{array}$$

∴ Value of k should be 9.

19. (c) Let $f(x)=x^{40}+2$

putting $x^4=-1$

$f(x)=(-1)^{10}+2=3$

20. (c) Given, $a^x=b^y=c^z=k$

$\Rightarrow$ $a=k^{1/x}$, $b=k^{1/y}$ and $c=k^{1/z}$

∴ $abc=k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$

$\Rightarrow$ $1=k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$

$\Rightarrow$ $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

$\Rightarrow$ $xy+yz+zx=0$

21. (c) $a=\frac{1+x}{2-x}$

∴ $\frac{1}{a+1}+\frac{2a+1}{a^2-1}=\frac{3a}{a^2-1}=\frac{3\left(\frac{1+x}{2-x}\right)}{\left(\frac{1+x}{2-x}\right)^2-1}$

$$=\frac{3(1+x)(2-x)}{1+x^2+2x-(4+x^2-4x)}$$

$$=\frac{3(1+x)(2-x)}{6x-3}$$

$$=\frac{(1+x)(2-x)}{2x-1}$$

22. (b) Given, $pq + qr + rp = 0$

$\Rightarrow -qr = pq + rp$ etc.

$$\therefore \frac{p^2}{p^2-qr}+\frac{q^2}{q^2-rp}+\frac{r^2}{r^2-pq}$$

$$=\frac{p^2}{p^2+rp+pq}+\frac{q^2}{q^2+pq+qr}+\frac{r^2}{r^2+qr+rp}$$

$$=\frac{p}{p+q+r}+\frac{q}{p+q+r}+\frac{r}{p+q+r}=\frac{p+q+r}{p+q+r}=1$$

23. (d) $x + y + z = 0$

$\Rightarrow (x+y)^2 = (-z)^2$

$\Rightarrow x^2 + y^2 - z^2 = -2xy$ etc.

$$\therefore \frac{1}{x^2+y^2-z^2}+\frac{1}{y^2+z^2-x^2}+\left(\frac{1}{z^2+x^2-y^2}\right)$$

$$=\frac{1}{-2xy}+\frac{1}{-2yz}+\frac{1}{-2zx}=-\frac{1}{2}\left\{\frac{z+x+y}{xyz}\right\}=0 \qquad (\because x+y+z=0)$$

24. (c) We know that, $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Here, $x - y + y - z + z - x = 0$

$$\therefore \frac{(x-y)^3+(y-z)^3+(z-x)^3}{4(x-y)(y-z)(z-x)}=\frac{3(x-y)(y-z)(z-x)}{4(x-y)(y-z)(z-x)}=\frac{3}{4}$$

25. (d) $\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\therefore 6^2 = 14 + 2(ab + bc + ca)$

$\Rightarrow ab + bc + ca = \frac{36-14}{2} = 11$

Now, $a^3 + b^3 + c^3 - 3abc = (a + b + c)$ from, $(a^2 + b^2 + c^2 - ab - bc - ca)$

$36 - 3abc = 6(14 - 11) = 18$

or, $3abc = 36 - 18 = 18$

$\Rightarrow abc = 6$

Second method :

By inspection (trial) $a = 1, b = 2, c = 3$

26. (c) Given, $x(x + y + z) = 9$, $y(x + y + z) = 16$ and $z(x + y + z) = 144$

Adding all the equations, $(x + y + z)(x + y + z) = 9 + 16 + 144$

$\Rightarrow (x + y + z)^2 = 169$

$\Rightarrow x + y + z = 13$

$\because x(x + y + z) = 9$

$\Rightarrow x(13) = 9 \qquad \therefore x = \frac{9}{13}$

27. (c) Given, $(u)^3 + (-2v)^3 + (-3w)^3 = 3 \times (-2)(-3)\,uvw$

$\therefore \quad u + (-2v) + (-3w) = 0$

$\Rightarrow \quad u - 2v - 3w = 0$

$\Rightarrow \quad u - 2v = 3w$

28. (d) $\because \quad a + b + c = 0$

$\Rightarrow \quad a^3 + b^3 + c^3 = 3abc$

$\Rightarrow \quad \frac{a^3}{abc} + \frac{b^3}{abc} + \frac{c^3}{abc} = 3$

$\Rightarrow \quad \frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} = 3$

29. (d) $\because \quad x^4 + \frac{1}{x^4} = 322$

$\Rightarrow \quad \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 322$

$\Rightarrow \quad \left(x^2 + \frac{1}{x^2}\right)^2 = 324 = 18^2$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 18$

$\Rightarrow \quad \left(x - \frac{1}{x}\right)^2 + 2 = 18$

$\therefore \quad x - \frac{1}{x} = 4$

30. (c) $\because \quad x \propto y^m, \qquad y \propto z^n, \qquad x \propto z^p$

$\Rightarrow \quad z \propto x^{\frac{1}{p}}$

$\Rightarrow \quad y \propto y^{\frac{n}{p}}$

$\Rightarrow \quad x \propto x^{\frac{mn}{p}}$

$\Rightarrow \quad 1 = \frac{mn}{p}$

$\Rightarrow \quad p = mn$

31. (c) $\because \quad x = (b-c)(a-d), y = (c-a)(b-d)$ and $z = (a-b)(c-d)$

$\therefore \quad x + y + z = (b-c)(a-d) + (c-a)(b-d) + (a-b)(c-d) = 0$

Hence, $x^3 + y^3 + z^3 = 3xyz$

32. (d) Given, $a + b + c = 6$

and $\quad a^2 + b^2 + c^2 = 26$

We know that, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\Rightarrow (a+b+c)^2 = (a^2+b^2+c^2)+2(ab+bc+ca)$

$\Rightarrow (ab+bc+ca) = \frac{1}{2}\{(a+b+c)^2-(a^2+b^2+c^2)\}$

$= \frac{1}{2}\{(6)^2-26\} = \frac{1}{2}\{36-26\} = \frac{1}{2}\times 10 = 5$

33. (a) Given expression $= 3x^3-2x^2y-13xy^2+10y^3$

$\because$ $x-2y$ is factor of given expression, the expression will be zero when $x-2y=0$

Putting, $x = 2y$ in the given expression

$= 3(2y)^3-2(2y)^2y-13(2y)y^2+10y^3$

$= 3(8y^3)-2(4y^2)y-13(2y^3)+10y^3$

$= 24y^3-8y^3-26y^3+10y^3 = 34y^3-34y^3$

$= 0$, which is remainder.

34. (b) $a^2+\frac{1}{a^2}+2=3 \Rightarrow a^2+\frac{1}{a^2}=1$

$\Rightarrow \frac{a^4+1}{a^2}=1$

$\Rightarrow a^4-a^2+1=0$

Multiplying both sides by a^2+1, $(a^2+1)(a^4-a^2+1) = 0\times(a^2+1)$

$\Rightarrow (a^2)^3+1^3=0$ $\quad (\because (x+y)(x^2-xy+y^2)=x^3+y^3)$

$\Rightarrow a^6=-1$

$\therefore$ $a^{12}=1, a^{18}=-1, a^{84}=1, a^{90}=-1$

$a^{200}=a^{198}\cdot a^2=(a^6)^{33}a^2=-a^2$ and $a^{206}=a^{204}\cdot a^2=(a^6)^{34}a^2=+a^2$

Required sum $= 1-1+1-1+1-1-a^2+a^2=0$

35. (a) Given, $x+y+z=0$ ... (i)

Expression $= \frac{xyz}{(x+y)(y+z)(z+x)}$

$= \frac{xyz}{\{(x+y+z)-z\}\{(x+y+z)-x\}\{(x+y+z)-y\}}$

$= \frac{xyz}{(0-z)(0-x)(0-y)}$ [from (i)]

$= \frac{xyz}{-xyz} = -1$

... (i)

(ii)

36. (c) From first two ratio,

$8(5x-7y+10) = 1\,(3x+2y+1)$

or, $37x - 58y = -79$ ... (i)

From first and third ratio, $9(5x-7y+10) = 1\,(11x+4y-10)$

or, $45x - 63y + 90 = 11x + 4y - 10$

or, $34x - 67y = -100$ ... (ii)

from, (i) – (ii), $3x + 9y = 21$

or, $x + 3y = 7$ ... (iii)

putting, $x = 7 - 3y$ in (i),

$37(7-3y) - 58y = -79$

or, $259 - 111y - 58y = -79$

or, $338 = 169y$

or, $y = 2$

Again from (iii), $x + 6 = 7$

$\Rightarrow\ x = 1$

$\therefore\ x + y = 1 + 2 = 3$

37. (d) **From I and II term,**

$$\frac{2x-3y+1}{2} = \frac{x+4y+8}{3}$$

$\Rightarrow\ 6x - 9y + 3 = 2x + 8y + 16$

$\Rightarrow\ 4x - 17y = 13$ ... (i)

From I and III term,

$$\Rightarrow\ \frac{x+4y+8}{3} = \frac{4x-7y+2}{5}$$

$\Rightarrow\ 5x + 20y + 40 = 12x - 21y + 6$

$7x - 41y = 34$ ... (ii)

Solving equation, (i) & (ii), $x = -1,\ y = -1$

$\therefore\ x + y = -1 - 1 = -2$

38. (c) $\frac{x}{a} = b - c,\ \frac{y}{b} = c - a,\ \frac{z}{c} = a - b$

Again, $b - c + c - a + a - b = 0$

$$\therefore\ \left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$$

$= (b-c)^3 + (c-a)^3 + (a-b)^3$

$= 3(b-c)(c-a)(a-b) = \frac{3xyz}{abc}$

39. (a) $x^4 - x^3 + x^2 + 2 = x^2 \cdot x^2 - x \cdot x^2 + (x^2 + 2)$

$= (2x - 2)^2 - x(2x - 2) + 2x$ $\quad (\because x^2 + 2 = 2x)$

... (i) $= 4x^2 - 8x + 4 - 2x^2 + 2x + 2x$

$= 2x^2 - 4x + 4 = 2(x^2 + 2) - 4x$

$= 2(2x) - 4x = 0$

... (ii)

40. (a) $2^x = 3^y = 6^{-z} = k$

... (iii) $\Rightarrow 2 = k^{\frac{1}{x}}; \quad 3 = k^{\frac{1}{y}}; \quad 6 = k^{\frac{-1}{z}} \quad \because 2 \times 3 = 6$

$\Rightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{y}}$

$\Rightarrow k^{\frac{1}{x}+\frac{1}{y}} = k^{\frac{1}{z}}$

$\Rightarrow \frac{1}{x} + \frac{1}{y} = -\frac{1}{z}$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

41. (a) $\because \quad \frac{1}{x+y} = \frac{1}{x} + \frac{1}{y} = \frac{y+x}{xy}$

$\Rightarrow (x + y)^2 = xy$

$\Rightarrow x^2 + 2xy + y^2 = xy$

$\Rightarrow x^2 + xy + y^2 = 0$

$\therefore x^3 - y^3 = (x - y)(x^2 + xy + y^2) = 0$

... (i) 42. (b) $\because \quad a^2 + b^2 + c^2 = ab + bc + ca$

$\Rightarrow \frac{1}{2}\left\{(a-b)^2 + (b-c)^2 + (c-a)^2\right\} = 0$

$\Rightarrow a = b = c$

... (ii) $\therefore \quad \frac{a+c}{b} = \frac{a+a}{a} = 2$

43. (b) $\frac{2x}{3x^2 - 5x + 3} = \frac{2x}{x\left(3x - 5 + \frac{3}{x}\right)} = \frac{2}{3\left(x + \frac{1}{x}\right) - 5} = \frac{2}{3 \cdot 5 - 5} = \frac{2}{10} = \frac{1}{5}$

44. (d) $\because \quad x^4 + \frac{1}{x^4} = 119$

$x^2 + \frac{1}{x^2} = \sqrt{119 + 2} = 11$

$\therefore \quad x - \frac{1}{x} = \sqrt{11 - 2} = 3$

Hence, $x^3 - \frac{1}{x^3} = (3)^3 + 3 \times 3 = 27 + 9 = 36$

45. (d) $\because$ $(x+y-z)^2+(y+z-x)^2+(z+x-y)^2=0$, It is possible only

When, $x+y-z=0$, $y+z-x=0$ and $z+x-y=0$

Adding, $(x+y+z)=0$

46. (a) From ratio proportion, $\frac{x-y}{1}=\frac{x+y}{7}=\frac{xy}{4}$

$$=\frac{(x-y)+(x+y)}{1+7}$$

$$=\frac{(x-y)-(x+y)}{1-7}$$

$\Rightarrow$ $\frac{x-y}{1}=\frac{x+y}{7}=\frac{xy}{4}=\frac{x}{4}=\frac{y}{3}$ (Multiplication rule of ratio)

Again, $\frac{xy}{4}=\frac{x}{4}=\frac{y}{3}=\frac{\sqrt{xy}}{\sqrt{4\times 3}}$

or, $\frac{xy}{4}=\frac{\sqrt{xy}}{\sqrt{12}}$

or, $\frac{(xy)^2}{16}=\frac{xy}{12}$

$\Rightarrow$ $xy=\frac{16}{12}=\frac{4}{3}$

47. (d) We know that when $x+y+z=0$, then, $x^3+y^3+z^3=3xyz$,

$\therefore$ $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}=\frac{x^3+y^3+z^3}{xyz}$

$\Rightarrow$ $\frac{3xyz}{xyz}=3$

48. (c) $x^3-x^2y-xy^2+y^3$

$=x^3+y^3-x^2y-xy^2=(x+y)^3-3xy(x+y)-xy(x+y)$

$=(x+y)^3-4xy(x+y)=a^3-4b^2a$

49. (a) $\because$ $(3a+1)^2+(b-1)^2+(2c-3)^2=0$

$\Rightarrow$ $3a+1=0$

$\Rightarrow$ $3a=-1$

$b-1=0$

$\Rightarrow$ $b=1$

$2c-3=0$

$\Rightarrow$ $2c=3$

$\therefore$ $3a+b+2c=-1+1+3=3$

50. (c) ...

$$\Rightarrow \quad x + y = \frac{1}{xy}$$

On cubing both sides, $x^3 + y^3 + 3xy(x + y) = \frac{1}{x^3y^3}$

$$\Rightarrow \quad \frac{1}{x^3y^3} - x^3 - y^3 = 3xy(x + y) = 3 \times 1 = 3$$

51. **(b)** $\frac{x}{2x^2+5x+2} = \frac{x}{x\left(2x+5+\frac{2}{x}\right)} = \frac{1}{2\left(x+\frac{1}{x}\right)+5}$

$$= \frac{1}{2t+5} \qquad \left(\text{Let } x + \frac{1}{x} = t\right)$$

According to question, $\frac{1}{2t+5} = \frac{1}{6}$

$$\Rightarrow \quad 6 = 2t + 5$$

$$\Rightarrow \quad t = \frac{1}{2}$$

$$\therefore \quad x + \frac{1}{x} = \frac{1}{2}$$

52. **(b)** $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)}$

$$= \frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)} = \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

[Since $(a - b) + (b - c) + (c - a) = a - b + b - c + c - a = 0$

$\therefore \quad (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$]

53. **(b)** $3x + \frac{1}{2x} = 5$

or, $\frac{3}{2}\left(2x+\frac{1}{3x}\right) = 5$

or, $2x + \frac{1}{3x} = \frac{10}{3}$

Cubing both sides, $8x^3 + \frac{1}{27x^3} + 3\left(2x+\frac{1}{3x}\right)\left(2x\cdot\frac{1}{3x}\right) = \frac{1000}{27}$

or, $8x^3 + \frac{1}{27x^3} + 3\cdot\frac{10}{3}\cdot\frac{2}{3} = \frac{1000}{27}$

or, $8x^3 + \frac{1}{27x^3} = \frac{1000}{27} - \frac{20}{3}$

$$= \frac{1000-180}{27} = \frac{820}{27} = 30\frac{10}{27}$$

54. (b) $(a-3)^2 + (b-4)^2 + (c-9)^2 = 0$

is possible only when each of $(a-3)^2$, $(b-4)^2$ and $(c-9)^2$ is zero.

$\therefore \quad (a-3)^2 = 0, (b-4)^2 = 0, (c-9)^2 = 0$

$\Rightarrow \quad a-3 = 0, b-4 = 0, c-9 = 0$

$\therefore \quad a = 3, b = 4$ and $c = 9$

Hence, $\sqrt{a+b+c} = \sqrt{3+4+9} = \sqrt{16} = 4$

55. (c) Average of x and $\frac{1}{x} = M$

$\therefore \quad \frac{x+\frac{1}{x}}{2} = M$

$\Rightarrow \quad x+\frac{1}{x} = 2M$

Squaring both sides, $\left(x+\frac{1}{x}\right)^2 = (2M)^2$

$\Rightarrow \quad x^2+\frac{1}{x^2}+2\cdot x\cdot\frac{1}{x} = 4M^2$

$\Rightarrow \quad x^2+\frac{1}{x^2}+2 = 4M^2$

$\Rightarrow \quad x^2+\frac{1}{x^2} = 4M^2-2 \quad$ ...(i)

Average of x^2 and $\frac{1}{x^2} = \frac{x^2+\frac{1}{x^2}}{2} = \frac{4M^2-2}{2}$ [from (i)]

$= \frac{2(2M^2-1)}{2} = 2M^2-1$

56. (c) $a^2+b^2+c^2 = 2(a-b-c)-3$

or, $a^2-2a+1+b^2+2b+1+c^2-2c+1 = 0$

or, $(a-1)^2+(b+1)^2+(c+1)^2 = 0$

It is possible only when, $a = 1, b = -1, c = -1$

Hence, $2a-3b+4c = 2+3-4 = 1$

57. (b) $\because \quad a+b+c = 0$

then, $\frac{1}{(a+b)(b+c)}+\frac{1}{(a+c)(b+a)}+\frac{1}{(c+a)(c+b)}$

$= \frac{(c+a)+(b+c)+(a+b)}{(a+b)(b+c)(c+a)}$

$= \frac{c+a+b+c+a+b}{(a+b)(b+c)(c+a)}$

$$= \frac{2a+2b+2c}{(a+b)(b+c)(c+a)}$$

$$= \frac{2(a+b+c)}{(a+b)(b+c)(c+a)}$$

$$= \frac{2\times 0}{(a+b)(b+c)(c+a)} = 0$$

58. **(d)** Given expression $= 5x^2 - 4xy + y^2 - 2x + 1 = 0$

$\Rightarrow \quad 4x^2 - 4xy + y^2 + x^2 - 2x + 1$

$\Rightarrow \quad (2x - y)^2 + (x - 1)^2 = 0$

It is possible only when $2x - y = 0$ and $x - 1 = 0$

$\therefore \quad x = 1$ and $y = 2$

$x + y = 1 + 2 = 3$

59. **(a)** $x + y + z = b + c - 2a + c + a - 2b + a + b - 2c$

$\Rightarrow \quad x + y + z = 0$

$\Rightarrow \quad x + y = -z$

Now, $x^2 + y^2 - z^2 + 2xy = (x + y)^2 - z^2 = (-z)^2 - z^2 = z^2 - z^2 = 0$

60. **(c)** $(y - z) + (z - x) + (x - y) = 0$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Hence, $(y - z)^3 + (z - x)^3 + (x - y)^3 = 3(y - z)(z - x)(x - y)$

61. **(c)** $\because \quad x = 2 - 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

$\Rightarrow \quad x - 2 = 2^{\frac{2}{3}} - 2^{\frac{1}{3}}$

Cubing both sides,

$$x^3 - 3x^2 \times 2 + 3x \times 4 - 8 = \left(2^{\frac{2}{3}}\right)^3 - \left(2^{\frac{1}{3}}\right)^3 - 3\cdot 2^{\frac{2}{3}}\cdot 2^{\frac{1}{3}}\left(2^{\frac{2}{3}} - 2^{\frac{1}{3}}\right)$$

$\Rightarrow \quad x^3 - 6x^2 + 12x - 8 = 4 - 2 - 6(x - 2)$

$\Rightarrow \quad x^3 - 6x^2 + 12x - 8 = 2 - 6x + 12$

$\Rightarrow \quad x^3 - 6x^2 + 18x + 18 = 2 + 12 + 8 + 18 = 40$

62. **(d)** $a^3 + b^3 + c^3 - 3abc = 0$, if $a + b + c = 0$

$\therefore \quad a^3 - b^3 - c^3 - 3abc = 0$

$\Rightarrow \quad a - b - c = 0$

$\Rightarrow \quad a = b + c$

63. **(b)** $\because \quad x^2 - 3x + 1 = 0$ ... (i)

$\Rightarrow \quad x^2 + 1 = 3x$

Squaring both sides, $(x^2+1)^2=(3x)^2$

$\Rightarrow \quad x^4+1+2x^2=9x^2$

$\Rightarrow \quad x^4+1=7x^2$ ...(

Now, $x^3+\frac{1}{x^3}=x^3+\left(\frac{1}{x}\right)^3=\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$

[$\because a^3+b^3=(a+b)(a^2+b^2-$

$=\left(\frac{x^2+1}{x}\right)\left(\frac{x^4+1}{x^2}-1\right)=\left(\frac{3x}{x}\right)\left(\frac{7x^2}{x^2}-1\right)$ *[From equation (i) & (*

$=3(7-1)=3\times 6=18$

64. **(d)** $\because \quad 2x+\frac{1}{3x}=5$

$\Rightarrow \quad \frac{6x^2+1}{3x}=5 \qquad \therefore \quad 6x^2+1=15x$ ...(

Now, $\frac{5x}{6x^2+20x+1}=\frac{5x}{(6x^2+1)+20x}$

$=\frac{5x}{15x+20x}=\frac{5x}{35x}=\frac{1}{7}$ (from (i)

65. **(c)** $\because \quad m+\frac{1}{m-2}=4$

$\Rightarrow \quad (m-2)+\frac{1}{m-2}=4-2=2$

Squaring both sides, $(m-2)^2+\frac{1}{(m-2)^2}+2(m-2)\left(\frac{1}{m-2}\right)=4$

$\Rightarrow \quad (m-2)^2+\frac{1}{(m-2)^2}=4-2=2$

66. **(c)** $\because \quad a^2=b+c$

$\Rightarrow \quad a^2+a=a+b+c$

$\Rightarrow \quad a(a+1)=a+b+c$

$\Rightarrow \quad (a+1)=\frac{a+b+c}{a}$

$\Rightarrow \quad \frac{1}{(a+1)}=\frac{a}{a+b+c}$

Similarly, $b^2=c+a$

$\Rightarrow \quad \frac{1}{b+1}=\frac{b}{a+b+c}$ and $c^2=a+b$

$\Rightarrow \quad \frac{1}{c+1}=\frac{c}{a+b+c}$

$\therefore \quad \frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}=\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}=\frac{a+b+c}{a+b+c}=1$

67. (d) $1-a \quad 1-b \quad 1-c = 1$

$$\Rightarrow \left(\frac{a}{1-a}+1\right)+\left(\frac{b}{1-b}+1\right)+\left(\frac{c}{1-c}+1\right)=3+1=4$$

$$\Rightarrow \frac{a+1-a}{1-a}+\frac{b+1-b}{1-b}+\frac{c+1-c}{1-c}=4$$

$$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=4$$

68. (c) $\because \quad a^2+b^2+2b+4a+5=0$

$$\Rightarrow a^2+4a+b^2+2b+5=0$$

$$\Rightarrow a^2+4a+4+b^2+2b+1=0$$

$$\Rightarrow (a+2)^2+(b+1)^2=0$$

It is possible only when, $a+2=0$

$$\Rightarrow a=-2$$

and $b+1=0$

$$\Rightarrow b=-1$$

$$\therefore \quad \frac{a-b}{a+b}=\frac{-2+1}{-2-1}=\frac{-1}{-3}=\frac{1}{3}$$

69. (c) Let each ratio $= k$ then

$$x=k(b-c)(b+c-2a)=k(b^2-c^2)-k\,2a(b-c)$$

$$y=k(c-a)(c+a-2b)=k(c^2-a^2)-k\,2b(c-a)$$

$$z=k(a-b)(a+b-2c)=k(a^2-b^2)-k\,2c(a-b)$$

Adding, $x+y+z=0$

70. (a) From componendo and dividendo

$$\frac{(x^3+3x)+(3x^2+1)}{(x^3+3x)-(3x^2+1)}=\frac{189+61}{189-61}$$

or, $$\frac{(x+1)^3}{(x-1)^3}=\frac{250}{128}=\frac{125}{64}.$$

or, $$\left(\frac{x+1}{x-1}\right)^3=\left(\frac{5}{4}\right)^3$$

or, $$\frac{x+1}{x-1}=\frac{5}{4}$$

$\therefore$ Solving $x=9$

71.

$\Rightarrow \quad x^2p^2 + x^2q^2 + y^2p^2 + y^2q^2 = x^2p^2 + y^2q^2 + 2xpyq$

$\Rightarrow \quad x^2q^2 + y^2p^2 - 2xpyq = 0$

$\Rightarrow \quad (xq - yp)^2 = 0$

$\therefore \quad xq = yp$

72. **(a)** $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$ (ratio proportion law)

Each ratio $= \dfrac{x+y+z}{(2x+y+z)+(x+2y+z)+(x+y+2z)}$

$= \dfrac{x+y+z}{4(x+y+z)} = \dfrac{1}{4}$

73. **(d)** $a + b + c = 0$

$\Rightarrow \quad a + b = -c$

Squaring, $a^2 + b^2 + 2ab = c^2$

or, $a^2 + b^2 = c^2 - 2ab$ ...(i)

Given expression $= \dfrac{a^2+b^2+c^2}{c^2-ab} = \dfrac{(c^2-2ab)+c^2}{c^2-ab}$ [from equation (i)]

$= \dfrac{2(c^2-ab)}{(c^2-ab)} = 2$

74. **(c)** $x^2 + 8y^2 + 9z^2 - 4xy - 12xz = 0$

or, $x^2 + 4y^2 - 4xy + 4y^2 + 9z^2 - 12xz = 0$

or, $(x-2y)^2 + (2y-3z)^2 = 0$

$\therefore \quad x - 2y = 0$

$\Rightarrow \quad 2y - 3z = 0$

$\Rightarrow \quad x = 2y = 3z$

75. **(c)** Given expression

$= p^2a^2 + p^2b^2 + p^2c^2 - 2pab - 2pbc - 2pcd + b^2 + c^2 + d^2 \le 0$

or, $(p^2a^2 - 2pab + b^2) + (p^2b^2 - 2pbc + c^2) + (p^2c^2 - 2pcd + d^2) \le 0$

or, $(pa-b)^2 + (pb-c)^2 + (pc-d)^2 \le 0$

It is possible only when, $pa - b = 0$, $pb - c = 0$ and $pc - d = 0$

or, $p = \frac{b}{a}$, $p = \frac{c}{b}$ and $p = \frac{d}{c}$

$\therefore \quad \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

(ratio proportion)

76. (a) We know that,

$(a+b+c)^3 = a^3+b^3+c^3+3(b+c)(c+a)(a+b)$

$\therefore \quad (30)^3 = a^3+b^3+c^3+3\times 10\times 20\times 30$

$(\because b+c+c+a+a+b = 10+20+30 \Rightarrow a+b+c=30)$

$a^3+b^3+c^3 = 27000 - 3\times 10\times 20\times 30$

$= 27000 - 18000 = 9000$

77. (c) We know that $(a+b+c)^3 = a^3+b^3+c^3+3(b+c)(c+a)(a+b)$

$\because \quad 2x+2y+2z = b+c+c+a+a+b \Rightarrow x+y+z = a+b+c$

$\therefore \quad (x+y+z)^3 = a^3+b^3+c^3+3(2x)(2y)(2z)$

$\therefore \quad a^3+b^3+c^3 = (x+y+z)^3 - 24xyz$

78. (b) We know that, $a+b+c=0$ ... (i)

$\Rightarrow \quad a^3+b^3+c^3 = 3abc$

but, $a+b+c=0$, $b=-(a+c)$, $c=-(a+b)$

$\therefore \quad$ from (i), $a^3+b^3+c^3 = 3a(-(a+c))(-(a+b))$

$= 3a(a+b)(c+a)$

from equation

79. (c) If $a=x-y$, $b=y-z$ and $c=z-x$ then

$a+b+c = x-y+y-z+z-x = 0$

$\therefore \quad a^3+b^3+c^3 = 3abc = 3(x-y)(y-z)(z-x)$

80. (b) $(ab+bc+ca)^2 = a^2b^2+b^2c^2+c^2a^2+2abbc+2abca+2bcca$

or, $\quad 0 = a^2b^2+b^2c^2+c^2a^2+2abc(b+a+c)$

$0 = a^2b^2+b^2c^2+c^2a^2+2pq$

$\therefore \quad a^2b^2+b^2c^2+c^2a^2 = -2pq$

81. (c) $49c^2+9(a+b)^2-42(a+b)c = (7c-3(a+b))^2$

$= (7\times 8 - 3(89-69))^2$

$= (56-60)^2 = (-4)^2 = 16$

82. (c) $x^2+y^2+z^2-2xy-2xz+2yz = (x-y-z)^2$

$= (q+r+s-r-s+p-p-q-r)^2$

$= (-r)^2 = r^2$

83. (d) $(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$

or, $\quad 10^2 = 60+2(xy+yz+zx)$

$\therefore \quad \dfrac{100-60}{2} = xy+yz+zx$

or, $\quad xy+yz+zx = 20$

84. (a) $(ac-bd)^2+(ad+bc)^2 = a^2c^2+b^2d^2-2acbd+a^2d^2+b^2c^2+2adbc$

$$= a^2c^2+a^2d^2+b^2d^2+b^2c^2$$
$$= a^2(c^2+d^2)+b^2(d^2+c^2)$$
$$= a^2\cdot 2+b^2\cdot 2 = 2(a^2+b^2) \quad (\because c^2+d^2=2)$$
$$= 2\times 1 = 2$$

85. (c) Let $x-a=p$, $x-b=q$ and $x-c=r$

then, $q-p=a-b$, $r-q=b-c$, $p-r=c-a$

Putting these values,

Given expression $= pq(q-p)+qr(r-p)+rp(p-r)$

$$= -(q-p)(r-p)(p-r)$$

[See result 7 of theory part]

$$= -(a-b)(b-c)(c-a)$$

$(\because a-b=q-p,\ b-c=r-q$ and $c-a=p-r)$

86. (d) $\left(x+\frac{1}{x}\right)\left(y+\frac{1}{y}\right)+\left(x-\frac{1}{x}\right)\left(y-\frac{1}{y}\right) = xy+\frac{x}{y}+\frac{y}{x}+\frac{1}{xy}+xy-\frac{x}{y}-\frac{y}{x}+\frac{1}{xy}$

$$= 2\left(xy+\frac{1}{xy}\right)$$

$\therefore\ xy+\frac{1}{xy} = \frac{1}{2}\left\{\left(x+\frac{1}{x}\right)\left(y+\frac{1}{y}\right)+\left(x-\frac{1}{x}\right)\left(y-\frac{1}{y}\right)\right\}$

$$= \frac{1}{2}[2a\cdot 2c+2b\cdot 2d]$$
$$= 2(ac+bd)$$

87. (d) $x^8+x^4+1 = x^8+2x^4+1-x^4$

$$= (x^4+1)^2-x^4 = (x^4+1+x^2)(x^4+1-x^2)$$
$$= (x^4+2x^2+1-x^2)(x^4-x^2+1)$$
$$= ((x^2+1)^2-x^2)(x^4-x^2+1)$$
$$= (x^2+1-x)(x^2+1+x)(x^4-x^2+1)$$
$$= (x^2-x+1)(x^2+2x+1-x)(x^4-x^2+1)$$
$$= (x^2-x+1)((x+1)^2-x)(x^4-x^2+1)$$
$$= (x^2-x+1)(x+1+\sqrt{x})(x+1-\sqrt{x})(x^4-x^2+1)$$

Among given options x^2-2x+1 is not a factor.

88. (a) $a^4 - 11a^2b^2 + b^4 = a^4 - 2a^2b^2 + b^4 - 9a^2b^2$

$$= (a^2 - b^2)^2 - (3ab)^2$$

$$= (a^2 - b^2 - 3ab)(a^2 - b^2 + 3ab)$$

89. (b) $a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2$

$$a^4 + b^4 = (a^2 + b^2 - \sqrt{2}\,ab)(a^2 + b^2 + \sqrt{2}\,ab)$$

or, $\dfrac{a^4 + b^4}{a^2 + b^2 - \sqrt{2}\,ab} = a^2 + b^2 + \sqrt{2}\,ab = x + \sqrt{2}\,y$

90. (d) Given, $x = \dfrac{a}{a+b}$

$$\Rightarrow x^2 = \frac{a^2}{(a+b)^2}$$

$$y = \frac{b}{a+b}$$

$$\Rightarrow y^2 = \frac{b^2}{(a+b)^2}$$

$$\therefore \frac{x^2}{y^2} = \frac{a^2}{b^2}$$

By componendo and dividendo, $\dfrac{x^2 + y^2}{x^2 - y^2} = \dfrac{a^2 + b^2}{a^2 - b^2}$

91. (b) $\dfrac{x}{y} = \dfrac{(p+q)^2}{(p-q)^2}$

By componendo and dividendo,

$$\frac{x - y}{x + y} = \frac{(p+q)^2 - (p-q)^2}{(p+q)^2 + (p-q)^2} = \frac{4pq}{2(p^2 + q^2)} = \frac{2pq}{p^2 + q^2}$$

92. (a) $x + \dfrac{a}{x} = 1 \Rightarrow x - 1 = \dfrac{a}{x}$

Now, $\dfrac{x^3 - x^2}{x^2 + x + a} = \dfrac{x^2(x-1)}{x\left(x + \frac{a}{x}\right) + x} = \dfrac{x^2 \cdot \frac{a}{x}}{x \cdot 1 + x}$

putting $x - 1 = \dfrac{a}{x}$ and $x + \dfrac{a}{x} = 1$ $= \dfrac{ax}{2x} = \dfrac{a}{2}$

93. (a) $\because \quad a = \dfrac{xy}{x+y}, b = \dfrac{xz}{x+z}$ and $c = \dfrac{yz}{y+z}$

$$\therefore \quad \frac{x+y}{xy} = \frac{1}{a}, \frac{x+z}{xz} = \frac{1}{b}, \frac{y+z}{yz} = \frac{1}{c}$$

or, $\frac{1}{y}+\frac{1}{x}=\frac{1}{a}, \frac{1}{z}+\frac{1}{x}=\frac{1}{b}, \frac{1}{z}+\frac{1}{y}=\frac{1}{c}$

$\therefore \quad \left(\frac{1}{y}+\frac{1}{x}\right)+\left(\frac{1}{z}+\frac{1}{x}\right)-\left(\frac{1}{z}+\frac{1}{y}\right)=\frac{1}{a}+\frac{1}{b}-\frac{1}{c}$

or, $\frac{2}{x}=\frac{bc+ca-ab}{abc}$

or, $x=\frac{2abc}{bc+ca-ab}$

94. **(c)** HCF of expression $=(a+1)$

LCM of expression $=a^3+a^2-a-1=(a+1)(a+1)(a-1)$

First expression $=a^2-1=(a-1)(a+1)$

We know that,

$$\text{Second expression}=\frac{\text{HCF}\times\text{LCM}}{\text{First expression}}$$

$$=\frac{(a+1)(a+1)(a+1)(a-1)}{(a-1)(a+1)}=(a+1)^2$$

95. **(c)** $\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2$

$\Rightarrow \left(x+\frac{1}{x}\right)^2=p+2$

$\Rightarrow x+\frac{1}{x}=\sqrt{p+2}$

Now, $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)\left(x\cdot\frac{1}{x}\right)$

$=(\sqrt{p+2})^3-3(\sqrt{p+2})$

$=(\sqrt{p+2})((\sqrt{p+2})^2-3)$

$=(\sqrt{p+2})(p+2-3)$

$=(p-1)\sqrt{p+2}$

96. **(c)** We know that, when $a+b+c=0$ then

$\Rightarrow a^2+b^2+c^2=-(2ab+2bc+2ca)$ ... (i)

$\therefore$ Given expression, $\frac{a^2+b^2+c^2}{(a-b)^2+(b-c)^2+(c-a)^2}$

$=\frac{a^2+b^2+c^2}{2(a^2+b^2+c^2)-(2ab+2bc+2ca)}$

$=\frac{a^2+b^2+c^2}{2(a^2+b^2+c^2)+(a^2+b^2+c^2)}$ *[from equation (i)]*

$=\frac{a^2+b^2+c^2}{3(a^2+b^2+c^2)}=\frac{1}{3}$

97. (c) Given $x^4 + x^3 - 4x^2 + x + 1 = 0$ dividing by x^2

$\Rightarrow \quad x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0$

$\Rightarrow \quad x^2 + \frac{1}{x^2} + x + \frac{1}{x} - 4 = 0$

$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2 + x + \frac{1}{x} - 6 = 0$

$\Rightarrow \quad \left(x + \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right) - 6 = 0$

$\Rightarrow \quad y^2 + y - 6 = 0 \qquad (\because y = x + \frac{1}{x})$

Which is required expression.

98. (c) Given expression,

$= (2 + 1)(2^2 + 1)(2^4 + 1) \ldots (2^{64} + 1)$

$= \frac{(2-1)(2+1)(2^2+1)(2^4+1)\ldots(2^{64}+1)}{(2-1)}$

$= (2^2 - 1)(2^2 + 1)(2^4 + 1) \ldots (2^{64} + 1)(2^4 - 1)(2^4 + 1) \ldots (2^{64} + 1)$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$= (2^{64} - 1)(2^{64} + 1) = 2^{128} - 1$

99. (a) First polynomial

$$p(x) = x^3 + 3x^2y + 2xy^2$$
$$= x(x^2 + 3xy + 2y^2) = x(x^2 + 2xy + xy + 2y^2)$$
$$= x[x(x + 2y) + y(x + 2y)] = x(x + y)(x + 2y)$$

Second polynomial

$$= x^4 + 6x^3y + 8x^2y^2$$
$$= x^2(x^2 + 6xy + 8y^2) = x^2(x^2 + 2xy + 4xy + 8y^2)$$
$$= x^2[x(x + 2y) + 4y(x + 2y)] = x^2(x + 2y)(x + 4y)$$

$\therefore \quad$ HCF $= x(x + 2y)$

00. (a) $\because pqr = 1 \qquad \therefore \quad r = p^{-1}q^{-1}$ and $r^{-1} = pq$

Eliminating r from given expression,

Given expression $= \frac{1}{1+p+q^{-1}} + \frac{1}{1+q+pq} + \frac{1}{1+p^{-1}q^{-1}+p^{-1}}$

$$= \frac{q}{q+pq+1} + \frac{1}{1+q+pq} + \frac{pq}{pq+1+q} = \frac{q+1+pq}{q+pq+1} = 1$$

101.(a) $\because \quad x + y + z = 2s$

and $(s - x) + (s - y) - z = 2s - (x + y + z) = 2s - 2s = 0$

$\therefore \quad (s - x)^3 + (s - y)^3 - z^3 + 3(s - x)(s - y)(z) = 0$

$\Rightarrow \quad (s - x)^3 + (s - y)^3 + 3(s - x)(s - y)(z) = z^3$

102.(a) $x^2 = y + z \Rightarrow x^2 + x = x + y + z$

$\Rightarrow \quad \dfrac{x}{x+y+z} = \dfrac{1}{x+1}$

Similarly $\dfrac{1}{y+1} = \dfrac{y}{x+y+z}$ and $\dfrac{1}{z+1} = \dfrac{z}{x+y+z}$

$\therefore \quad \dfrac{1}{x+1} + \dfrac{1}{y+1} + \dfrac{1}{z+1} = \dfrac{x}{x+y+z} + \dfrac{y}{x+y+z} + \dfrac{z}{x+y+z} = \dfrac{x+y+z}{x+y+z} = 1.$

103.(b) $\because \quad p + q + (-r) = 0$

$\therefore \quad p^3 + q^3 + (-r)^3 = 3pq(-r) = 3 \times (-30) = -90$

(Recall that $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$)

104.(a) $\dfrac{x^5-1}{x-1} + x^3 + 2x^2 + x$

$= \dfrac{x^5 - 1 + x^4 + 2x^3 + x^2 - x^3 - 2x^2 - x}{x-1} = \dfrac{x^5 + x^4 + x^3 - x^2 - x - 1}{x-1}$

$= \dfrac{x^3(x^2+x+1) - 1(x^2+x+1)}{(x-1)} = \dfrac{(x^2+x+1)(x^3-1)}{x-1}$

$= \dfrac{(x^2+x+1)(x-1)(x^2+x+1)}{(x-1)} = (x^2 + x + 1)^2$

$\therefore$ Required square root $= \sqrt{(x^2+x+1)^2} = x^2 + x + 1$

Second method,

$\because \quad \dfrac{x^5-1}{x-1} = x^4 + x^3 + x^2 + x + 1$

$\therefore$ Given expression, $= x^4 + x^3 + x^2 + x + 1 + x^3 + 2x^2 + x$

$= x^4 + x^2 + 2 \cdot x^3 + 2x^2 + x = (x^2 + x + 1)^2$

105.(b) $\dfrac{x^8+4}{x^4+2x^2+2} = \dfrac{x^8 + 4x^4 + 4 - 4x^4}{x^4 + 2x^2 + 2}$

$= \dfrac{(x^4+2)^2 - (2x^2)^2}{x^4 + 2x^2 + 2}$

$= \dfrac{(x^4+2x^2+2)(x^4-2x^2+2)}{(x^4+2x^2+2)}$

$= x^4 - 2x^2 + 2$

$\frac{z}{z} = 1.$

106.(d) Given, $x + \frac{1}{x} = p$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = p^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = p^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = p^2 - 2$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^3 = (p^2 - 2)^3$$

$$\Rightarrow x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right) = p^6 - 8 - 6p^2(p^2 - 2)$$

$$\Rightarrow x^6 + \frac{1}{x^6} + 3(p^2 - 2) = p^6 - 8 - 6p^4 + 12p^2$$

$$\Rightarrow x^6 + \frac{1}{x^6} = p^6 - 6p^4 + 9p^2 - 2$$

107.(c) Given, $\left(\frac{y-z-x}{2}\right)^3 + \left(\frac{z-x-y}{2}\right)^3 + \left(\frac{x-y-z}{2}\right)^3$

$$= \left(\frac{y-(z+x)}{2}\right)^3 + \left(\frac{z-(x+y)}{2}\right)^3 + \left(\frac{x-(y+z)}{2}\right)^3$$

$$= \left(\frac{y-(-y)}{2}\right)^3 + \left(\frac{z-(-z)}{2}\right)^3 + \left(\frac{x-(-x)}{2}\right)^3$$

$(\because x + y + z = 0, \Rightarrow x + z = -y$ etc)

$$= \left(\frac{2y}{2}\right)^3 + \left(\frac{2z}{2}\right)^3 + \left(\frac{2x}{2}\right)^3 = y^3 + z^3 + x^3$$

$= 3xyz$ $\quad (\because$ if $a + b + c = 0,$ then $a^3 + b^3 + c^3 = 3abc)$

Exercise–1B

1. If $a + b + 1 = 0$ then what is the value of $(a^3 + b^3 + 1 - 3ab)$?

(a) -1 (b) 1 (c) 3 (d) 0

[SSC Tier-I 2012]

2. If $x = (0.08)^2, y = \frac{1}{(0.08)^2}$ and $z = (1 - 0.08)^2 - 1$ then which of the following relation is true ?

(a) $y < z < x$ (b) $z < x < y$

(c) $y < x$ and $x = z$ (d) $x < y$ and $x = z$ *[SSC Tier-I 2012]*

3. If $x^4 + \frac{1}{x^4} = 23$ then what is the value of $\left(x - \frac{1}{x}\right)^2$?

(a) -3 (b) 3 (c) 7 (d) -7

[SSC Tier-I 2012]

4. If $x + \frac{1}{x} = 3$ then what is the value of $x^5 + \frac{1}{x^5}$?

(a) 113 (b) 129 (c) 123 (d) 126

[SSC Tier-I 2012]

5. If $a + b = 6$, $a - b = 2$ then what is the value of $2(a^2 + b^2)$?

(a) 20 (b) 30 (c) 40 (d) 10

[SSC Tier-I 2012]

6. If $2a - \frac{2}{a} + 3 = 0$, then value of $\left(a^3 - \frac{1}{a^3} + 2\right)$ is—

(a) 5 (b) $-\frac{35}{8}$ (c) $-\frac{40}{7}$ (d) $\frac{-47}{8}$

[SSC Tier-I 2012]

7. If factors of $x^3 + (a+1)x^2 - (b-2)x - 6$ are $(x + 1)$ and $(x - 2)$ then values of a and b are respectively is—

(a) 2 and 8 (b) 1 and 7 (c) 5 and 3 (d) 3 and 7

[SSC Tier-I 2012]

8. If x is real and $x^4 + \frac{1}{x^4} = 119$, then value of $\left(x - \frac{1}{x}\right)$ is

(a) ± 4 (b) ± 9 (c) ± 3 (d) ± 2

[SSC Tier-I 2012]

9. If $x^3 + y^3 = 35$ and $x + y = 5$, then value of $\left(\frac{1}{x} + \frac{1}{y}\right)$ is

(a) $\frac{4}{7}$ (b) $\frac{3}{8}$ (c) $\frac{5}{6}$ (d) $\frac{3}{5}$

[SSC Tier-I 2012]

10. If $\frac{x^2}{by+cz} = \frac{y^2}{cz+ax} = \frac{z^2}{ax+by} = 1$, then value of $\frac{a}{a+x} + \frac{b}{b+y} + \frac{c}{c+z}$ is

(a) -1 (b) 2 (c) 1 (d) -2

[SSC Tier-I 2012]

11. Value of a and b $(a > 0, b < 0)$ for which $8x^3 - ax^2 + 54x + b$ is a perfect cube is

(a) $a = 12, b = -9$ (b) $a = 36, b = -27$

(c) $a = 18, b = -27$ (d) $a = 16, b = -9$

[SSC Tier-I 2012]

12. If $x = \frac{4ab}{a+b}$, then value of $\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$ is

(a) a (b) b (c) 0 (d) 2

[SSC Tier-I 2012]

d) -7
[SSC Tier-I 2012]

d) 126
[SSC Tier-I 2012]

) ?
) 10
[SSC Tier-I 2012]

$\frac{-47}{8}$
[SSC Tier-I 2012]

-2) then values

3 and 7
[SSC Tier-I 2012]

± 2
[SSC Tier-I 2012]

$\frac{3}{5}$
[SSC Tier-I 2012]

$\frac{c}{+z}$ is

-2
[SSC Tier-I 2012]

b is a perfect

[SSC Tier-I 2012]

[SSC Tier-I 2012]

13. If $a + b + c = 8$, then value of $(a-4)^3 + (b-3)^3 + (c-1)^3 - 3(a-4)(b-3)(c-1)$ is
(a) 2 (b) 4 (c) 1 (d) 0
[SSC Tier-I 2012]

14. If $x = \sqrt{a} + \frac{1}{\sqrt{a}}, y = \sqrt{a} - \frac{1}{\sqrt{a}}$, then value of $x^4 + y^4 - 2x^2y^2$ is
(a) 16 (b) 20 (c) 10 (d) 5
[SSC Tier-I 2012]

15. If $5a + \frac{1}{3a} = 5$, then value of $9a^2 + \frac{1}{25a^2}$ is
(a) $\frac{51}{5}$ (b) $\frac{29}{5}$ (c) $\frac{52}{5}$ (d) $\frac{39}{5}$
[SSC Tier-I 2012]

16. If $a + b + c = 0$, then value of $\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right)$ is
(a) 2 (b) 3 (c) 4 (d) 5
[SSC Tier-I 2012]

17. If a, b, c are real, $a^3 + b^3 + c^3 = 3abc$ and $a + b + c \neq 0$, then relation between a, b, c will be
(a) $a + b = c$ (b) $a + c = b$ (c) $a = b = c$ (d) $b + c = a$
[SSC Tier-I 2012]

18. If $a^2 + \frac{1}{a^2} = 98$ $(a > 0)$ then what is the value of $a^3 + \frac{1}{a^3}$?
(a) 535 (b) 1030 (c) 790 (d) 970
[SSC Tier-I 2012]

19. If $x + \frac{1}{x} = 5$ then what is the value of $\frac{x^4 + \frac{1}{x^2}}{x^2 - 3x + 1}$?
(a) 70 (b) 50 (c) 110 (d) 55
[SSC Tier-I 2012]

20. If $a^2 + b^2 + c^2 = 2(a - b - c) - 3$, then what is the value of $2a - 3b + 4c$?
(a) 3 (b) 1 (c) 2 (d) 4
[SSC Tier-I 2012]

21. If $2x - \frac{1}{2x} = 6$ then what is the value of $x^2 + \frac{1}{16x^2}$?
(a) $\frac{19}{2}$ (b) $\frac{17}{2}$ (c) $\frac{18}{3}$ (d) $\frac{15}{2}$
[SSC Tier-I 2012]

22. If $(5x^2 - 3y^2) : xy = 11 : 2$ then what is the positive value of $\frac{x}{y}$?
(a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{3}$ (d) $\frac{7}{2}$
[SSC Tier-I 2012]

23. If $ax + by = 6$, $bx - ay = 2$ and $x^2 + y^2 = 4$ then what is $(a^2 + b^2)$?
(a) 2 (b) 4 (c) 5 (d) 10
[SSC Tier-I 2012]

24. If $a + \frac{1}{a+2} = 0$ then what is the value of $(a+2)^3 + \frac{1}{(a+2)^3}$?

(a) 6 (b) 4 (c) 3 (d) 2

[SSC Tier-I 2012]

25. If $a^3 - b^3 = 56$ and $a - b = 2$ then what is the value of $(a^2 + b^2)$?

(a) -12 (b) 20 (c) 18 (d) -10

[SSC Tier-I 2012]

26. If $a + \frac{1}{a} = 1$ then what is the value of a^3 ?

(a) -2 (b) 2 (c) -1 (d) 4

[SSC Tier-I 2012]

27. If $(a - b) = 3$, $(b - c) = 5$ and $(c - a) = 1$ then what is the value of $\frac{a^3 + b^3 + c^3 - 3abc}{a + b + c}$?

(a) 17.5 (b) 20.5 (c) 10.5 (d) 15.5

[SSC Tier-I 2012]

28. If $x = 2t$ and $y = \frac{2t-1}{3}$, then for what value of $t, x = y$ is correct ?

(a) $\frac{1}{3}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{4}$ (d) $-\frac{1}{2}$

[SSC Tier-I 2012]

29. If x and y are two real numbers and $x + y = 8$, then maximum value of xy is

(a) 16 (b) 18 (c) 12 (d) 15

[SSC Tier-I 2012]

Answer–1B

1. (d)	2. (b)	3. (b)	4. (a)	5. (c)	6. (d)	7. (b)	8. (c)
9. (c)	10. (c)	11. (b)	12. (d)	13. (d)	14. (a)	15. (d)	16. (b)
17. (c)	18. (d)	19. (d)	20. (b)	21. (a)	22. (b)	23. (d)	24. (d)
25. (b)	26. (c)	27. (★)	28. (b)	29. (a)			

Explanation

1. (d) $\because a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 - 3abc = 0$, here $c = 1$.

2. (b) $z = (1 - 0.08)^2 - 1 = (1 - 0.08 - 1)\ (1 - 0.08 + 1)$

$= (-0.08)\ (1.92) =$ a negative quantity.

$\because\ x = (0.08)^2 \qquad \because 0 < x < 1$

$\because\ y = \frac{1}{(0.08)^2} \qquad \therefore\ y > 1$

Hence, $z < x < y$

3. **(b)** Trick, if $x - \frac{1}{x} = a$ then $x^4 + \frac{1}{x^4} = a^4 + 4a^2 + 2$

According to question, $a^4 + 4a^2 + 2 = 23$

$$\text{or, } a^4 + 4a^2 - 21 = 0$$
$$\text{or, } a^4 - 3a^2 + 7a^2 - 21 = 0$$
$$\text{or, } (a^2 - 3)(a^2 + 7) = 0$$
$$\text{or, } a^2 = 3 \qquad (\because a^2 + 7 \neq 0)$$
$$\therefore \left(x - \frac{1}{x}\right)^2 = 3$$

4. **(a)** Trick, if $x + \frac{1}{x} = a$ then $x^5 + \frac{1}{x^5} = a^5 - 5a^3 + 5a$

Here, $x + \frac{1}{x} = 3$

$$\therefore \quad x^5 + \frac{1}{x^5} = 3^5 - 5 \times 3^3 + 5 \times 3$$
$$= 243 - 145 + 15$$
$$= 243 - 130 = 113$$

5. **(c)** $(a + b)^2 + (a - b)^2 = 6^2 + 2^2$

$2(a^2 + b^2) = 40$

$a^2 + b^2 = 20$

6. **(d)** $2\left(a - \frac{1}{a}\right) + 3 = 0$

$\Rightarrow a - \frac{1}{a} = \frac{-3}{2}$

Cubing both side, $a^3 - \frac{1}{a^3} - 3a^2\frac{1}{a} + 3a\frac{1}{a^2} = \frac{-27}{8}$

$$\text{or, } a^3 - \frac{1}{a^3} - 3\left(a - \frac{1}{a}\right) = \frac{-27}{8}$$
$$\text{or, } a^3 - \frac{1}{a^3} - 3\left(-\frac{3}{2}\right) = \frac{-27}{8}$$
$$\text{or, } a^3 - \frac{1}{a^3} = \frac{-9}{2} - \frac{27}{8} = \frac{-63}{8}$$
$$\therefore \quad a^3 - \frac{1}{a^3} + 2 = \frac{-63}{8} + 2 = \frac{-63 + 16}{8} = \frac{-47}{8}$$

7. **(b)** Given expression $= x^3 + (a + 1)x^2 - (b - 2)x - 6$

$\because$ $x + 1$ and $x - 2$ are its factor, then $x = -1$ and $x = 2$ are its zeros.

putting $x = -1$, $-1 + (a + 1) + b - 2 - 6 = 0$ ... (i)

or, $a + b = 8$

putting $x = 2$, $8 + 4(a+1) - 2(b-2) - 6 = 0$ ... (ii)

or, $4a - 2b = -10$

Solving (1) and (2), $a = 1, b = 7$

9. (c) $x^3 + y^3 = 35$

$\Rightarrow (x+y)(x^2 - xy + y^2) = 35$

$\Rightarrow 5(x^2 - xy + y^2) = 35$

$\Rightarrow x^2 - xy + y^2 = 7$

or, $(x+y)^2 - 3xy = 7$ $(\because x + y = 5)$

or, $25 - 3xy = 7$

or, $3xy = 18$

or, $xy = 6$

$\therefore \frac{1}{x} + \frac{1}{y} = \frac{y+x}{xy} = \frac{5}{6}$

10. (c) From $\frac{x^2}{by+cz} = \frac{y^2}{cz+ax} = \frac{z^2}{ax+by} = 1$

$x^2 = by + cz,\ y^2 = cz + ax,\ z^2 = ax + by$ ... (i)

Given expression $= \frac{a}{a+x} + \frac{b}{b+y} + \frac{c}{c+z}$

$= \frac{a}{a+x} \times \frac{x}{x} + \frac{b}{b+y} \times \frac{y}{y} + \frac{c}{c+z} \times \frac{z}{z}$

$= \frac{ax}{ax+x^2} + \frac{by}{by+y^2} + \frac{cz}{cz+z^2}$

$= \frac{ax}{ac+by+cz} + \frac{by}{by+cz+ax} + \frac{cz}{cz+ax+by}$

Putting value of x^2, y^2, z^2 from (1) $\frac{ax+by+cz}{ax+by+cz} = 1$

11. (b) $8x^3 - ax^2 + 54x + b = (2x)^3 - 3\cdot(2x)^2\left(\frac{a}{12}\right) + 3(2x)(3)^2 + b$

Clearly $\frac{a}{12} = 3$ and $b = -3^3$

or, $a = 36, b = -27$

12. (d) $x = \frac{4ab}{a+b}$

$\Rightarrow \frac{x}{2a} = \frac{2b}{a+b}$

$\Rightarrow \frac{x+2a}{x-2a} = \frac{2b+a+b}{2b-(a+b)}$ (by componendo and dividendo)

$\Rightarrow \frac{x+2a}{x-2a} = \frac{3b+a}{b-a}$ (i)

Again from $x = \frac{4ab}{a+b}, \frac{x}{2b} = \frac{2a}{a+b}$

or, $\frac{x+2b}{x-2b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b}$ ... (ii)

Adding (1) and (2),

$$\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = \frac{(3b+a)-(3a+b)}{b-a} = \frac{2(b-a)}{b-a} = 2$$

tricky approach,

putting $a = 1, b = 3, x = \frac{4 \times 1 \times 3}{1+3} = 3$

$$\therefore \quad \frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = \frac{3+2}{3-2} + \frac{3+6}{3-6} = 5 - 3 = 2$$

13. (d) $a + b + c = 8$

$\Rightarrow \quad (a-4) + (b-3) + (c-1) = 8 - 4 - 3 - 1 = 0$

$\because \quad x + y + z = 0$

$\Rightarrow \quad x^3 + y^3 + z^3 - 3xyz = 0$

putting, $x = a - 4, y = b - 3, z = c - 1,$

$$(a-4)^3 + (b-3)^3 + (c-1)^3 - 3(a-4)(b-3)(c-1) = 0$$

14. (a) $x^4 + y^4 - 2x^2y^2 = (x^2 - y^2)^2 = ((x+y)(x-y))^2$

$$= \left(2\sqrt{a} \cdot \frac{2}{\sqrt{a}}\right)^2 = 16 \qquad (\because x + y = 2\sqrt{a} \text{ and } x - y = \frac{2}{\sqrt{a}})$$

15. (d) $5a + \frac{1}{3a} = 5$

$\Rightarrow \quad \frac{5}{3}\left(3a + \frac{1}{5a}\right) = 5$

$\Rightarrow \quad 3a + \frac{1}{5a} = 3$ now squaring both sides.

16. (b) $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc} = \frac{3abc}{abc} = 3$

$(\because \; a + b + c = 0, a^3 + b^3 + c^3 = 3abc)$

17. (c) $a^3 + b^3 + c^3 - 3abc = (a + b + c) \frac{1}{2} \{(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

$\because \quad a + b + c \neq 0$

$\therefore \quad \frac{1}{2}\left\{(a-b)^2 + (b-c)^2 = (c-a)^2\right\} = 0$

It is possible only when $(a - b) = 0, b - c = 0$ and $c - a = 0$

$\therefore \quad a = b, b = c, c = a$ or, $a = b = c$

18. (d) $a^2 + \frac{1}{a^2} = 98$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 - 2a \cdot \frac{1}{a} = 98$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^2 = 100$$

$$\Rightarrow a + \frac{1}{a} = 10$$

Now, $a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3a \cdot \frac{1}{a}\left(a + \frac{1}{a}\right)$

$$= 10^3 - 3 \cdot 10 = 1000 - 30 = 970$$

19. (d) $\frac{x^4 + \frac{1}{x^2}}{x^2 - 3x + 1} = \frac{x\left(x^3 + \frac{1}{x^3}\right)}{x\left(x - 3 + \frac{1}{x}\right)} = \frac{x^3 + \frac{1}{x^3}}{\left(x + \frac{1}{x} - 3\right)}$

$$= \frac{\left(x + \frac{1}{x}\right)^3 - 3x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right)}{\left(x + \frac{1}{x}\right) - 3}$$

$$= \frac{5^3 - 3 \cdot 1 \cdot 5}{5 - 3} = \frac{110}{2} = 55$$

20. (b) Given, $a^2 + b^2 + c^2 - 2a + 2b + 2c + 3 = 0$

or, $(a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$

or, $(a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$

It is possible only when, $a - 1 = 0$, $b + 1 = 0$ and $c + 1 = 0$

$$\therefore a = 1, b = -1, c = -1$$

Required value= $2a - 3b + 4c = 2 + 3 - 4 = 1$

21. (a) $2x - \frac{1}{2x} = 6$

$$\Rightarrow 2\left(x - \frac{1}{4x}\right) = 6$$

$$\Rightarrow x - \frac{1}{4x} = 3$$

Squaring, $x^2 + \frac{1}{16x^2} - 2 \cdot x \cdot \frac{1}{4x} = 9$

or, $x^2 + \frac{1}{16x^2} = 9 + \frac{1}{2} = \frac{19}{2}$

22. (b) $\dfrac{5x^2-3y^2}{xy}=\dfrac{11}{2}$

$5\dfrac{x}{y}-\dfrac{3y}{x}=\dfrac{11}{2}$

If $\dfrac{x}{y}=t$ then $5t-\dfrac{3}{t}=\dfrac{11}{2}$

or, $\dfrac{5t^2-3}{t}=\dfrac{11}{2}$

or, $10t^2-6=11t$

or, $10t^2-11t-6=0$

or, $10t^2-15t+4t-6=0$

or, $5t(2t-3)+2(2t-3)=0$

$\therefore\ t=\dfrac{3}{2},\dfrac{-2}{5}$

Since $\dfrac{x}{y}$ is positive, $\therefore\ t=\dfrac{x}{y}=\dfrac{3}{2}$

23. **(d)** $ax+by=6$ and $bx-ay=2$

Squaring and adding,

$a^2x^2+b^2y^2+2axby+b^2x^2+a^2y^2-2bxay=36+4$

or, $a^2x^2+b^2y^2+b^2x^2+a^2y^2=40$

or, $(a^2+b^2)x^2+(b^2+a^2)y^2=40$

or, $(a^2+b^2)(x^2+y^2)=40$

or, $(a^2+b^2)\times 4=40$

or, $a^2+b^2=10$

24. **(d)** Adding 2 both sides of the equation $a+\dfrac{1}{a+2}=0$

$a+2+\dfrac{1}{a+2}=2$

Cubing, $(a+2)^3+3(a+2)^2\left(\dfrac{1}{a+2}\right)+3(a+2)\left(\dfrac{1}{a+2}\right)^2+\left(\dfrac{1}{a+2}\right)^3=8$

or, $(a+2)^3+3\left\{(a+2)+\dfrac{1}{a+2}\right\}+\dfrac{1}{(a+2)^3}=8$

or, $(a+2)^3+3\times 2+\dfrac{1}{(a+2)^3}=8$

$\therefore\ (a+2)^3+\dfrac{1}{(a+2)^3}=8-6=2$

25. (b) $\because$ $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

$\therefore$ $56 = 2(a^2 + b^2 + ab)$

$\Rightarrow$ $28 = a^2 + b^2 + ab = (a-b)^2 + 3ab$

$\Rightarrow$ $28 = 4 + 3ab$

$\Rightarrow$ $ab = 8$

Now, $a^2 + b^2 = (a-b)^2 + 2ab = 2^2 + 2 \times 8 = 20$

26. (c) $a + \frac{1}{a} = 1$ Cubing, $a^3 + 3a + 3 \cdot \frac{1}{a} + \frac{1}{a^3} = 1$

or, $a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = 1$

or, $a^3 + \frac{1}{a^3} + 3 \times 1 = 1$

or, $t + \frac{1}{t} = 1 - 3 = -2$ (where $t = a^3$)

or, $t^2 + 1 + 2t = 0$

$\Rightarrow$ $(t+1)^2 = 0$

$\Rightarrow$ $t = -1$

or, $a^3 = -1$

27. (*) $(a-b) + (b-c) = 3 + 5 = 8$

or, $a - c = 8$ or, $c - a = -8$

but in next relation $c - a = 1$

$\therefore$ question is wrong

28. (b) $x = y \Rightarrow 2t = \frac{2t-1}{3}$

$\Rightarrow 6t = 2t - 1$

$\Rightarrow t = \frac{-1}{4}$

29. (a) We know that, $(x-y)^2 \geq 0$

or, $x^2 + y^2 - 2xy \geq 0$

or, $(x+y)^2 - 4xy \geq 0$

or, $64 - 4xy \geq 0$

or, $xy \leq 16$

★★★

02

Indices and Surds

e $t = a^3$)

Meaning of index :

1. $a \times a \times a \times a \ldots$ to m terms is written as a^m. It is read as a raise to power m.

   Clearly $(a \times a \times a \times \ldots$ to m term$) \times (a \times a \times a \ldots$to n term$)$

   $= a \times a \times a \times \ldots$ to $(m + n)$ terms

   $\therefore \boxed{a^m \times a^n = a^{m+n}}$

   $$\frac{a \times a \times a \times \ldots \text{ to } m \text{ terms}}{a \times a \times a \times \ldots \text{ to } n \text{ terms}} = a \times a \times a \times \ldots \text{ to } m - n \text{ terms}$$

   $\therefore \boxed{\frac{a^m}{a^n} = a^{m-n}}$ etc.

   Similarly more formulae can be established on index which are given below.

2. Important formulae about indices :

   If $a > 0, a \neq 1$, m and n are integers then

   2.1. $a^m \times a^n = a^{m+n}$

   2.2. $a^m \times a^n \times a^p = a^{m+n+p}$

   2.3. $(a^m)^n = a^{mn}$

   2.4. $\frac{a^m}{a^n} = a^{m-n}$

   2.5. $a^0 = 1$

   2.6. $a^{-m} = \frac{1}{a^m}$

   2.7. (i) $a^{m^n} = a^{(m^n)}$ (ii) $a^{m^{n^p}} = a^{m^{(n^p)}} = a^{\left(m^{(n^p)}\right)}$

   2.8. $(ab)^n = a^n b^n$, $(abc)^n = a^n b^n c^n$

3. Surd : If square root, cube root etc. of a number cannot be expressed in the rational form $\left(\frac{p}{q}, q \neq 0\right)$ then it is called a surd.

   e.g. $\sqrt{2}, \sqrt[3]{4}, \sqrt[4]{18}$, etc are surds as they cannot be expressed as a rational number. Contrary to this $\sqrt[3]{27}, \sqrt[2]{25}, \sqrt[4]{\frac{162}{32}}$ etc. are not surds as their values are respectively $3, 5, \frac{3}{2}$ and they are rational numbers. Clearly

every number expressed in a surd is an irrational number $\sqrt{2}$ is also written as $2^{1/2}$, $\sqrt[3]{4}$ is written as $(4)^{1/3}$, $\sqrt[4]{18}$ is written $(18)^{1/4}$ etc.

4. Type of surds :

4.1. Pure Surd : $\sqrt{7}, \sqrt[3]{11}, \sqrt[4]{25}$ etc. are pure surds.

4.2. Mixed Surd : $3\sqrt{2}, 7\sqrt[2]{11}, \sqrt{32} = 4\sqrt{2}$ etc. are mixed surds.

4.3. Similar Surds: Two or more surds are said to be similar surds if their surd part (irrational part) are same. e.g. $\sqrt{27}$ and $\sqrt{75}$ are similar surds as $\sqrt{27} = 3\sqrt{3}$ and $\sqrt{75} = 5\sqrt{3}$ and their surd part '$\sqrt{3}$' is same.

$\sqrt{12}$ and $\sqrt{8}$ are distinct surds as $\sqrt{12} = 2\sqrt{3}$ and $\sqrt{8} = 2\sqrt{2}$ and their surds part are respectively $\sqrt{3}$ and $\sqrt{2}$.

5. Order of the surd : $\sqrt{7}, \sqrt[3]{8}, \sqrt[4]{9}, \sqrt[5]{15}$ etc. are respectively surds of order two three, four, five.

$3^{3/2}$ is a surd of order 2 while $3^{2/3}$ is a surd of order 3.

6. Conjugate of surds : If a and b are rational numbers where b is not a perfect square then conjugate of $a + \sqrt{b}$ is $a - \sqrt{b}$.

e.g. conjugate of $3 + \sqrt{5}$ is $3 - \sqrt{5}$; conjugate of $\sqrt{6} - 4$ is $-\sqrt{6} - 4$.

Be careful that sign of surd part should be changed while writing conjugate.

7. Condition for two surds to be equal : If a, b, c, d are all rational numbers and b and d are not perfect square then $\boxed{a + \sqrt{b} = c + \sqrt{d} \Leftrightarrow a = c \text{ and } b = d}$

Thus two surds are said to be equal if their rational parts are equal and their irrational part are also equal.

8. Rationalization of Denominator of a surd :

To rationalize a surd, whose denominator is the of the form $a + \sqrt{b}$

9. Square root of surd of $a+\sqrt{b}$ form :

$$\sqrt{a+\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}+\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

and $$\sqrt{a-\sqrt{b}}=\sqrt{\frac{a+\sqrt{a^2-b}}{2}}-\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

10. Some important formulae for surd : **Formula of surd is same as that of Indices. If p, q, r, s are rational numbers then**

10.1. $a^p \times a^q = a^{p+q}$

10.2. $\dfrac{a^p}{a^q} = a^{p-q}$

10.3. $\left(a^p\right)^q = a^{pq}$

10.4. $a^{-p} = \dfrac{1}{a^p}$

10.5. **If** $a^n = y$ **then** $a = y^{1/n}$

10.6. **If** $a^x = b^y$ **then** $a = b^{y/x}$

10.7. **If** $a^x = b^y$ **then** $a^{1/y} = b^{1/x}$ **etc.**

11. Formulae in Radical Notations :

11.1. $x^n = a \Leftrightarrow x = \sqrt[n]{a}$, $(a \in R, a \geq 0)$

11.2. **If** n **is an odd positive integer and** $a > 0$ **then** $\sqrt[n]{-a} = -\sqrt[n]{a}$
(here $m, n \geq 2$, **and** $a, b > 0$ **then)**

11.3. $\sqrt[n]{a} = a^{\frac{1}{n}}$

11.4. $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} = a^{\frac{m}{n}}$

11.5. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

11.6. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

11.7. $\sqrt[n]{\sqrt[m]{a}} = \left(a^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{1}{mn}} = \sqrt[mn]{a}$

11.8. $\sqrt[n]{a} \cdot \sqrt[m]{a} = a^{\frac{1}{n}} \times a^{\frac{1}{m}} = a^{\frac{1}{n}+\frac{1}{m}} = a^{\frac{m+n}{mn}} = \sqrt[mn]{a^{m+n}}$

11.9. $\dfrac{\sqrt[n]{a}}{\sqrt[m]{a}} = \dfrac{a^{\frac{1}{n}}}{a^{\frac{1}{m}}} = a^{\frac{1}{n}-\frac{1}{m}} = a^{\frac{m-n}{mn}} = \sqrt[mn]{a^{m-n}}$

12. A special property : **If x is a positive integer th** 1

12.1. See solved example 15 to see the application of above property.

12.2. If x is a negative quantity then $x + \frac{1}{x} \le -2$

Solved Examples

1. Find the least positive integer x, y, z greater than one for which $\sqrt{xy^{-2}z^{3}} \div \left(\sqrt[3]{x^{3}y^{2}z^{-3}}\right)^{-1}$ is an integer.

Solution : Given expression $= \dfrac{(xy^{-2}z^{3})^{\frac{1}{2}}}{\left(\left(x^{3}y^{2}z^{-3}\right)^{\frac{1}{3}}\right)^{-1}} = \dfrac{x^{\frac{1}{2}}y^{-1}z^{\frac{3}{2}}}{(xy^{2/3}z^{-1})^{-1}}$

$$= (x^{1/2}y^{-1}z^{3/2}) \times (xy^{2/3}z^{-1})$$

$$= x^{\frac{1}{2}+1}y^{-1+\frac{2}{3}}z^{\frac{3}{2}-1}$$

$$= x^{\frac{3}{2}}y^{\frac{-1}{3}}z^{\frac{1}{2}} = \frac{\left(x^{3}\right)^{\frac{1}{2}}\left(z^{\frac{1}{2}}\right)}{y^{\frac{1}{3}}}$$

Clearly it is an integer if least value of x, y, z (> 1) are respectively $x = 4, z = 4, y = 8$.

For these values $= \dfrac{(64)^{\frac{1}{2}}4^{\frac{1}{2}}}{8^{\frac{1}{3}}} = \dfrac{8 \times 2}{2} = 8$ which is an integer.

2. If $a^{b} = b^{a}$ then prove that $\left(\frac{a}{b}\right)^{\frac{a}{b}} = a^{\frac{a}{b}-1}$

Solution : $a^{b} = b^{a} \Rightarrow a = \left(b^{a}\right)^{\frac{1}{b}} \Rightarrow a = b^{\frac{a}{b}}$ ... (i)

Now, $\left(\frac{a}{b}\right)^{\frac{a}{b}} = \dfrac{a^{\frac{a}{b}}}{b^{\frac{a}{b}}} = \dfrac{a^{\frac{a}{b}}}{a} = a^{\frac{a}{b}-1}$ ($\because$ from (i) $b^{\frac{a}{b}} = a$)

3. Prove that $\dfrac{1}{1+a^{x-y}+a^{x-z}} + \dfrac{1}{1+a^{y-z}+a^{y-x}} + \dfrac{1}{1+a^{z-x}+a^{z-y}} = 1$

Solution : First term $= \dfrac{1}{1+a^{x-y}+a^{x-z}} \times \dfrac{a^{-x}}{a^{-x}} = \dfrac{a^{-x}}{a^{-x}+a^{-y}+a^{-z}}$

Second term $= \dfrac{1}{1+a^{y-z}+a^{y-x}} \times \dfrac{a^{-y}}{a^{-y}} = \dfrac{a^{-y}}{a^{-y}+a^{-z}+a^{-x}}$

Third term $= \dfrac{1}{1+a^{z-x}+a^{z-y}} \times \dfrac{a^{-z}}{a^{-z}} = \dfrac{a^{-z}}{a^{-z}+a^{-x}+a^{-y}}$

Here, denominator of each term is same, so adding them

Given expression $= \dfrac{a^{-x}+a^{-y}+a^{-z}}{a^{-x}+a^{-y}+a^{-z}} = 1$

4. If $x^{\frac{1}{m}} = y^{\frac{1}{n}} = z^{\frac{1}{p}}$ and $xyz = 1$ then prove that $m + n + p = 0$

Solution : Let $x^{\frac{1}{m}} = y^{\frac{1}{n}} = z^{\frac{1}{p}} = k$

Then $x = k^m, y = k^n, z = k^p$

Now, from $xyz = 1$

$$k^m \times k^n \times k^p = 1$$

or, $k^{m+n+p} = 1 = k^0$

$\therefore \quad m + n + p = 0$

5. If $x = \dfrac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} - \sqrt{a-2b}}$ then prove that $bx^2 - ax + b = 0$

Solution : On rationalization,

$$x = \frac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} - \sqrt{a-2b}} \times \frac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} + \sqrt{a-2b}}$$

$$= \frac{(a+2b) + (a-2b) + 2\sqrt{(a+2b)(a-2b)}}{(a+2b) - (a-2b)}$$

$$= \frac{2a + 2\sqrt{a^2 - 4b^2}}{4b} = \frac{a + \sqrt{a^2 - 4b^2}}{2b}$$

or, $2bx = a + \sqrt{a^2 - 4b^2}$

or, $2bx - a = \sqrt{a^2 - 4b^2}$

Squaring both sides

$$4b^2x^2 + a^2 - 2.2bx.a = a^2 - 4b^2$$

or, $4b^2x^2 - 4abx = -4b^2$

or, $4(b^2x^2 - abx + b^2) = 0$

or, $4b(bx^2 - ax + b) = 0$

or, $bx^2 - ax + b = 0$

6. If $\dfrac{A}{a} = \dfrac{B}{b} = \dfrac{C}{c} = \dfrac{D}{d}$ then prove that

$$\sqrt{Aa} + \sqrt{Bb} + \sqrt{Cc} + \sqrt{Dd} = \sqrt{(a+b+c+d)(A+B+C+D)}$$

Solution : Let $\dfrac{A}{a} = \dfrac{B}{b} = \dfrac{C}{c} = \dfrac{D}{d} = k$ then

$$A = ak, B = bk, C = ck \text{ and } D = dk$$

$\therefore \quad \sqrt{Aa} + \sqrt{Bb} + \sqrt{Cc} + \sqrt{Dd} = \sqrt{aka} + \sqrt{bkb} + \sqrt{ckc} + \sqrt{dkd}$

$= \sqrt{k}\ (a + b + c + d)$ ... (i

Again, $\sqrt{(a+b+c+d)(A+B+C+D)} = \sqrt{(a+b+c+d)(ak+bk+ck+dk)}$

$$= \sqrt{(a+b+c+d)k(a+b+c+d)}$$

$$= \sqrt{k}\ (a+b+c+d)$$

From (i) and (ii) $\sqrt{Aa}+\sqrt{Bb}+\sqrt{Cc}+\sqrt{Dd} = \sqrt{(a+b+c+d)(A+B+C+D)}$

7. If $x = \frac{1}{2}\left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)$ then prove that $\frac{2a\sqrt{1+x^2}}{x+\sqrt{1+x^2}} = a+b$

Solution : $\frac{2a\sqrt{1+x^2}}{x+\sqrt{1+x^2}} = \frac{2a\sqrt{1+x^2}}{x+\sqrt{1+x^2}} \times \frac{x-\sqrt{1+x^2}}{x-\sqrt{1+x^2}}$

$$= \frac{2a\sqrt{1+x^2}\left(x-\sqrt{1+x^2}\right)}{x^2-(1+x^2)}$$

$$= -2ax\sqrt{1+x^2}+2a(1+x^2)$$

But squaring $x = \frac{1}{2}\left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)$

$$x^2 = \frac{1}{4}\left(\frac{a}{b}+\frac{b}{a}-2\sqrt{\frac{a}{b}}\sqrt{\frac{b}{a}}\right) = \frac{1}{4}\left(\frac{a}{b}+\frac{b}{a}-2\right)$$

$\therefore\quad 1+x^2 = 1+\frac{1}{4}\left(\frac{a}{b}+\frac{b}{a}-2\right) = \frac{1}{4}\left(\frac{a}{b}+\frac{b}{a}+2\right) = \frac{1}{4}\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)^2$

or, $\sqrt{1+x^2} = \frac{1}{2}\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)$

Now from (i)

Given expression $= -2a\frac{1}{2}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)\frac{1}{2}\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)+2a\frac{1}{4}\cdot\left(\frac{a}{b}+\frac{b}{a}+2\right)$

$$= -2a\frac{1}{4}\left(\frac{a}{b}-\frac{b}{a}\right)+2a\cdot\frac{1}{4}\left(\frac{a}{b}+\frac{b}{a}+2\right) = 2a\left(\frac{1}{2}+\frac{1}{2}\cdot\frac{b}{a}\right) = a$$

8. Simplify $\frac{(\sqrt{3}+\sqrt{5})(\sqrt{5}+\sqrt{2})}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

Solution : Given expression $= \frac{(\sqrt{3}+\sqrt{5})(\sqrt{5}+\sqrt{2})}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

$$= \frac{(\sqrt{3}+\sqrt{5}+\sqrt{2}-\sqrt{2})(\sqrt{5}+\sqrt{2})}{\sqrt{2}+\sqrt{3}+\sqrt{5}} = \left(1-\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}\right)(\sqrt{5}+\sqrt{2})$$

$$= \left\{1-\frac{\sqrt{2}}{(\sqrt{5}+\sqrt{2})+\sqrt{3}} \times \frac{(\sqrt{5}+\sqrt{2})-\sqrt{3}}{(\sqrt{5}+\sqrt{2})-\sqrt{3}}\right\}(\sqrt{5}+\sqrt{2})$$

$$= \left\{1 - \frac{\sqrt{2} \times ((\sqrt{5}+\sqrt{2}) - \sqrt{3})}{(\sqrt{5}+\sqrt{2})^2 - (\sqrt{3})^2}\right\}(\sqrt{5}+\sqrt{2})$$

$$= \left\{1 - \frac{\sqrt{2}\,((\sqrt{5}+\sqrt{2}) - \sqrt{3})}{5+2+2\sqrt{10}-3}\right\}(\sqrt{5}+\sqrt{2})$$

$$= \left\{1 - \frac{\sqrt{2}\,((\sqrt{5}+\sqrt{2}) - \sqrt{3})}{4+2\sqrt{10}}\right\}(\sqrt{5}+\sqrt{2})$$

$$= \left\{\frac{4+2\sqrt{10} - \sqrt{2}\,((\sqrt{5}+\sqrt{2}) - \sqrt{3}}{4+2\sqrt{10}}\right\}(\sqrt{5}+\sqrt{2})$$

$$= \left\{\frac{4+2\sqrt{10} - \sqrt{10} - 2 + \sqrt{6}}{2\sqrt{2}\,(\sqrt{2}+\sqrt{5})}\right\}(\sqrt{5}+\sqrt{2})$$

$$= \frac{2+\sqrt{10}+\sqrt{6}}{2\sqrt{2}} = \frac{\sqrt{2}\,(\sqrt{2}+\sqrt{5}+\sqrt{3})}{2\sqrt{2}} = \frac{1}{2}(\sqrt{2}+\sqrt{3}+\sqrt{5})$$

9. Find the square root of $6 - \sqrt{35}$

Solution : First method, $6 - \sqrt{35} = 6 - \sqrt{5}\,\sqrt{7} = \frac{1}{2}(12 - 2\sqrt{5}\,\sqrt{7})$

$$= \frac{1}{2}(5+7-2\sqrt{5}\,\sqrt{7}) = \frac{1}{2}(\sqrt{7}-\sqrt{5})^2$$

$\therefore$ Required square root $= \pm\,\frac{1}{\sqrt{2}}(\sqrt{7}-\sqrt{5})$

Second Method :

$$\therefore \quad a - \sqrt{b} = \left(\sqrt{\frac{a+\sqrt{a^2-b}}{2}} - \sqrt{\frac{a-\sqrt{a^2-b}}{2}}\right)^2$$

$$\therefore \quad 6 - \sqrt{35} = \left(\sqrt{\frac{6+\sqrt{36-35}}{2}} - \sqrt{\frac{6-\sqrt{36-35}}{2}}\right)^2 = \left(\sqrt{\frac{7}{2}} - \sqrt{\frac{5}{2}}\right)^2$$

$\therefore$ Square root $6 - \sqrt{35} = \pm\left(\sqrt{\frac{7}{2}} - \sqrt{\frac{5}{2}}\right)$

10. Find the value of $\sqrt{6-\sqrt{35}}$

12. Find the square root of $21-4\sqrt{5}+8\sqrt{3}-4\sqrt{15}$

Solution : Let $21-4\sqrt{5}+8\sqrt{3}-4\sqrt{15}=(\sqrt{x}+\sqrt{y}-\sqrt{z})^2$

$\therefore\ 21-4\sqrt{5}+8\sqrt{3}-4\sqrt{15}=x+y+z+2\sqrt{xy}-2\sqrt{xz}-2\sqrt{yz}$

$\therefore\ x+y+z=21$

$2\sqrt{xy}=8\sqrt{3}$

$2\sqrt{yz}=4\sqrt{5}$

$2\sqrt{zx}=4\sqrt{15}$

Multiplying these three terms

$2\times2\times2\,xyz=8\times4\times4\sqrt{3}\sqrt{5}\sqrt{15}$

$xyz=4\times4\times15=240$

$\Rightarrow\ \sqrt{xyz}=\sqrt{240}=4\sqrt{15}$

$\therefore\ \sqrt{x}=\dfrac{4\sqrt{15}}{\sqrt{yz}}=2\sqrt{3}\qquad \sqrt{y}=\dfrac{4\sqrt{15}}{\sqrt{zx}}=2\qquad \sqrt{z}=\dfrac{4\sqrt{15}}{\sqrt{xy}}=\sqrt{5}$

These values also satisfy $x+y+z=21$

Hence required square root $\pm(2\sqrt{3}+2-\sqrt{5})$

13. Find the cube root of $72-32\sqrt{5}$

Solution : Let $(72-32\sqrt{5})^{1/3}=x-\sqrt{y}$

$\therefore\ (72+32\sqrt{5})^{1/3}=x+\sqrt{y}$ (conjugate prope...

On multiplication $(72^2-(32\sqrt{5})^2)^{1/3}=x^2-y$

or, $(5184-1024\times5)^{1/3}=x^2-y$

or, $(64)^{1/3}=x^2-y$

or, $x^2-y=4$ ...

Again from (i), $72-32\sqrt{5}=(x-\sqrt{y})^3$

or, $72-32\sqrt{5}=x^3-3x^2\sqrt{y}+3xy-y\sqrt{y}$

or, $72-32\sqrt{5}=x^3+3xy-(3x^2+y)\sqrt{y}$ ...

From irrational part $y=5$

From (ii), $x^2-5=4\Rightarrow x^2=9\Rightarrow x=\pm3$

Here, $x=3, y=5$, also satisfy equation (iii).

$\therefore$ Required cube root $=3-\sqrt{5}$

14. Express $\dfrac{(4+\sqrt{15})^{3/2}+(4-\sqrt{15})^{3/2}}{(6+\sqrt{35})^{3/2}-(6-\sqrt{35})^{3/2}}$ in rational form.

Solution : $\because \quad 4+\sqrt{15}=\frac{1}{2}(3+5+2\sqrt{15})=\frac{1}{2}\,(3+5+2\sqrt{15})=\frac{1}{2}\left(\sqrt{3}+\sqrt{5}\right)^2$

$6+\sqrt{35}=\frac{1}{2}(12+2\sqrt{35})=\frac{1}{2}(7+5+2\sqrt{7}\,\sqrt{5})=\frac{1}{2}(\sqrt{7}+\sqrt{5})^2$ etc.

Hence, Given expression $=\dfrac{\left\{\frac{1}{2}(\sqrt{5}+\sqrt{3})^2\right\}^{\frac{3}{2}}+\left\{\frac{1}{2}(\sqrt{5}-\sqrt{3})^2\right\}^{\frac{3}{2}}}{\left\{\frac{1}{2}\left(\sqrt{7}+\sqrt{5}\right)^2\right\}^{\frac{3}{2}}-\left\{\frac{1}{2}\left(\sqrt{7}-\sqrt{5}\right)^2\right\}^{\frac{3}{2}}}$

$$=\frac{(\sqrt{5}+\sqrt{3})^3+(\sqrt{5}-\sqrt{3})^3}{(\sqrt{7}+\sqrt{5})^3-(\sqrt{7}-\sqrt{5})^3}$$

$$=\frac{2((\sqrt{5})^3+3\sqrt{5}\,(\sqrt{3})^2)}{2(3\left(\sqrt{7}\right)^2\sqrt{5}+\left(\sqrt{5}\right)^3)}$$

$(\because\ (a+b)^3+(a-b)^3=2\,(a^3+3ab^2)$ etc)

$$=\frac{5\sqrt{5}+9\sqrt{5}}{21\sqrt{5}+5\sqrt{5}}=\frac{5+9}{21+5}=\frac{14}{26}=\frac{7}{13}$$

15. Solve the equation $\sqrt{3-x^4+2x^2}=x^2+\frac{1}{x^2}$ for real value of x.

Solution : $\because$ Given equation is $\sqrt{3-x^4+2x^2}=x^2+\frac{1}{x^2}$

or, $\sqrt{4-\left(x^4-2x^2+1\right)}=x^2+\frac{1}{x^2}$

or, $\sqrt{4-\left(x^2-1\right)^2}=x^2+\frac{1}{x^2}$

Since value of $(x^2-1)^2$ is zero or greater than zero

therefore $\sqrt{4-\left(x^2-1\right)^2}\le\sqrt{4-0}$

i.e. LHS $\le\sqrt{4}\le 2$

Also, RHS $=x^2+\frac{1}{x^2}=\left(x-\frac{1}{x}\right)^2+2$

$\because\ \left(x-\frac{1}{x}\right)^2\ge 0$

$\therefore$ RHS ≥ 2

Hence both LHS and RHS are equal if each of them is 2, which occurs at $x=\pm 1$, which is required solution

Exercise—2A

1. Simplest form of $\sqrt[4p]{z^6} \div \sqrt[2p]{z^{-5}}$ is.

(a) $\sqrt[4]{z^p}$ (b) $\sqrt[p]{z^4}$ (c) $\sqrt{z^{4p}}$ (d) $\sqrt{z^{\frac{p}{4}}}$

2. Which among the following is simplest form of $\sqrt[4]{x^{-3}} \div \left(\sqrt[8]{x}\right)^{12}$?

(a) $x^{\frac{4}{3}}$ (b) $\sqrt[4]{x^5}$ (c) $\sqrt[4]{x^{9}}$ (d) $\sqrt[4]{x^3}$

3. For what value of a, b, c expression $\sqrt{a^3 b^{-2/3} c^{-7/6}} \div \sqrt[3]{a^4 b^{-1} c^{3/4}}$ is a integer ?

(a) $a = 16, b = 2, c = 4$ (b) $a = 32, b = 3, c = \frac{1}{2}$

(c) $a = 729, b = \frac{1}{3}, c = \frac{1}{6}$ (d) $a = 64, b, = 4, c = 3$

4. If $a^b = b^a$ and $a = 2b$ then value of b is

(a) 1 (b) -1 (c) 2 (d) -2

5. On simplification $\left(\frac{a^p}{a^q}\right)^{p+q} \times \left(\frac{a^q}{a^r}\right)^{q+r} \times \left(\frac{a^r}{a^p}\right)^{r+p}$ yeilds

(a) a^{p+q+r} (b) $a^{pq+qr+rp}$ (c) 0 (d) 1

6. Expression $\left(\frac{a^x}{a^y}\right)^{x^2+xy+y^2} \times \left(\frac{a^y}{a^z}\right)^{y^2+yz+z^2} \times \left(\frac{a^z}{z^x}\right)^{z^2+zx+x^2}$ in its simplest form is

(a) a (b) $\frac{1}{a}$ (c) 0 (d) 1

7. Expression $\dfrac{\left(a+\frac{1}{b}\right)^{\frac{1}{x}}\left(a-\frac{1}{b}\right)^{\frac{1}{x}}}{\left(b+\frac{1}{a}\right)^{\frac{1}{x}}\left(b-\frac{1}{a}\right)^{\frac{1}{x}}}$ is an integer if

(a) $a = 4, b = 2, x = 4$ (b) $a = 16, b = 2, x = 4$

(c) $a = 64, b = 2, x = 8$ (d) $a = 16, b = 4, x = 4$

8. The value of

$$\frac{x^d}{x^d + x^{d+b-a} + x^{d+c-a}} + \frac{x^d}{x^d + x^{d+a-b} + x^{d+c-b}} + \frac{x^d}{x^d + x^{d+a-c} + x^{d+b-c}} \text{ is}$$

(a) x^d (b) x^{a+b+c} (c) x^{-d} (d) 1

9. If $x = \sqrt[3]{a} - \frac{1}{\sqrt[3]{a}}$ then value of $x^3 + 3x$ is

(a) $a + a^{-1}$ (b) $a - a^{-1}$ (c) $a^2 + a^{-2}$ (d) 0

10. If $a=2+2^{\frac{1}{3}}+2^{\frac{2}{3}}$ then value of a^3-6a^2+6a is
(a) 2 (b) -2 (c) 4 (d) -4

11. If $a^x=b^y=c^z$ and $abc=1$ then value of $xy+yz+zx$ is
(a) abc (b) 1 (c) 0 (d) -1

12. Value of $\sqrt[yz]{\frac{a^{\frac{y}{z}}}{a^{\frac{z}{y}}}} \times \sqrt[zx]{\frac{a^{\frac{z}{x}}}{a^{\frac{x}{z}}}} \times \sqrt[xy]{\frac{a^{\frac{x}{y}}}{a^{\frac{y}{x}}}}$ is
(a) a^{xyz} (b) $a^{\frac{1}{xyz}}$ (c) 1 (d) a

13. If $a^{m^n}=\left(a^m\right)^n$ then which of the following relation is true ?
(a) $m=n^{\frac{1}{n-1}}$ (b) $n=m^{\frac{1}{m-1}}$ (c) $m=n^{n-1}$ (d) $n=m^{m-1}$

14. Simplified value of $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ is
(a) $\sqrt{2}-\sqrt{3}+\sqrt{6}$ (b) $4\sqrt{3}-6\sqrt{2}$
(c) 1 (d) None of these

15. Value of $\sqrt{4+\sqrt{15}}$ is
(a) $\frac{\sqrt{5}-\sqrt{3}}{2}$ (b) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{2}}$
(c) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{5}+\sqrt{3}}{2}$

16. If $n=7+4\sqrt{3}$ then value of $\left(\sqrt{n}+\frac{1}{\sqrt{n}}\right)$ is
(a) 2 (b) $2\sqrt{3}$ (c) 4 (d) $4\sqrt{3}$

17. Square root of $\frac{3}{2}(x-1)+\sqrt{2x^2-7x-4}$ is
(a) $\frac{1}{2}\left(\sqrt{2x+1}+\sqrt{x-4}\right)$ (b) $\frac{1}{\sqrt{2}}\left(\sqrt{2x+1}+\sqrt{x-4}\right)$
(c) $\frac{1}{\sqrt{2}}\left(\sqrt{2x-1}+\sqrt{x+4}\right)$ (d) $\frac{1}{2}\left(\sqrt{2x-1}+\sqrt{x+4}\right)$

18. Square root of $2a-\sqrt{3a^2-2ab-b^2}$, $(a>b>0)$ is
(a) $\frac{1}{\sqrt{2}}\left(\sqrt{3a+b}+\sqrt{a-b}\right)$ (b) $\frac{1}{\sqrt{2}}\left(\sqrt{3a-b}+\sqrt{a+b}\right)$
(c) $\frac{1}{\sqrt{2}}\left(\sqrt{3a+b}-\sqrt{a-b}\right)$ (d) $\frac{1}{\sqrt{2}}\left(\sqrt{3a-b}-\sqrt{a+b}\right)$

19. Value of $\sqrt{28-6\sqrt{3}}+\sqrt{28+6\sqrt{3}}$ is
(a) $6\sqrt{3}$ (b) 2 (c) $-6\sqrt{3}$ (d) -2

20. Value of $\sqrt{1+x^2+\left(1+x^2+x^4\right)^{\frac{1}{2}}}$ is

(a) $\frac{1}{\sqrt{2}}\left(\sqrt{1+x+x^2}+\sqrt{1-x+x^2}\right)$
(b) $\frac{1}{\sqrt{2}}\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)$
(c) $\frac{1}{\sqrt{2}}\left(\sqrt{1+x+x^2}-\sqrt{1+x-x^2}\right)$
(d) None of these

21. Square root of $x+y+z+2\sqrt{xz+yz}$ is

(a) $\sqrt{x}+\sqrt{y+z}$
(b) $\sqrt{x+y+z}$
(c) $\sqrt{x+y}+\sqrt{z}$
(d) None of these

22. If $\sqrt{3x-7}+\sqrt{3x+7}=4+\sqrt{2}$ then value of $x+\frac{1}{x}$ is

(a) $\frac{82}{9}$ (b) $\frac{10}{3}$ (c) $\frac{5}{2}$ (d) $\frac{4}{3}$

23. Square root of $6+\sqrt{12}-\sqrt{24}-\sqrt{8}$ is

(a) $\sqrt{3}+\sqrt{2}-1$
(b) $\sqrt{3}+1-\sqrt{2}$
(c) $1+\sqrt{2}-\sqrt{3}$
(d) None of these

24. If $a=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $b=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ then value of $\sqrt{3a^2-5ab+3b^2}$ is

(a) 289 (b) 17 (c) $\sqrt{17}$ (d) $17\sqrt{17}$

25. Simplest form of $\left(\frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}\right)^2$ is

(a) 3 (b) 9 (c) $\frac{1}{9}$ (d) $\frac{1}{3}$

26. Expression $\frac{12}{3+\sqrt{5}+2\sqrt{2}}$ is equal to

(a) $-\sqrt{5}+\sqrt{2}+\sqrt{10}$
(b) $1+\sqrt{5}+\sqrt{2}-\sqrt{10}$
(c) $1+\sqrt{5}-\sqrt{2}-\sqrt{10}$
(d) $1-\sqrt{5}-\sqrt{2}+\sqrt{10}$

27. Number of real root of the equation $x^2+\frac{1}{\sqrt{x^2-x+1}}+\sqrt{x^2-x+1}=2$

(a) 0 (b) 1 (c) 2 (d) 4

28. If $x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ then x^2+xy+y^2 is a multiple of

(a) 11
(b) 3
(c) 9
(d) All (a), (b) and (c)

29. If $(\sqrt{5}-\sqrt{2})p=\sqrt{5}+\sqrt{2}$ and $pq=(pq)^{3}$, then the value of $3p^2+4pq-3q^2$ is $(pq\neq 0)$

(a) $\frac{1}{9}(12+56\sqrt{10})$
(b) $\frac{1}{9}(12-56\sqrt{10})$
(c) $\frac{1}{3}(12-56\sqrt{10})$
(d) $\frac{1}{3}(12+56\sqrt{10})$

30. If $\sqrt{10+\sqrt{24}+\sqrt{40}+\sqrt{60}}=\sqrt{p}+\sqrt{q}+\sqrt{r}$ then value of $p+q+r$ is

(a) $\sqrt{10}$ (b) 10 (c) 11 (d) $\sqrt{10}$

31. The irrational part of cube root $72-32\sqrt{5}$ is

(a) $-2\sqrt{5}$ (b) $-\sqrt{5}$ (c) $2\sqrt{5}$ (d) $\sqrt{5}$

32. Value of $\frac{1}{\sqrt{11-2\sqrt{30}}}-\frac{3}{7-2\sqrt{10}}-\frac{4}{\sqrt{8-4\sqrt{3}}}$ is

(a) 0 (b) 1 (c) 2 (d) 4

33. Value of $\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\frac{1}{3}$ (d) 3

34. If $a=\sqrt{\frac{\sqrt{5}+1}{\sqrt{5}-1}}$ then $\sqrt{a^2-a-1}$ is equal to

(a) 5 (b) $\sqrt{5}$ (c) 0 (d) 1

35. If $\left(a+\sqrt{a^2+b^3}\right)^{\frac{1}{3}}+\left(a-\sqrt{a^2+b^3}\right)^{\frac{1}{3}}$ then what is the value of $x^3+3bx-2a$?

(a) $2a^2$ (b) $-2a^3$ (c) 1 (d) 0

36. Value of $\sqrt{139-80\sqrt{3}}$ is

(a) $5\sqrt{3}-8$ (b) $8-5\sqrt{3}$ (c) $\pm(8-5\sqrt{3})$ (d) $(16-5\sqrt{3})$

37. If $(a+3)\sqrt{2}+3=b\sqrt{8}+a-1$ then value of $a+b$ is

(a) 3 (b) 6 (c) $\frac{13}{2}$ (d) $\frac{15}{2}$

38. If $x>2$ then what is the value of $\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}$?

(a) 2 (b) $2\sqrt{x-1}$ (c) -2 (d) $-2\sqrt{x-1}$

39. If $1<x<2$ then what is the value of $\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}$?

(a) 2 (b) $2\sqrt{x-1}$ (c) $2+2\sqrt{x-1}$ (d) $2-2\sqrt{x-1}$

40. If $x = \frac{\sqrt{3}}{2}$ then what is the value of $\frac{1+x}{1+\sqrt{1+x}} + \frac{1-x}{1-\sqrt{1-x}}$?

(a) 1 (b) 2 (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

41. If $\frac{x}{y} = \frac{y}{z} = \frac{z}{w}$ then $\frac{x^m + y^m + z^m + w^m}{x^{-m} + y^{-m} + z^{-m} + w^{-m}}$ is equals to—

(a) 0 (b) 1 (c) $(xyzw)^m$ (d) $(xyzw)^{m/2}$

42. Which of the following quantity is an integer ?

(a) $[(\sqrt{2}+\sqrt{3})/(\sqrt{3}-\sqrt{2})] + \sqrt{6}$ (b) $[(\sqrt{2}+\sqrt{3})/(\sqrt{3}-\sqrt{2})] + 2\sqrt{6}$

(c) $[(\sqrt{2}+\sqrt{3})/(\sqrt{2}-\sqrt{3})] + 2\sqrt{6}$ (d) $[(\sqrt{2}+\sqrt{3})/(\sqrt{2}-\sqrt{3})] + \sqrt{6}$

43. What is the real value of $(256)^{0.16} \times (16)^{0.18}$?

(a) 2 (b) 4 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

44. $2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} - \frac{1}{2-\sqrt{2}}$ is equals to—

(a) 2 (b) $2-\sqrt{2}$ (c) $4+\sqrt{2}$ (d) $2\sqrt{2}$

45. If $x = \frac{2ab}{b^2+1}$ then value of $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}$ is

(a) $a+b$ (b) $a-b$ (c) a (d) b

Answers—2A

1. (b)	2. (d)	3. (c)	4. (c)	5. (d)	6. (d)	7. (d)	8. (d
9. (a)	10. (a)	11. (c)	12. (c)	13. (a)	14. (d)	15. (c)	16. (c
17. (b)	18. (c)	19. (a)	20. (a)	21. (c)	22. (b)	23. (b)	24. (b
25. (d)	26. (b)	27. (b)	28. (d)	29. (d)	30. (b)	31. (b)	32. (a
33. (b)	34. (c)	35. (d)	36. (a)	37. (d)	38. (b)	39. (a)	40. (a
41. (d)	42. (c)	43. (b)	44. (a)	45. (d)			

Explanation

1. **(b)** Given Expression $= \frac{(z^6)^{\frac{1}{4p}}}{(z^{-5})^{\frac{1}{2p}}} = z^{\frac{6}{4p}} \times z^{\frac{5}{2p}} = z^{\frac{3+5}{2p}} = z^{\frac{4}{p}} = (z^4)^{\frac{1}{p}} = \sqrt[p]{z^4}$

2. **(d)** Given Expression $= \frac{(x^{-3})^{\frac{1}{4}}}{\left(x^{\frac{1}{8}}\right)^{-12}} = \frac{x^{-\frac{3}{4}}}{x^{-\frac{3}{2}}} = x^{-\frac{3}{4}+\frac{3}{2}} = x^{\frac{3}{4}} = (x^3)^{\frac{1}{4}} = \sqrt[4]{x^3}$

3. (c) Given Expression $= \dfrac{\left(a^{3} b^{-\frac{2}{3}} c^{\frac{-7}{6}}\right)^{\frac{1}{2}}}{\left(a^{4} b^{-1} c^{\frac{5}{4}}\right)^{\frac{1}{3}}}$

$$= \frac{a^{\frac{3}{2}} b^{-\frac{1}{3}} c^{-\frac{7}{12}}}{a^{\frac{4}{3}} b^{-\frac{1}{3}} c^{\frac{5}{12}}}$$

$$= a^{\frac{3}{2}-\frac{4}{3}} c^{-\frac{7}{12}-\frac{5}{12}}$$

$$= a^{\frac{9-8}{6}} c^{\frac{-7-5}{12}} = a^{\frac{1}{6}} c^{-1} = \frac{a^{1/6}}{c}.$$

putting values given in each option one by one, we find that (c) is the correct option

4. (c) Given, $a^{b} = b^{a}$

or, $(2b)^{b} = b^{2b}$ $(\because a = 2b)$

or, $(2b)^{b} = (b^{2})^{b}$

$\therefore$ $2b = b^{2}$

or, $b = 2$

5. (d) Given Expression $= \left(a^{p-q}\right)^{p+q} \times \left(a^{q-r}\right)^{q+r} \times \left(a^{r-p}\right)^{r+p}$

$$= a^{p^2-q^2} \times a^{q^2-r^2} \times a^{r^2-p^2}$$

$$= a^{\left(p^2-q^2\right)+\left(q^2-r^2\right)+\left(r^2-p^2\right)}$$

$$= a^{0} = 1$$

6. (d) Solved as in above question $(x-y)(x^2+xy+y^2) = x^3 - y^3$

7. (d) Given Expression $= \dfrac{\left(\dfrac{ab+1}{b}\right)^{\frac{1}{x}} \left(\dfrac{ab-1}{b}\right)^{\frac{1}{x}}}{\left(\dfrac{ab+1}{a}\right)^{\frac{1}{x}} \left(\dfrac{ab-1}{a}\right)^{\frac{1}{x}}}$

$$= \left(\frac{a}{b}\right)^{\frac{1}{x}} \left(\frac{a}{b}\right)^{\frac{1}{x}} = \left(\frac{a}{b}\right)^{\frac{2}{x}}$$

Clearly it is integer at $a = 16, b = 4, x = 4$

Required value $= \left(\dfrac{16}{4}\right)^{\frac{2}{4}} = 4^{\frac{1}{2}} = 2$

8. (d) Do as in solved example (3). Multiply numerator and Denominator of first term by x^a that of second term by x^b and that of third by x^c etc.

9. (a) Cube both sides

10. (a) Given $a = 2^{\frac{1}{3}} + 2^{\frac{2}{3}} + 2$ or, $a - 2 = 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$

Cubing both sides, $a^3 - 6a^2 + 12a - 8 = 2 + 4 + 3 2^{\frac{1}{3}} 2^{\frac{2}{3}}\left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)$

or, $a^3 - 6a^2 + 12a - 8 = 6 + 3.2(a-2)$

or, $a^3 - 6a^2 + 12a - 8 = 6 + 6a - 12$

or, $a^3 - 6a^2 + 6a = -6 + 8 = 2$

11. (c) Do as in solved example (4)

Here, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$\Rightarrow \frac{yz + zx + xy}{xyz} = 0$

$\Rightarrow xy + yz + zx = 0$

13. (a) $a^{m^n} = \left(a^m\right)^n \Rightarrow a^{m^n} = a^{mn}$

$\Rightarrow m^n = mn$

$\Rightarrow \frac{m^n}{m} = n$

$\Rightarrow m^{n-1} = n$

$\Rightarrow m = n^{\frac{1}{n-1}}$

14. (d) Required value $= \frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} \times \frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

$+ \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{3\sqrt{12} - 3\sqrt{6}}{6-3} - \frac{4\sqrt{18} - 4\sqrt{6}}{6-2} + \frac{\sqrt{18} - \sqrt{12}}{3-2}$

$= \left(\sqrt{12} - \sqrt{6}\right) - \left(\sqrt{18} - \sqrt{6}\right) + \left(\sqrt{18} - \sqrt{12}\right) = 0$

15. (c) $\sqrt{4+\sqrt{15}} = \frac{1}{\sqrt{2}}\sqrt{8 + 2\sqrt{15}}$

$= \frac{1}{\sqrt{2}}\left(5 + 3 + 2\sqrt{15}\right)$

$= \frac{1}{\sqrt{2}}\sqrt{\left(\sqrt{5}+\sqrt{3}\right)^2} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{2}}$

16. (c)

∴

∴

17.

18

16. (c) $n = 7 + 4\sqrt{3} = 4 + 3 + 2\cdot 2\cdot\sqrt{3} = \left(2+\sqrt{3}\right)^2$

$\therefore \quad \sqrt{n} = 2 + \sqrt{3}$

$\frac{1}{n} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{49-48}$

$= 7 - 4\sqrt{3} = \left(2-\sqrt{3}\right)^2$

$\therefore \quad \frac{1}{\sqrt{n}} = 2 - \sqrt{3}$

Hence, $\sqrt{n} + \frac{1}{\sqrt{n}} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

17. (b) Given expression $= \frac{1}{2}\left\{(3x-3) + 2\sqrt{(2x+1)(x-4)}\right\}$

$= \frac{1}{2}\left\{(2x+1) + (x-4) + 2\sqrt{(2x+1)(x-4)}\right\}$

$= \frac{1}{2}\left(\sqrt{2x+1} + \sqrt{x-4}\right)^2$

$\therefore$ Required square root $= \pm\frac{1}{\sqrt{2}}\left(\sqrt{2x+1} + \sqrt{x-4}\right)$

18. (c) Given expression $= 2a - \sqrt{3a^2 - 2ab - b^2}$

$= 2a - \sqrt{(3a+b)(a-b)}$

$= \frac{1}{2}\left\{4a - 2\sqrt{(3a+b)(a-b)}\right\}$

$= \frac{1}{2}(3a+b) + (a-b) - 2\sqrt{(3a+b)(a-b)}$

$= \frac{1}{2}\left(\sqrt{3a+b} - \sqrt{a-b}\right)^2$

$\therefore$ Required square root $= \pm\frac{1}{\sqrt{2}}\left(\sqrt{3a+b} - \sqrt{a-b}\right)$

19. (a) $28 - 6\sqrt{3}$

$= 1 + 3\sqrt{3} - 2\cdot 1\cdot 3\sqrt{3}$

$= \left(3\sqrt{3} - 1\right)^2$ etc.

$\therefore \quad \sqrt{28-6\sqrt{3}} + \sqrt{28+6\sqrt{3}}$

$= \left(3\sqrt{3} - 1\right) + 3\sqrt{3} + 1 = 6\sqrt{3}$

20. (a) $1 + x^2 + \left(1 + x^2 + x^4\right)^{\frac{1}{2}}$

$= 1 + x^2 + \left(\left(1+x^2\right)^2 - x^2\right)^{\frac{1}{2}}$

$$= 1 + x^2 + \left((1+x^2+x)(1+x^2-x)\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(2+2x^2+2\sqrt{1+x^2+x}\sqrt{1+x^2-x}\right)$$

$$= \frac{1}{2}\left\{(1+x^2+x)+(1+x^2-x)+2\sqrt{1+x^2+x}\times\sqrt{1+x^2-x}\right\}$$

$$= \frac{1}{2}\left(\sqrt{1+x+x^2}+\sqrt{1+x^2-x}\right)^2$$

$$\therefore \quad \sqrt{1+x^2+(1+x^2+x^4)^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}\left(\sqrt{1+x+x^2}+\sqrt{1+x^2-x}\right)$$

21. (c) Given expression $= x + y + z + 2\sqrt{(x+y)z}$

$$= (x+y) + z + 2\sqrt{x+y}\sqrt{z}$$

$$= \left(\sqrt{x+y}+\sqrt{z}\right)^2 \text{ etc.}$$

22. (b) $\sqrt{3x-7}+\sqrt{3x+7} = 4+\sqrt{2}$

squaring both sides,

$$3x-7+3x+7+2\sqrt{3x-7}\sqrt{3x+7} = 16+2+8\sqrt{2}$$

or, $6x + 2\sqrt{9x^2-49} = 18+8\sqrt{2}$

equation rational parts, $6x = 18$

$$\Rightarrow \quad x = 3$$

at, $x = 3$, irrational part

$= 2\sqrt{9\times9-49}$

$= 2\sqrt{32} = 8\sqrt{2}$ which is correct

$$\therefore \quad x+\frac{1}{x} = 3+\frac{1}{3} = \frac{10}{3}$$

23. (b) $6+\sqrt{12}-\sqrt{24}-\sqrt{8} = 6+2\sqrt{3}-2\sqrt{6}-2\sqrt{2}$

$$= 6+2\sqrt{3}-2\sqrt{3}\sqrt{2}-2\sqrt{2}$$

$$= 3+2+1+2\sqrt{3}-2\sqrt{3}\sqrt{2}-2\sqrt{2}$$

$$= \left(\sqrt{3}+1-\sqrt{2}\right)^2 \text{ etc.}$$

24. (b) $a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= 5-2\sqrt{6}$ and $b = 5+2\sqrt{6}$

$$\therefore \quad ab = 25-24 = 1$$

Now, $3a$ ... $= 3(a-b)^2 + ab$

$$= 3\left(5-2\sqrt{6}-5-2\sqrt{6}\right)^2+1$$

$$= 3(96) + 1 = 289$$

$\therefore$ $$\sqrt{3a^2-5ab+3b^2} = \sqrt{289} = 17$$

25. (d) $$\sqrt{26-15\sqrt{3}} = \sqrt{\frac{52-30\sqrt{3}}{2}}$$

$$= \sqrt{\frac{\left(3\sqrt{3}-5\right)^2}{2}} = \frac{3\sqrt{3}-5}{\sqrt{2}}$$

$$\sqrt{38+5\sqrt{3}} = \sqrt{\frac{76+10\sqrt{3}}{2}}$$

$$= \sqrt{\frac{\left(5\sqrt{3}+1\right)^2}{2}} = \frac{5\sqrt{3}+1}{\sqrt{2}}$$

Hence given expression $$= \left(\frac{\frac{3\sqrt{3}-5}{\sqrt{2}}}{5\sqrt{2}-\frac{5\sqrt{3}+1}{\sqrt{2}}}\right)^2 = \left(\frac{3\sqrt{3}-5}{9-5\sqrt{3}}\right)^2$$

$$= \left(\frac{3\sqrt{3}-5}{\sqrt{3}\left(3\sqrt{3}-5\right)}\right)^2 = \frac{1}{3}$$

26. (b) Given expression $$= \frac{12}{3+\sqrt{5}+2\sqrt{2}} \times \frac{\left(3+2\sqrt{2}\right)-\sqrt{5}}{\left(3+2\sqrt{2}\right)-\sqrt{5}}$$

$$= \frac{12}{\left(3+2\sqrt{2}\right)^2-\left(\sqrt{5}\right)^2} \times \left(\left(3+2\sqrt{2}\right)-\sqrt{5}\right)$$

$$= \frac{12}{12+12\sqrt{2}}\left(3+2\sqrt{2}-\sqrt{5}\right)$$

$$= \frac{1}{\sqrt{2}+1}\left(3+2\sqrt{2}-\sqrt{5}\right) \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= 3\sqrt{2}-3+4-2\sqrt{2}-\sqrt{10}+\sqrt{5}$$

$$= \sqrt{2}+1-\sqrt{10}+\sqrt{5}$$

27. (b) $$\frac{1}{\sqrt{x^2-x+1}} + \sqrt{x^2-x+1} = 2-x^2$$

Here, LHS is sum of a positive quantity and its reciprocal

$\therefore$ LHS ≥ 2

And RHS $= 2 - x^2 \le 2$

$\therefore$ Both sides are equal $\frac{1}{\sqrt{x^2-x+1}} + \sqrt{x^2-x+1} = 2$ and $2x^2 = 2$

clearly, $2 - x^2 = 2 \Rightarrow x = 0$

and at, $x = 0$, LHS $= 2$

Hence, $x = 0$ is only solution of the equation.

Thus equation has only one real root

28. (d) Do as in question no. 21. Required value is 99, which is a multiple of all 11, 3, 9

29. (d) $(pq) = (pq)^3$

$\Rightarrow pq = 1$

$\because p = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{7+2\sqrt{10}}{3}$

$\therefore q = \frac{1}{p} = \frac{\sqrt{5}-\sqrt{2}}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}} = \frac{7-2\sqrt{10}}{3}$

Now $3p^2 + 4pq - 3q^2 = 3(p+q)(p-q) + 4pq$

$$= 3 \times \frac{14}{3} \times \frac{4\sqrt{10}}{3} + 4 \times 1$$

$$= \frac{1}{3}(12 + 56\sqrt{10})$$

30. (b) $10 + \sqrt{24} + \sqrt{40} + \sqrt{60}$

$$= 10 + 2\sqrt{2}\sqrt{3} + 2\sqrt{2}\sqrt{5} + 2\sqrt{3}\sqrt{5}$$

$$= (\sqrt{2})^2 + (\sqrt{3})^2 + (\sqrt{5})^2 + 2\sqrt{2}\sqrt{3} + 2\sqrt{2}\sqrt{5} + 2\sqrt{3}\sqrt{5}$$

$$= (\sqrt{2} + \sqrt{3} + \sqrt{5})^2$$

$\therefore \sqrt{10 + \sqrt{24} + \sqrt{40} + \sqrt{60}} = \sqrt{2} + \sqrt{3} + \sqrt{5}$

$\therefore p + q + r = 2 + 3 + 5 = 10$

31. (b) Do as in solved example 13. Required cube root is $3 - \sqrt{5}$.

32. (a) $11 - 2\sqrt{30} = (\sqrt{6} - \sqrt{5})^2, 7 - 2\sqrt{10}$

$$= (\sqrt{5} - \sqrt{2})^2, 8 + 4\sqrt{3}$$

$$= (\sqrt{6} + \sqrt{2})^2 \text{ etc.}$$

33. (b) Given expression $= \dfrac{3+\sqrt{6}}{5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}} = \dfrac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}}$

$$= \frac{\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)}{\sqrt{3}+\sqrt{2}} = \sqrt{3}$$

34. (c) $a = \sqrt{\dfrac{\sqrt{5}+1}{\sqrt{5}-1} \times \dfrac{\sqrt{5}+1}{\sqrt{5}+1}}$

$$= \sqrt{\frac{\left(\sqrt{5}+1\right)^2}{5-1}} = \frac{\sqrt{5}+1}{2}$$

$$\frac{1}{a} = \frac{2}{\sqrt{5}+1}$$

$$= \frac{2}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$$

$$= \frac{\sqrt{5}-1}{2}$$

Now, $a^2 - a - 1 = a\left(a - 1 - \frac{1}{a}\right)$

$$= \frac{\sqrt{5}+1}{2}\left(\frac{\sqrt{5}+1}{2} - 1 - \frac{\sqrt{5}-1}{2}\right)$$

$$= \frac{\sqrt{5}+1}{2}\left(\frac{1}{2} - 1 - \frac{1}{2}\right) = 0$$

$\therefore \quad \sqrt{a^2 - a - 1} = \sqrt{0} = 0$

35. (d) Given $x = (a + \sqrt{a^2+b^3})^{1/3} + (a - \sqrt{a^2+b^3})^{1/3}$

cubing both sides,

$$x^3 = (a + \sqrt{a^2+b^3}) + (a - \sqrt{a^2+b^3}) + 3\,(a + \sqrt{a^2+b^3})^{1/3}(a - \sqrt{a^2+b^3})^{1/3}$$
$$\{(a + \sqrt{a^2+b^3})^{1/3} + (a - \sqrt{a^2+b^3})^{1/3}\}$$

$\Rightarrow \quad x^3 = 2a - 3b\,(x)$

$\Rightarrow \quad x^3 + 3bx - 2a = 0$

36. (a) $139 - 80\sqrt{3} = 139 - 2 \times 8 \times 5\sqrt{3}$

$$= 8^2 + \left(5\sqrt{3}\right)^2 - 2 \times 8 \times 5\sqrt{3}$$

$$= \left(5\sqrt{3} - 8\right)^2$$

Note that : $5\sqrt{3} > 8$

$\therefore \quad \sqrt{139 - 80\sqrt{3}} = 5\sqrt{3} - 8$

37. (d) $(a+3)\sqrt{2}+3=b2\sqrt{2}+a-1$

equation rational and irrational part,

$$a+3=2b \text{ and } a-1=3$$

$$\therefore \quad a=2b-3 \text{ and } a=4$$

$$\therefore \quad 4=2b-3 \text{ and } a=4$$

or, $b=\frac{7}{2}$ and $a=4$

Hence, $a+b=\frac{15}{2}$

38. (b) $\sqrt{x+2\sqrt{x-1}}=\sqrt{(x-1)+1+2\sqrt{x-1}}=\sqrt{\left(\sqrt{x-1}+1\right)^2}=\sqrt{x-1}+1$

$$\sqrt{x-2\sqrt{x-1}}=\sqrt{(x-1)+1-2\sqrt{x-1}}=\sqrt{\left(\sqrt{x-1}-1\right)^2}=\sqrt{x-1}-1$$

$\therefore$ Required value $=\left(\sqrt{x-1}+1\right)+\left(\sqrt{x-1}-1\right)=2\sqrt{x-1}$

39. (a) $\sqrt{x+2\sqrt{x-1}}=\sqrt{(x-1)+1+2\sqrt{x-1}}=\sqrt{\left(\sqrt{x-1}+1\right)^2}=\sqrt{x-1}+1$

$$\sqrt{x-2\sqrt{x-1}}=\sqrt{(x-1)+1-2\sqrt{x-1}}=\sqrt{\left(1-\sqrt{x-1}\right)^2}=1-\sqrt{x-1}$$

($\because 1<x<2$ hence value of $x-1$ is less then 1. Therefore $1-\sqrt{x-1}$ is a positive quantity)

Hence, Required sum $=\sqrt{x-1}+1+1-\sqrt{x-1}=2$

40. (a) $\sqrt{1+x}=\sqrt{1+\frac{\sqrt{3}}{2}}=\sqrt{\frac{2+\sqrt{3}}{2}}=\sqrt{\frac{2+\sqrt{3}}{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$$=\sqrt{\frac{4+2\sqrt{3}}{4}}=\frac{1}{2}\left(\sqrt{3}+1\right)^2=\frac{1}{2}(\sqrt{3}+1)$$

$$\sqrt{1-x}=\sqrt{1-\frac{\sqrt{3}}{2}}=\sqrt{\frac{2-\sqrt{3}}{2}}=\sqrt{\frac{4-2\sqrt{3}}{4}}=\frac{1}{2}\left(\sqrt{3}-1\right)$$

$$\therefore \quad \frac{1+x}{1+\sqrt{1+x}}+\frac{1-x}{1-\sqrt{1-x}}$$

$$=\frac{1+\frac{\sqrt{3}}{2}}{1+\frac{1}{2}\left(\sqrt{3}+1\right)}+\frac{1-\frac{\sqrt{3}}{2}}{1-\frac{1}{2}\left(\sqrt{3}-1\right)}$$

$$=\frac{2+\sqrt{3}}{3+\sqrt{3}}+\frac{2-\sqrt{3}}{3-\sqrt{3}}$$

$$=\frac{6-2\sqrt{3}+3\sqrt{3}-3+6+2\sqrt{3}-3\sqrt{3}-3}{\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)}$$

$$=\frac{6-3+6-3}{9-3}=\frac{6}{6}=1$$

41. (d) Let $\frac{x}{y} = \frac{y}{z} = \frac{z}{w} = k$

then, $x = y\,k,\ y = zk,\ z = wk$

$\therefore \quad x = y\,k = (zk)k = (wk)kk = wk^3$

$y = zk = (wk)k = wk^2$

$$\text{Hence Given expression} = \frac{\left(wk^3\right)^m + \left(wk^2\right)^m + \left(wk\right)^m + w^m}{\left(wk^3\right)^{-m} + \left(wk^2\right)^{-m} + \left(wk\right)^{-m} + w^{-m}}$$

$$= \frac{w^m\left(k^{3m} + k^{2m} + k^m + 1\right)}{w^{-m}\left(k^{-3m} + k^{-2m} + k^{-m} + 1\right)}$$

$$= \frac{w^{2m}\left(k^{3m} + k^{2m} + k^m + 1\right)}{k^{-3m}\left(1 + k^m + k^{2m} + k^{3m}\right)}$$

$$= w^{2m}k^{3m} = \left(w^4k^6\right)^{\frac{m}{2}}$$

$$= \left(wk^3 \cdot wk^2 \cdot wk \cdot w\right)^{\frac{m}{2}} = \left(xyzw\right)^{\frac{m}{2}}$$

42. (c) (a) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}} + \sqrt{6} = \frac{(\sqrt{2}+\sqrt{3})^2}{3-2} + \sqrt{6}$

$$= 2 + 3 + 2\sqrt{6} + \sqrt{6}$$

$$= 5 + 3\sqrt{6}$$

Which is not an integer,

(b) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}} + 2\sqrt{6} = \frac{(\sqrt{2}+\sqrt{3})^2}{3-2} + 2\sqrt{6}$

$$= 2 + 3 + 2\sqrt{6} + 2\sqrt{6}$$

$$= 5 + 4\sqrt{6}$$

Which is not an integer,

(c) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} + 2\sqrt{6} = \frac{(\sqrt{2}+\sqrt{3})^2}{2-3} + 2\sqrt{6}$

$$= -(2 + 3 + 2\sqrt{6}) + 2\sqrt{6}$$

$$= -5 - 2\sqrt{6} + 2\sqrt{6}$$

$$= -5$$

Which is an integer.

Hence, option (c) is correct

43. (b) $(256)^{0.16} \times (16)^{0.18} = (4^4)^{0.16} \times (4^2)^{0.18}$

$$= 4^{0.64} + 4^{0.36}$$

$$= 4^{0.64 + 0.36}$$

$$= 4^1 = 4$$

44. (a) $2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} - \frac{1}{2-\sqrt{2}} = 2 + \sqrt{2} + \frac{2-\sqrt{2}-2-\sqrt{2}}{4-2}$

$$= 2 + \sqrt{2} + \frac{(-2\sqrt{2})}{2}$$

$$= 2 + \sqrt{2} - \sqrt{2} = 2$$

45. (d) $a + x = a + \frac{2ab}{b^2+1}$

$$= \frac{a(b^2+1)+2ab}{b^2+1}$$

$$= \frac{a(b^2+1+2b)}{b^2+1} = \frac{a(b+1)^2}{b^2+1} \text{ etc.}$$

Exercise—2B

If $a = \frac{2+\sqrt{3}}{2-\sqrt{3}}$ and $b = \frac{2-\sqrt{3}}{2+\sqrt{3}}$ then the value of $(a^2 + b^2 + ab)$ is

(a) 185 (b) 195 (c) 200 (d) 175

[SSC Tier-I 201

2. If $x = \frac{2\sqrt{6}}{\sqrt{3}+\sqrt{2}}$ then the value of $\frac{x+\sqrt{2}}{x-\sqrt{2}} + \frac{x+\sqrt{3}}{x-\sqrt{3}}$ is

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{6}$ (d) 2

[SSC Tier-I 201

3. If $x = \frac{1}{2+\sqrt{3}}$, $y = \frac{1}{2-\sqrt{3}}$ then the value of $\frac{1}{x+1} + \frac{1}{y+1}$ is

(a) $\frac{1}{2}$ (b) $\sqrt{3}$ (c) 1 (d) $\frac{1}{\sqrt{3}}$

[SSC Tier-I 201

4. If $x = \frac{\sqrt{3}}{2}$, then the value of $\sqrt{1+x} + \sqrt{1-x}$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) 2

[SSC Tier-I 201

5. If $a = \sqrt{2} + 1$, $b = \sqrt{2} - 1$, then the value of $\frac{1}{a+1} + \frac{1}{b+1}$ is

(a) 9 (b) 3 (c) 1 (d) 2

[SSC Tier-I 201

6. If $a = 3 + 2\sqrt{2}$, then the value of $\frac{a^6 + a^4 + a^2 + 1}{a^3}$ is

(a) 192 (b) 240 (c) 204 (d) 212

[SSC Tier-I 2012]

7. If $x = 1 + \sqrt{2} + \sqrt{3}$ then the value of $(2x^4 - 8x^3 - 5x^2 + 26x - 28)$ is

(a) $6\sqrt{6}$ (b) 0 (c) $3\sqrt{6}$ (d) $2\sqrt{6}$

[SSC Tier-I 2012]

8. If $x = 2 + \sqrt{3}$, $y = 2 - \sqrt{3}$ then the value of $\frac{x^2 + y^2}{x^3 + y^3}$ is

(a) $\frac{7}{38}$ (b) $\frac{7}{40}$ (c) $\frac{7}{19}$ (d) $\frac{7}{26}$

[SSC Tier-I 2012]

9. If $2\sqrt{x} = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} - \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$ then what is the value of x ?

(a) 30 (b) $\sqrt{15}$ (c) 15 (d) 6

[SSC Tier-I 2012]

10. If $a = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$, $b = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$ then what is the value of $\frac{a^2}{b} + \frac{b^2}{a}$?

(a) 970 (b) 1030 (c) 930 (d) 900

[SSC Tier-I 2012]

11. If $x = 2 + \sqrt{3}$ then the value of $\sqrt{x} + \frac{1}{\sqrt{x}}$ is

(a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) 6 (d) $\sqrt{3}$

[SSC Tier-I 2012]

12. What is the value of $\sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$?

(a) 3 (b) 4 (c) 1 (d) 2

[SSC Tier-I 2012]

13. If $\sqrt{\frac{x-a}{x-b}} + \frac{a}{x} = \sqrt{\frac{x-b}{x-a}} + \frac{b}{x}$, $b \neq a$ then what is the value of x ?

(a) $\frac{ab}{a+b}$ (b) 1 (c) $\frac{a}{a+b}$ (d) $\frac{b}{a+b}$

[SSC Tier-I 2012]

14. If $\sqrt{4x - 9} + \sqrt{4x + 9} = 5 + \sqrt{7}$ then what is the value of x ?

(a) 5 (b) 7 (c) 3 (d) 4

[SSC Tier-I 2012]

Answers—2B

1. (b) 2. (d) 3. (c) 4. (c) 5. (c) 6. (c) 7. (a) 8. (d)
9. (c) 10. (a) 11. (a) 12. (d) 13. (a) 14. (d)

Explanation

1. **(b)** $a = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = (2+\sqrt{3})^2$

$b = \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = (2-\sqrt{3})^2$

$\therefore \quad a^2 + b^2 + ab = (a+b)^2 - ab = \left((2+\sqrt{3})^2 + (2-\sqrt{3})^2\right)^2 - 1$ $(\because ab = 1)$

$\left(2\left(2^2 + (\sqrt{3})^2\right)\right)^2 - 1 = 14^2 - 1 = 196 - 1 = 195$

2. **(d)** $x = \frac{2\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3}\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ ...(i)

or, $\frac{x}{\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}}$

By, Componendo-dividendo

$\frac{x+\sqrt{2}}{x-\sqrt{2}} = \frac{2\sqrt{3}+\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{3}-\sqrt{2}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ ...(ii)

Again from (i), $\frac{x}{\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

or, $\frac{x+\sqrt{3}}{x-\sqrt{3}} = \frac{2\sqrt{2}+\sqrt{3}+\sqrt{2}}{2\sqrt{2}-\sqrt{3}-\sqrt{2}} = \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$ ...(iii)

Adding (ii) and (iii),

$$\frac{x+\sqrt{2}}{x-\sqrt{2}} + \frac{x+\sqrt{3}}{x-\sqrt{3}} = \frac{(3\sqrt{3}+\sqrt{2}) - (3\sqrt{2}+\sqrt{3})}{\sqrt{3}-\sqrt{2}} = \frac{2\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 2$$

3. **(c)** $x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

and $y = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$

$\therefore \quad \frac{1}{x+1} + \frac{1}{y+1} = \frac{1}{2-\sqrt{3}+1} + \frac{1}{2+\sqrt{3}+1}$

$= \frac{1}{3-\sqrt{3}} + \frac{1}{3+\sqrt{3}}$

$= \frac{3+\sqrt{3}+3-\sqrt{3}}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{6}{9-3} = 1$

4. (c) $\sqrt{1+x}+\sqrt{1-x}=\sqrt{1+\frac{\sqrt{3}}{2}}+\sqrt{1-\frac{\sqrt{3}}{2}}$

$$=\sqrt{\frac{2+\sqrt{3}}{2}}+\sqrt{\frac{2-\sqrt{3}}{2}}$$

$$=\sqrt{\frac{4+2\sqrt{3}}{4}}+\sqrt{\frac{4-2\sqrt{3}}{4}}$$

$$=\sqrt{\left(\frac{1+\sqrt{3}}{2}\right)^2}+\sqrt{\left(\frac{\sqrt{3}-1}{2}\right)^2}$$

$$=\frac{1+\sqrt{3}}{2}+\frac{\sqrt{3}-1}{2}=\sqrt{3}$$

5. (c) $\frac{1}{a+1}+\frac{1}{b+1}=\frac{1}{\sqrt{2}+2}+\frac{1}{\sqrt{2}}$

$$=\frac{1}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}}+\frac{\sqrt{2}}{2}$$

$$=\frac{2-\sqrt{2}}{4-2}+\frac{\sqrt{2}}{2}$$

$$=\frac{2-\sqrt{2}+\sqrt{2}}{2}=1$$

6. (c) $\frac{a^6+a^4+a^2+1}{a^3}=a^3+a+\frac{1}{a}+\frac{1}{a^3}$ ... (i)

$\because\ a=3+2\sqrt{2}$

$\therefore\ \frac{1}{a}=\frac{1}{3+2\sqrt{2}}=\frac{1}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}=\frac{3-2\sqrt{2}}{9-8}=3-2\sqrt{2}$

Hence, $a+\frac{1}{a}=3+2\sqrt{2}+3-2\sqrt{2}=6$

cubing, $a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)=6^3$

or, $a^3+\frac{1}{a^3}+3\times6=216$

or, $a^3+\frac{1}{a^3}=216-18=198$

From (i), Required value $=\left(a^3+\frac{1}{a^3}\right)+\left(a+\frac{1}{a}\right)$

$$=198+6=204$$

7. (a) $x-1=\sqrt{2}+\sqrt{3}$

Squaring both sides, $x^2-2x+1=2+3+2\sqrt{6}$ ... (i)

or, $x^2-2x-4=2\sqrt{6}$

Squaring both sides $x^4+4x^2+16-4x^3-8x^2+16x=24$

$$x^4-4x^3-4x^2+16x-8=0$$

(Multiply by ...

$\therefore\ 2x^4 - 8x^3 - 8x^2 + 32x - 16 = 0$

or, $(2x^4 - 8x^3 - 5x^2 + 26x - 28) + (-3x^2 + 6x + 12) = 0$

$\therefore$ Required value $-3(x^2 - 2x - 4) = 0$

or, $2x^4 - 8x^3 - 5x^2 + 26x - 28 = -(-3x^2 + 6x + 12)$

or, Required value $= 3(x^2 - 2x - 4) = 3\left(2\sqrt{6}\right)$ (From ...

$= 6\sqrt{6}$

8. (d) Given, $x + y = (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$

and $xy = (2 + \sqrt{3})(2 - \sqrt{3}) = 1$

$\therefore\ x^2 + y^2 = (x + y)^2 - 2xy = 4^2 - 2 = 14$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$= (x + y)\{(x + y)^2 - 3xy\}$

$= 4\{4^2 - 3 \times 1\} = 52$

$\therefore\ \dfrac{x^2+y^2}{x^3+y^3} = \dfrac{14}{52} = \dfrac{7}{26}$

9. (c) $2\sqrt{x} = \dfrac{\left(\sqrt{5}+\sqrt{3}\right)^2 - \left(\sqrt{5}-\sqrt{3}\right)^2}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)} = \dfrac{4\left(\sqrt{5}\sqrt{3}\right)}{5-3}$

or, $\sqrt{x} = \dfrac{4\sqrt{15}}{2\times 2} = \sqrt{15}$

or, $x = 15$

10. (a) $a = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \dfrac{\left(\sqrt{3}-\sqrt{2}\right)^2}{3-2} = 5 - 2\sqrt{6}$

Similarly $b = \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 5 + 2\sqrt{6}$

and $a \cdot b = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 1$

$\therefore\ \dfrac{a^2}{b} + \dfrac{b^2}{a} = \dfrac{a^3+b^3}{ab}$

$= a^3 + b^3$

$= (5 - 2\sqrt{6})^3 + (5 + 2\sqrt{6})^3$

$= 2[5^3 + 3\cdot 5\left(2\sqrt{6}\right)^2]$

$= 2(125 + 360)$

$= 970$

$(\because (x + y)^3 + (x - y)^3 = 2(x^3 + 3x\ldots$

ply by 2)

rom (i))

11. (a) Let $t = \sqrt{x} + \frac{1}{\sqrt{x}}$

then, $t^2 = x + \frac{1}{x} + 2$

$$= 2 + \sqrt{3} + \frac{1}{2+\sqrt{3}} + 2$$

$$= 2 + \sqrt{3} + \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} + 2$$

$$= 2 + \sqrt{3} + \frac{2-\sqrt{3}}{4-3} + 2$$

$$= 2 + \sqrt{3} + 2 - \sqrt{3} + 2 = 6$$

$\therefore \quad t = \sqrt{6}$

or, $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6}$

12. (d) $\sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$

$$= \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{(2+\sqrt{3})^2}}} \qquad \left(\because (2+\sqrt{3})^2 = 2^2 + 3 + 4\sqrt{3} = 7 + 4\sqrt{3}\right)$$

$$= \sqrt{-\sqrt{3} + \sqrt{3 + 8(2+\sqrt{3})}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{19 + 8\sqrt{3}}}$$

$$= \sqrt{-\sqrt{3} + \sqrt{(4+\sqrt{3})^2}} \qquad \left(\because (4+\sqrt{3})^2 = 16 + 3 + 8\sqrt{3}\right)$$

$$= \sqrt{-\sqrt{3} + 4 + \sqrt{3}} = \sqrt{4} = 2$$

13. (a) $\sqrt{\frac{x-a}{x-b}} + \frac{a}{x} = \sqrt{\frac{x-b}{x-a}} + \frac{b}{x}$

or, $\sqrt{\frac{x-a}{x-b}} - \sqrt{\frac{x-b}{x-a}} = \frac{b}{x} + \frac{-a}{x}$

$$\frac{(x-a)-(x-b)}{\sqrt{x-b}\sqrt{x-a}} = \frac{b-a}{x}$$

or, $\frac{-a+b}{\sqrt{x-b}\sqrt{x-a}} = \frac{b-a}{x}$

or, $\frac{1}{\sqrt{x-b}\sqrt{x-a}} = \frac{1}{x}$

Squaring both sides, $(x-b)(x-a) = x^2$

$$x^2 - (a+b)x + ab = x^2$$

$$\therefore \quad x = \frac{ab}{a+b}$$

$3xy^2))$

14. (d) By trial, $x = 4$ satisfies the equation. If can be solved as follows

$$\left(\sqrt{4x-9}+\sqrt{+x+9}\right)\left(\sqrt{4x-9}-\sqrt{4x+9}\right) = 4x-9-4x-9$$

or, $\left(5+\sqrt{7}\right)\left(4x-9-\sqrt{4x+9}\right) = -18$

or, $\sqrt{4x-9}-\sqrt{4x+9} = \frac{-18}{5+\sqrt{7}}\times\frac{5-\sqrt{7}}{5-\sqrt{7}}$

or, $\sqrt{4x-9}-\sqrt{4x+9} = \frac{-18\left(5-\sqrt{7}\right)}{25-7}$

or, $\sqrt{4x-9}-\sqrt{4x+9} = -\left(5-\sqrt{7}\right)$ ... (i)

Given, $\sqrt{4x-9}+\sqrt{4x+9} = 5+\sqrt{7}$ ... (ii)

Adding, $2\sqrt{4x-9} = 2\sqrt{7}$

or, $4x-9=7$

or, $4x=16$

or, $x=4$

★★★

03

Graphical Solution of Linear Equation

1. **Equation of a straight line :** General equation of a straight line in xy plane is given by $ax + by + c = 0$. In various situations its graph is drawn as follows :

Case (i) : When $a \neq 0, b \neq 0, c \neq 0$

This straight line intersects x–axis at $\left(-\frac{c}{a}, 0\right)$ and y–axis at $\left(0, -\frac{c}{b}\right)$

Explanation & Putting $y = 0$ in the equation $ax + by + c = 0$ we get $ax + c = 0$

or, $x = -\frac{c}{a}$ i.e. straight line $ax + by + c = 0$ cuts x–axis at $\left(-\frac{c}{a}, 0\right)$

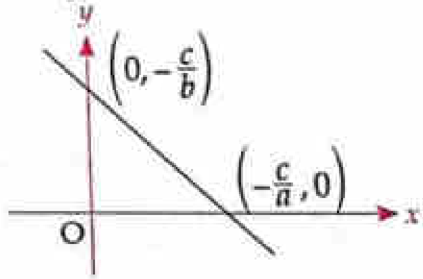
Again, putting $x = 0$ in the equation $ax + by + c = 0$, we get $by + c = 0$ or $y = -\frac{c}{b}$ i.e. y–axis intersects straight line $ax + by + c = 0$ at $\left(0, -\frac{c}{b}\right)$

It can be learned as follows, $ax + by + c = 0$

or, $ax + by = -c$

or, $\frac{ax}{-c} + \frac{by}{-c} = 1$

or, $\frac{x}{\left(-\frac{c}{a}\right)} + \frac{y}{\left(-\frac{c}{b}\right)} = 1$



This is known as intercept forms of a straight line. The terms in denominator are respectively known as x–intercept and y–intercept Clearly,

Length intercepted by the straight line $ax + by + c = 0$ between the axes is $\sqrt{\left(\frac{-c}{a}\right)^2 + \left(-\frac{c}{b}\right)^2}$. Here $\left(-\frac{c}{a}\right)$ and $\left(-\frac{c}{b}\right)$ are intercepts made by the line respectively on the x–axis and y–axis.

Case (ii) : When $a \neq 0, b \neq 0, c = 0$ i.e. equation of the straight line is $ax + by = 0$

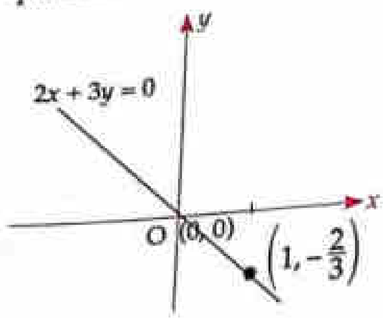
This straight line always passes through origin. If a and b are of opposite sign, it passes through first and third quadrant while when a and b are of same sign, it passes through second and fourth quadrant.

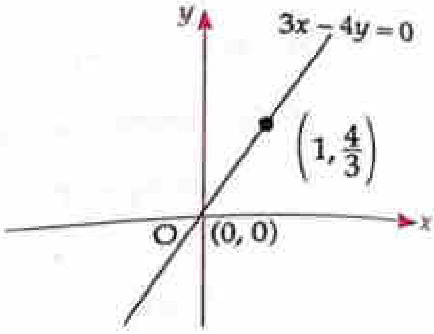
For example. Draw the graph of straight lines

(a) $2x + 3y = 0$ (b) $4x - 3y = 0$

Soln : For the straight line $2x + 3y = 0$

When $x = 0, y = 0$ and when $x = 1, y = -\frac{2}{3}$, thus the line passes through origin (0, 0) and another point $\left(1, -\frac{2}{3}\right)$. Clearly it lies in second and fourth quadrant.





For the straight line $4x - 3y = 0$

When $x = 0, y = 0$ and when $x = 1, y = \frac{4}{3}$ i.e. the straight line goes through origin (0, 0) and another point $\left(1, \frac{4}{3}\right)$. Clearly it lies in the first and third quadrant.

Case (iii) : When $a = 0, b \neq 0, c \neq 0$ i.e. equation of the line is $by + c = 0$.

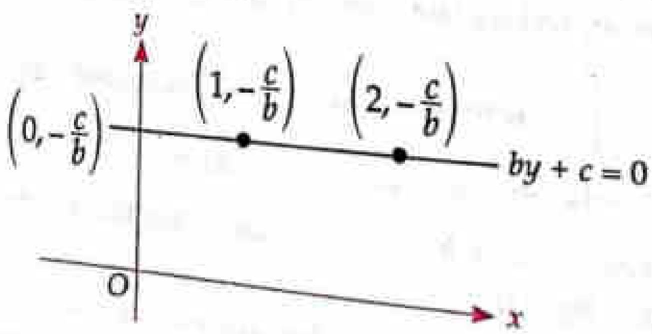
This line is parallel to x–axis and intersects y–axis at $\left(0, -\frac{c}{b}\right)$

Explanation : From $by + c = 0$, we have $y = -\frac{c}{b}$

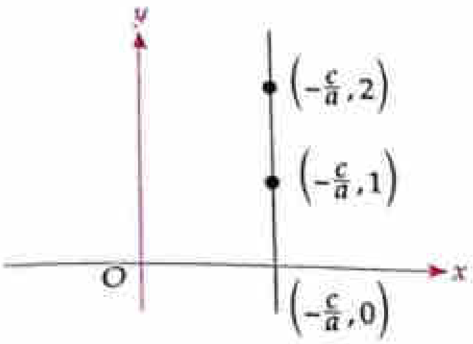
Now, when $x = 0, y = -\frac{c}{b}$, $\quad x = 1, y = -\frac{c}{b}$

$x = 2, y = -\frac{c}{b}$ etcetera.

Thus this line passes through the points $\left(0, -\frac{c}{b}\right), \left(1, -\frac{c}{b}\right), \left(2, -\frac{c}{b}\right)$ etcetera. Clearly it is parallel to x–axis



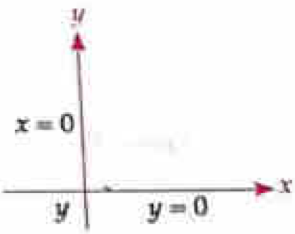
Case (iv) : When $a \neq 0, b = 0$ and $c \neq 0$ i.e. equation of straight line is of the form $ax + c = 0$



This line is parallel to y–axis and cuts x–axis at $\left(-\frac{c}{a},0\right)$.

2. Equation of axes : Equation of x–axis is $y = 0$ because y-coordinates of all the points lie on x-axis are zero.

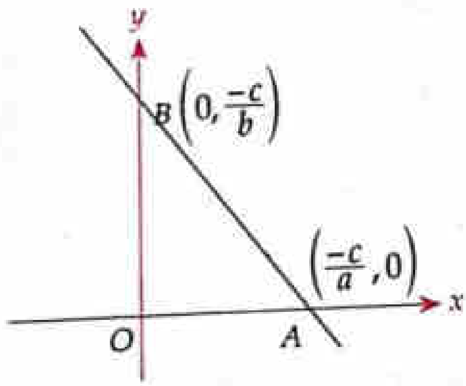
   Equation of y–axis is $x = 0$ because x-coordinates of all the points lie on y-axis are zero.



3. Point of intersection : To find the coordinates of point of intersection of two straight lines, solve their equations.

4. Area of triangle formed by straight lines

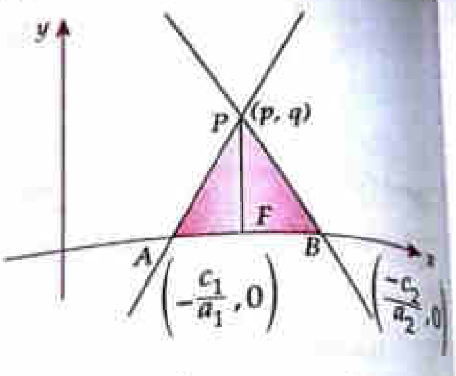
   4.1. Area of triangle formed by straight lines $ax + by + c = 0, a \neq 0, b \neq 0, c \neq 0$ with coordinate axes is $\left|\frac{1}{2}\frac{c^2}{ab}\right|$.



Explanation : Since straight lines $ax + by + c = 0$ cuts x–axis at $A\left(-\frac{c}{a},0\right)$ and cuts y–axis at $B\left(0,-\frac{c}{b}\right)$, then $OA = -\frac{c}{a}$ and $OB = -\frac{c}{b}$.

Hence, area of triangle formed by straight lines $ax + by + c = 0$ with x–axis and y–axis $= \left|\frac{1}{2}(OA)(OB)\right| = \left|\frac{1}{2}\left(\frac{-c}{a}\right)\left(\frac{-c}{b}\right)\right| = \left|\frac{1}{2}\frac{c^2}{ab}\right|$

4.2. Area of triangle formed by two straight lines either with x–axis or with y–axis :

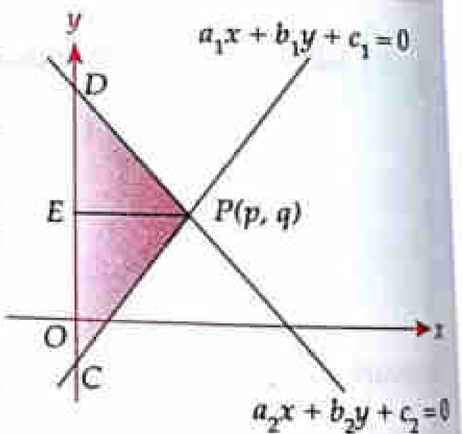


If two straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ intersect at point $P(p, q)$ then area of the triangle formed by these lines with x–axis $= \left|\frac{1}{2}(AB)(PF)\right|$

(see figure)

Here $PF = q$ is the y coordinate of point P and $A\left(\frac{-c_1}{a_1}, 0\right)$ and B $\left(\frac{-c_2}{a_2}, 0\right)$ are respectively point of intersection of given lines on the x–axis

Hence area of triangle formed by two straight lines with x–axis $= \frac{1}{2}$ (difference of x–intercept of the two lines) × (y coordinate of point of intersection of two lines)



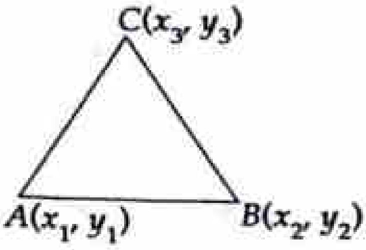
Similarly, straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ intersect y–axis respectively at $C\left(0, \frac{-c_2}{b_2}\right)$ and $D\left(0, \frac{-c_2}{b_2}\right)$ and if they intersect each other at $P(p, q)$ *[see figure]*, then

Area of triangle $CPD = \left|\frac{1}{2}(CD)(PE)\right|$

Here, $PE = p$ is the x–coordinate of point P.

Hence area of triangle formed by two straight lines with y–axis $= \frac{1}{2}$ (difference of y–intercept of two lines) × (x coordinate of point of intersection of two lines)

4.3. If three straight lines intersect each other at the points $A\ (x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ then from coordinate Geometry.

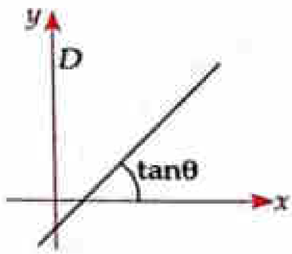


Area of ΔABC,

$$= \frac{1}{2}\left|x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right|$$

4.4. If $x_1(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3\,(y_1 - y_2) = 0$ then three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear.

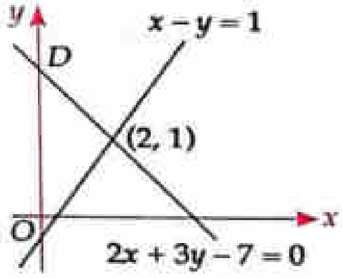
4.5. Slope of the line : If a straight line makes angle θ with x–axis in positive direction (anti-clock-wise direction) then slope of the line is $\tan\theta$.



Slope of the straight line $ax + by + c = 0$ is $\frac{-a}{b}$

[For more details see exercise on coordinate Geometry]

5. Solution of corresponding equation of straight lines : If two straight lines intersect at a point then x–coordinate and y–coordinate of the point are called solution of equation of the straight lines.



For example, solving $2x + 3y - 7 = 0$ and $x - y = 1$, we get $x = 2$, $y = 1$. Hence two straight lines intersect at (2, 1).

6. Consistent and Inconsistent system of equations : A system of equation $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has

(i) a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) Infinitely many solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(iii) no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

to understand it clearly, consider the following examples :

6.1. Consider the system of equations $2x + 3y = 7$ and $3x - y = 5$

here, $a_1 = 2, b_1 = 3, c_1 = -7$

and $a_2 = 3, b_2 = -1, c_2 = -5$

$\because \quad \frac{a_1}{a_2} = \frac{2}{3}$ and $\frac{b_1}{b_2} = -3$

$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

So we can conclude that the system of equation has unique solution. Solving these equations we get $x = 2, y = 1$ (do yourself). Geometrically, the two lines of the system (having unique solution) intersect each other at a unique point.

6.2. Consider the system of equation $2x + y = 10$ and $4x + 2y = 20$

here, $a_1 = 2, b_1 = 1, c_1 = -10$

and, $a_2 = 4, b_2 = 2, c_2 = -20$

$\because \quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-10}{-20} = \frac{1}{2}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

From this we can conclude that the system of equations has infinitely many solutions.

To solve the equation, multiply first equation by 2 and subtract second equation from it

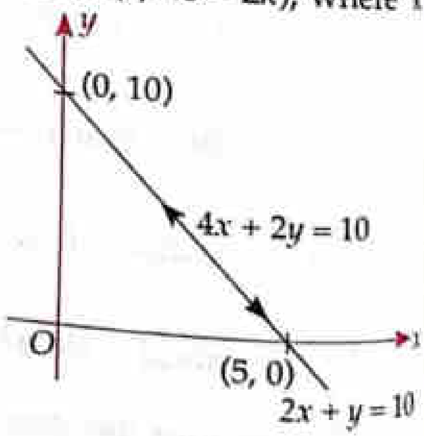
$$\begin{array}{r} [2x + \ y = 10\,] \times 2 \\ 4x + 2y = 20 \\ - \quad - \quad - \quad \\ \hline 0 \ = 0 \end{array}$$

here $0 = 0$ indicates that system of equations has infinitely many solutions. To find its solution, proceed as follows—

From first equation $y = 10 - 2x$ when $x = k, y = 10 - 2k$

Clearly $x = k, y = 10 - 2k$ also satisfy the second equation.

Hence, solution of the system of equation is $(k, 10 - 2k)$, where k is a real number. For each real value of k, the system has a solution. Putting $k = 1, 2, 3, 4, \ldots\ldots\ldots$ we get the solution as (1, 8), (2, 6), (3, 4), (4, 2)… etcetera, which are infinitely many is counting



Geometrically, these two lines are coincident. Both lines cut x–axis at (5, 0) and y–axis at (0, 10).

6.3. Consider the system of equations $2x + y = 6$ and $4x + 2y = 16$.

Here, $a_1 = 2, b_1 = 1, c_1 = -6$

$a_2 = 4, b_2 = 2, c_2 = -16$

$\because \quad \dfrac{a_1}{a_2} = \dfrac{2}{4} = \dfrac{1}{2}, \dfrac{b_1}{b_2} = \dfrac{1}{2}, \dfrac{c_1}{c_2} = \dfrac{-6}{-16} = \dfrac{3}{8}$

$\therefore \quad \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

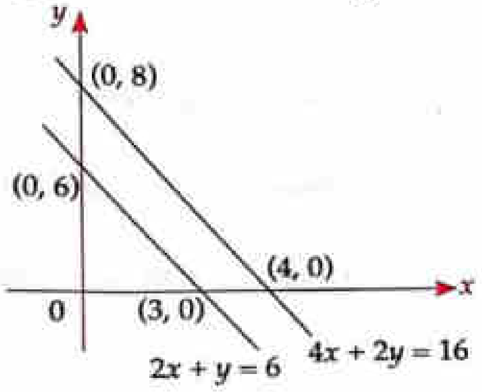
From this, we can conclude the system of equation has no solution.

Solving,

$$\begin{array}{r} 2x + y = 6\ ] \times 2 \\ \underline{\overset{-}{}4x \underset{-}{+} 2y \underset{-}{=} 16} \\ 0 = -2 \end{array}$$

$0 = -2$ indicates that solution is not possible. Geometrically, these two straight lines are parallel

[see figure]



The system of equations having solution is called **consistent**. It is of two types—

(i) Unique solution

(ii) Infinitely many solution

The system of equations having no solution is called **inconsistent**

Conclusion : For the system of equations $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$

$\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$	Unique Solution	Consistent (independent)	Intersecting lines
$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$	Infinitely many Solution	Consistent (dependent)	Coincident lines
$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$	No Solution	Inconsistent	Parallel lines

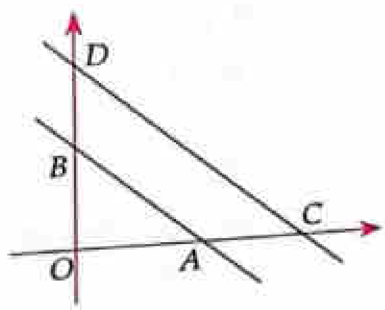
7. Area of trapezium between two parallel lines and axes :

Suppose $ax + by + c = 0$

and $ax + by + d = 0$ are two parallel lines.

First line cuts x–axis at A, y–axis at B while second line cuts x–axis at C and y–axis at D.

[see figure]



Hence, Area of trapezium $ACBD$

$= \text{area of } \Delta OCD - \text{area of } \Delta OAB$

$$= \frac{1}{2}\left|\frac{d^2}{ab}\right| - \frac{1}{2}\left|\frac{c^2}{ab}\right| = \frac{1}{2}\left(\left|\frac{d^2}{ab}\right| - \left|\frac{c^2}{ab}\right|\right)$$

Note : Donot write it as $\frac{1}{2}\left|\frac{d^2-c^2}{ab}\right|$. In the above figure, above fact ca also be used if AB and CD are not parallel.

8. Some important points about coordinate Geometry regarding straig lines :

8.1. Distance between two points (x_1, y_1) and (x_2, y_2)

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

8.2. Distance between origin and $(x, y) = \sqrt{x^2 + y^2}$

8.3. Distance of the straight line $ax + by + c = 0$ from origin $= \left|\frac{c}{\sqrt{a^2+b^2}}\right|$

8.4. Distance of the straight line $ax + by + c = 0$ from the point $(x_1, y_1$

$$= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$$

8.5. If point P divides the line segment joining the points $(x_1, y$ and (x_2, y_2) in the ratio $m : n$ internally then coordinates of P

$$\left(\frac{mx_2 + nx_1}{m-n}, \frac{my_2 + ny_1}{m-n}\right)$$

8.6. If point Q divides the line joining the points (x_1, y_1) and (x_2, y_2) the ratio $m : n$ externally then coordinates of Q are

$$\left(\frac{mx_2 - nx_1}{m+n}, \frac{my_2 - ny_1}{m+n}\right)$$

8.7. If P be the midpoint of line segment joining the points (x_1, y_1) a (x_2, y_2) then coordinates of P are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

8.8. If points $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$ ... are collinear th

$\frac{y_2-y_1}{x_2-x_1} = \frac{y_3-y_2}{x_3-x_2} = \frac{y_4-y_3}{x_4-x_3}$ .... Here each term is slope of the strai line.

8.9. To find the point of intersection of two straight lines, solve th equations.

8.10. Equation of x-axis is $y = 0$ and Equation y-axis is $x = 0$.

8.11. Equation of a straight line parallel to x-axis is $y = c$. It cuts y-axis at $(0, c)$

8.12. Equation of a straight line parallel to y-axis is $x = k$. It cuts x-axis at $(k, 0)$.

8.13. Distance between two parallel lines $ax + by + c = 0$ and $ax + by + d = 0$ is equal to $= \left|\frac{d-c}{\sqrt{a^2+b^2}}\right|$

8.14. If point (α, β) lies on the line $ax + by + c = 0$ then $a\alpha + b\beta + c = 0$

8.15. Equation of a straight line passing through points (x_1, y_1) and (x_2, y_2) is $y - y_1 \; \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

where, $m = \frac{y_2-y_1}{x_2-x_1}$ = slope of the line.

9. Definition of Modulus and its graph :

$|x|$ shows the absolute value of x, it is therefore $|4| = 4$ and $|-4| = 4$.

But it is incorrect to write $|x| = \pm x$. $|x|$ is defined as follows

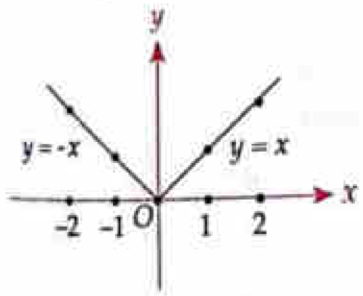
$$|x| = \begin{cases} x \text{ when } x \geq 0 \\ -x \text{ when } x < 0 \end{cases}$$

Hence, $|4| = 4$ and $|-4| = -(-4)$

$$\text{Similarly } |x-1| = \begin{cases} x-1 \text{ when } x-1 \geq 0 \\ -(x-1) \text{ when } x-1 < 0 \end{cases}$$

$$\text{or, } |x-1| = \begin{cases} x-1 \text{ when } x \geq 1 \\ 1-x \text{ when } x < 1 \end{cases}$$

Graph of $y = |x|$ is as follows :



x	-2	-1	0	1	2
y	2	1	0	1	2

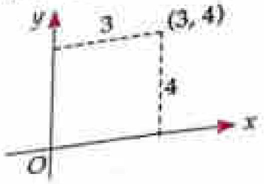
This graph contains two different lines (in fact rays). For $x \geq 0$, it shows $y = x$ and for $x < 0$ it shows $y = -x$

Solved Examples

1. Find the distance of point (3, 4) from (i) x–axis (ii) y–axis (iii) origin.

Solution : Point (3, 4) is at a distance of 4 unit from x–axis and at a distance of 3 unit from y–axis.

Its distance from origin $= \sqrt{3^2+4^2} = 5.$



2. What is the distance between points (–2, 5) and (6, –1).

Solution : From coordinate Geometry, distance between

points (x_1, y_1) and $(x_2, y_2) = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ (Note it)

$\because$ Required distance $= \sqrt{(-2-6)^2+(5-(-1))^2} = \sqrt{64+36} = \sqrt{100} = 10$

3. Find the point where the straight line $2x-3y=12$ cuts x–axis and y–axis. Also find the length intercepted by the line between the axes.

Solution : $2x-3y=12$

or, $\frac{2x}{12}-\frac{3y}{12}=1$

or, $\frac{x}{6}+\frac{y}{(-4)}=1$

Thus straight line cuts x–axis at (6, 0) and y–axis at (0, –4)

Length intercepted between axes $= \sqrt{6^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Second method : In the equation of line $2x-3y=12$

putting $y=0,\ 2x=12 \Rightarrow x=6$

putting $x=0\ \ -3y=12 \Rightarrow x=-4$

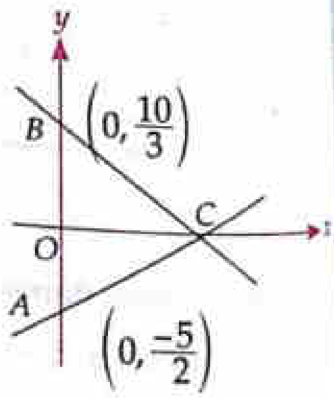
i.e. line cuts x–axis at (6, 0) and y–axis at (0, –4)

4. Find the area of triangle formed by lines $x-2y=5$ and $2x+3y=10$ with y–axis.

Solution : Solving equation $x-2y=5$

and $2x+3y=10$, $(x, y)=(5, 0)$

Let $C \equiv (5, 0)$



putting $x=0$ in $x-2y=5$ we get $y=\frac{-5}{2}$ i.e.

first line cuts y–axis at $A\left(0,\frac{-5}{2}\right)$

putting $x=0$ in $2x+3y=10$ we get $y=\frac{10}{3}$ i.e.

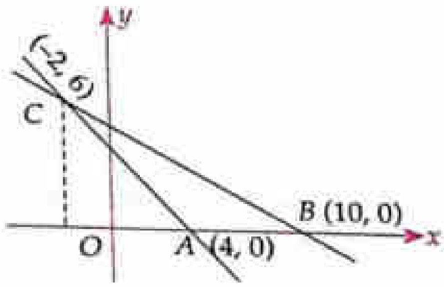
second line cuts y–axis at $B\left(0,\frac{10}{3}\right)$

Hence, Area of $\Delta ABC=\frac{1}{2}\cdot AB\cdot OC$

$$=\frac{1}{2}\left(\frac{10}{3}-\left(\frac{-5}{2}\right)\right)\times 5=\frac{1}{2}\left(\frac{20+15}{6}\right)\times 5=\frac{175}{12}$$

5. Find the area of triangle formed by straight lines $x+y-4=0, x+2y-10=0$ and $y=0$

Solution : $y=0$ represents x–axis.



Solving $x+y-4=0$ and $x+2y-10$ we get $(x,y)=(-2,6)$ i.e. two lines intersect at $C\equiv(-2,6)$

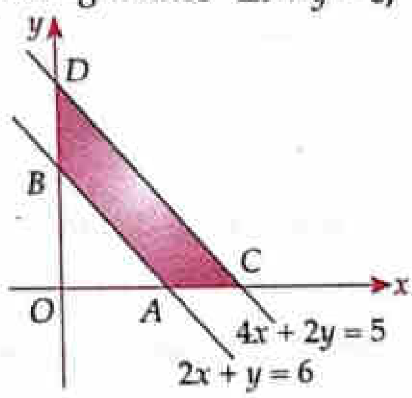
putting $y=0$ in $x+y-4=0$ we get $x=4$ i.e. first line cuts x–axis at $A(4,0)$

putting $y=0$ in $x+2y-10=0$ we get $x=10$ i.e. second line cuts x–axis at $B(10,0)$

$\therefore$ Required area $=\frac{1}{2}\times$ (difference of x–intercept of the two lines) $\times$ (y coordinate of point of intersection of two lines)

$$=\frac{1}{2}(10-4)\,6=\frac{1}{2}\times 6\times 6=18\text{ (unit)}^2$$

6. Find the area of quadrilateral intercepted by straight lines $2x+y=6$, $4x+2y=25$ between the axes.



Solution : For the given lines $2x+y-6=0$ and $4x+2y-25=0$, $\frac{2}{4}=\frac{1}{2}\neq\frac{-6}{25}$, thus two lines are parallel.

These two lines along with coordinate axes formed trapezium $ABCD$ (see fig.)

Area of trapezium $ABCD=$ area of ΔOCD $-$ area of ΔOAB

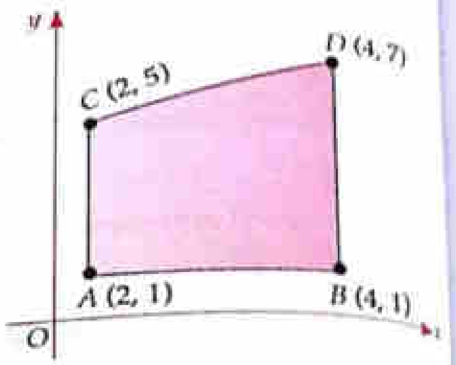
$$=\frac{1}{2}\left|\frac{c_2^2}{a_2b_2}\right|-\frac{1}{2}\left|\frac{c_1^2}{a_1b_1}\right|=\left|\frac{1}{2}\left(\frac{-25}{4\times 2}\right)\right|-\frac{1}{2}\left|\frac{(-6)}{2\times 1}\right|$$

$= \frac{25}{16} - \frac{6}{4} = \frac{25-24}{16} = \frac{1}{16}$ (unit)^2

(Recall that area of triangle formed by straight line $ax + by + c = 0$, with coordinate axes $= \frac{1}{2}\left|\frac{c^2}{ab}\right|$)

7. Find the area of quadritateral formed by joining points (2, 1), (4, 1), (2, 5) and (4, 7)

Solution : Let $A = (2, 1)$, $B = (4, 1)$,
$C = (2, 5)$ and $D = (4, 7)$



Clearly (see fig.) AB is parallel to x-axis while AC and BD are parallel to y-axis.
So $ABDC$ is a trapezium.

Area of trapezium $ABCD$

$= \frac{1}{2}(AC + BD) \times AB = \frac{1}{2}(4+6) \times 2$

$= 10$ square unit

What is the distance of the line $3x - 4y + 15 = 0$ from origin ? What is its distance from point $(-5, -1)$.

Solution : Distance of the line $ax + by + c = 0$ from origin $= \left|\frac{c}{\sqrt{a^2+b^2}}\right|$

Thus, distance of $3x - 4y + 15 = 0$ from origin $= \left|\frac{15}{\sqrt{3^2+4^2}}\right| = \left|\frac{15}{5}\right| = 3$

distance of point (x_1, y_1) from $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$

$\therefore$ distance of $(-5, -1)$ from $3x - 4y + 15 = 0$ is $\left|\frac{3(-5) - 4(-1) + 15}{\sqrt{3^2+4^2}}\right|$

$= \frac{4}{5}$ unit.

9. What is the distance between two parallel lines $2x + 3y + 13 = 0$ and $4x + 6y - 91 = 0$

Solution : Given lines are

$2x + 3y + 13 = 0$ and $4x + 6y - 91 = 0$

or, $4x + 6y + 26 = 0$ and $4x + 6y - 91 = 0$

(Make the coefficients of x and y same

Since, distance between $ax + by + c = 0$ and $ax + by + d = 0$ is $\left|\frac{d-c}{\sqrt{a^2+b^2}}\right|$

$\therefore$ Required distance $= \left|\frac{-91-26}{\sqrt{4^2+6^2}}\right| = \left|\frac{-117}{\sqrt{52}}\right| = \frac{13 \times 9}{2\sqrt{13}} = \frac{9\sqrt{13}}{2}$ unit

10. For what values of a and b points $(1, 1)$, $(2, 3)$, $(3, a)$ and $(b, 7)$ are collinear.

Solution : Points (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) are collinear i.e. lie on a straight line if $\frac{y_2-y_1}{x_2-x_1} = \frac{y_3-y_2}{x_3-x_2} = \frac{y_4-y_3}{x_4-x_3}$

$\therefore$ points $(1, 1)$, $(2, 3)$, $(3, a)$ and $(b, 7)$ are on a straight line if

$$\frac{3-1}{2-1} = \frac{a-3}{3-2} = \frac{7-a}{b-3}$$

or, $2 = a - 3 = \frac{7-a}{b-3}$

from first two relations $a - 3 = 2 \quad \Rightarrow a = 5$

from first and third relations $\frac{7-a}{b-3} = 2$

or, $\frac{7-5}{b-3} = 2$ $\qquad (\because a = 5)$

or, $2 = 2(b-3)$

or, $1 = b - 3$

or, $b = 4$

11. For what value of k given system of equations has infinitely many solution.

or, For what value of k following equations show coincident lines ?

or, For what value of k given system of equation is dependent ?

$kx + 3y = k - 3$, $12x + ky = k$

Solution : Given equations are

$$kx + 3y - (k-3) = 0 \text{ and } 12x + ky - k = 0$$

here, $a_1 = k$, $b_1 = 3$, $c_1 = -(k-3)$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$

$\therefore$ from, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\frac{k}{12} = \frac{3}{k} = \frac{-(k-3)}{-k}$$

or, $\frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$

from first two ratios, $\frac{k}{12} = \frac{3}{k}$

or, $k^2 = 36$ $\qquad \ldots$ (i)

or, $k = \pm 6$

from last two ratios, $\frac{3}{k}=\frac{k-3}{k}$

or, $3k=k^2-3k$

or, $k^2-6k=0$

or, $k=0, 6$

from (i) and (ii) we get that common value of k is 6.

Thus for $k=6$, given system of equation has infinitely many solution.

12. Find the value of a and b so that following system of equation has infinitely many solutions

$$2x-(a-4)\,y=2b+1$$

$$4x-(a-1)\,y=5b-1$$

Solution : System of equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ has infinitely many solution if

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Given equations are $2x-(a-4)\,y-(2b+1)=0$

and $4x-(a-1)\,y-(5b-1)=0$

here $a_1=2, b_1=-(a-4), c_1=-(2b+1)$

and $a_2=4, b_2=-(a-1), c_2=-(5b-1)$

Hence from (i), $\frac{2}{4}=\frac{-(a-4)}{-(a-1)}=\frac{-(2b+1)}{-(5b-1)}$

or, $\frac{1}{2}=\frac{a-4}{a-1}=\frac{2b+1}{5b-1}$

from first two ratio, $\frac{1}{2}=\frac{a-4}{a-1}$

or, $a-1=2a-8$

or, $8-1=2a-a$

or, $7=a$

or, $a=7$

from first and third ratio, $\frac{1}{2}=\frac{2b+1}{5b-1}$

or, $5b-1=4b+2$

or, $5b-4b=1+2$

or, $b=3$

Hence, $a=7,\ b=3$

13. For what value of k given pair of equation has no solution ?

or, For what value of k given lines are parallel ?

or, For what value of k given system of equation is inconsistent ?

$kx + 3y = 3$, $12x + ky = 6$

Solution : Pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ does not have a solution if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. In this situation lines are parallel

Given equations are $kx + 3y - 3 = 0$ and $12x + ky - 6 = 0$

here, $a_1 = k, b_1 = 3, c_1 = -3$ and $a_2 = 12, b_2 = k, c_2 = -6$

$\therefore$ from, $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ we get $\frac{k}{12} = \frac{3}{k}$

or, $k^2 = 12 \times 3 = 36$

or, $k = \pm 6$

taking $k = 6$, $\frac{a_1}{a_2} = \frac{6}{12} = \frac{1}{2} = \frac{b_1}{b_2}$

taking $k = -6$, $\frac{a_1}{a_2} = \frac{-6}{12} = \frac{-1}{2} = \frac{b_1}{b_2}$

also $\frac{c_1}{c_2} = \frac{-3}{-6} = \frac{1}{2}$

clearly at $k = 6$, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$ at $k = 6$ pair of equation has infinitely many solution.

Again at $k = -6$, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ at $k = -6$ pair of equation has no solution

$\therefore$ at $k = -6$ will be the required solution.

14. For what value of k given system of equation has unique solution.

or, For what value of k given lines are intersecting.

or, For what value of k pair of equation has independent solution ?

(i) $kx + 2y = 5$, $3x + y = 1$ (ii) $kx + 3y = 6$, $2x - ky = 10$

(iii) $2x + y = 5$, $6x + 3y = 2k$

Solution : System of equation $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution

if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

In this situation corresponding straight lines intersect each other and solution is consistant (independent)

(i) Given equations are

$$kx + 2y - 5 = 0 \text{ and } 3x + y - 1 = 0$$

here $a_1 = k, b_1 = 2, c_1 = -5$ and $a_2 = 3, b_1 = 1, c_2 = -1$

$\therefore$ from $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ from $\frac{k}{3} \neq \frac{2}{1}$

or, $k \neq 6$

(ii) Given equations are

$$kx + 3y - 6 = 0 \text{ and } 2x - ky - 10 = 0$$

here, $a_1 = k, b_1 = 3, c_1 = -6$ and $a_2 = 2, b_2 = -k, c_2 = -10$

$\therefore$ from $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, $\frac{k}{2} \neq \frac{3}{-k}$

or, $k^2 \neq -6$ which is true for all values of k.

Hence for any real value of k system of equation has unique solution

(iii) Given equations are

$$2x + y - 5 = 0 \text{ and } 6x + 3y - 2k = 0$$

here, $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$ and $\frac{b_1}{b_2} = \frac{1}{3}$

$\because$ $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\therefore$ there is no real value of k for which system of equation has unique solution

15. Find the area of region bounded by straight lines $|x| + |y| = k$

Solution : Given equation is $|x| + |y| = k$

when $x \geq 0,\ y \geq 0,\ x + y = k$ ...(i)

$x \leq 0,\ y \geq 0,\ -x + y = k$ ...(ii)

$x \leq 0,\ y \leq 0, -x - y = k$ ...(iii)

$x \geq 0, y \leq 0,\ x - y = k$ ...(iv)

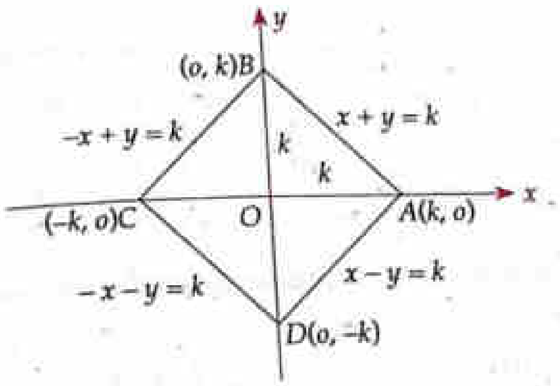
line (i) lies in the first quadrant and cuts axes respectively at (k, o) and (o, k)

line (ii) line in the second quadrant and cuts axes respectively at $(-k$ and (o, k)

line (iii) line in the third quadrant and cuts axes respectively at $(-k, o)$ and $(o, -k)$

line (iv) line in the fourth quadrant and cuts axes respectively at (k, o) and $(o, -k)$

Hence, graph of area enclosed by these lines are as follows



Clearly it is a square with each side $= \sqrt{k^2+k^2} = \sqrt{2}\,k$

Hence, Required Area $= \left(\sqrt{2}\,k\right)^2 = 2k^2$

Trick : Area enclosed by $|x| + |y| = k$ is $2k^2$.

Exercise—3A

1. What is the distance of point $(3, 4)$ from the x–axis ?

   (a) 3 (b) 4 (c) 5 (d) $\sqrt{5}$

2. If point $(a, a+2)$ lies on the line $y = 3x + 5$, then distance of the point from y–axis is

   (a) $\frac{3}{2}$ (b) 3 (c) $\frac{1}{2}$ (d) $\frac{7}{2}$

3. What is the distance of point of intersection of straight lines $2x + 3y = 6$ and $y = x + 7$ from origin ?

   (a) 7 (b) 3 (c) 4 (d) 5

4. What is the difference between distances of mid point of points $(-3, 5)$ and $(7, -6)$ from x–axis and y–axis ?

   (a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{4}$ (d) $\frac{3}{4}$

5. Area of triangle formed by coordinate axes and straight line $y = 3x - 14$ is

   (a) $\frac{196}{3}$ (b) $\frac{49}{3}$ (c) $\frac{98}{3}$ (d) $\frac{3}{98}$

6. The length intercepted by the straight line $12x - 9y = 108$ between the coordinate axes is
(a) 12 unit (b) 18 unit (c) 15 unit (d) 9 unit

7. The length intercepted by the straight line $y = mx + c$ between the coordinate axes is
(a) $\frac{c}{m}\sqrt{1+m^2}$ (b) $\frac{c}{m}$
(c) $\sqrt{c^2+m^2}$ (d) None of these

8. Length intercepted by the straight line $8x - 15y = 60$ between the coordinate axes is
(a) $\frac{23}{2}$ (b) $\frac{23}{4}$ (c) $\frac{17}{2}$ (d) $\frac{17}{4}$
[SSC Tier-I 2014]

9. Straight line $2x + 3y + 10 = 0$ intersects coordinate axes respectively at the points
(a) $(-5,0)\left(0,\frac{-10}{3}\right)$ (b) $\left(\frac{10}{3},0\right)$ (0, 5)
(c) (–5, 0), $\left(0,\frac{10}{3}\right)$ (d) $\left(\frac{-10}{3},0\right),\left(0,\frac{-5}{3}\right)$

10. Equation of the straight lines passing through points (4, 3) and respectively parallel to x–axis and y–axis are
(a) $x = 4, y = 3$ (b) $x = 3, y = 4$
(c) $x + 4 = 0, y + 3 = 0$ (d) $x + 3 = 0, y + 4 = 0$

11. Equation of straight line passing through the points (2, 0) and (0, –3) is
(a) $\frac{x}{2}-\frac{y}{3}=1$ (b) $\frac{y}{3}-\frac{x}{2}=1$ (c) $\frac{x}{3}-\frac{y}{2}=1$ (d) $\frac{y}{2}-\frac{x}{3}=1$

12. Area of triangle between the straight line $3x + 2y - 6 = 0$ and coordinate axes is ... square unit.
(a) 3 (b) $\frac{3}{2}$ (c) 6 (d) $\frac{9}{2}$

13. Area of triangle formed by the straight line $8x - 3y + 24 = 0$ and coordinate axes is
(a) 24 sq. unit (b) 12 sq. unit (c) 6 sq. unit (d) 18 sq. unit

14. Area of triangle formed by straight line $y = mx + c$ with coordinate axes is
(a) $\frac{c^2}{m}$ (b) $\frac{m}{c^2}$ (c) $\frac{c^2}{2m}$ (d) $\frac{2m}{c^2}$

15. Area of triangle formed by straight lines $2x + 3y = 5$ and $y = 3x - 11$ with x–axis is
(a) $\frac{11}{3}$ sq. unit (b) $\frac{22}{3}$ sq. unit (c) $\frac{11}{6}$ sq. unit (d) $\frac{11}{12}$ sq. unit

6. Area of triangle formed by straight lines $4x-3y+4=0$, $4x+3y-20=0$ and x–axis is

(a) 3 sq. unit (b) 6 sq. unit (c) 12 sq. unit (d) 24 sq. unit

7. Area of triangle formed by straight lines $3x-y=3$, $x-2y+4=0$ and $y=0$ is

(a) $\frac{15}{4}$ sq. unit (b) $\frac{15}{2}$ sq. unit

(c) 15 sq. unit (d) Cannot be determined

8. Area of triangle formed by straight lines $4x-y=4$, $3x+2y=14$ and y–axis is

(a) $\frac{11}{2}$ sq. unit (b) $\frac{11}{4}$ sq. unit (c) 22 sq. unit (d) 11 sq. unit

9. Ratio of area of triangle formed by straight lines $2x+3y=4$ and $3x-y+5=0$ with x–axis and y–axis is

(a) $1:2$ (b) $2:1$

(c) $4:1$ (d) None of these

0. What is the height of triangle formed by straight lines $3x+y=10$, $2x-3y=6$ and x–axis when x–axis is the base of the triangle ?

(a) 3 (b) 1 (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

1. Area of triangle formed by straight lines $2x-3y+6=0$, $2x+3y-18=0$ and $y-1=0$ is

(a) 27 sq. unit (b) $\frac{27}{2}$ sq. unit

(c) 9 sq. unit (d) None of these

2. Area of triangle formed by straight lines $x+y=4$, $2x-y=2$ and $x-2=0$ is

(a) 4 sq. unit (b) 9 sq. unit

(c) $\frac{7}{2}$ sq. unit (d) None of these

3. Area of quadrilateral formed by straight lines $x+y=2$, $3x+4y=24$ and coordinate axes is

(a) 22 sq. unit (b) 26 sq. unit (c) 44 sq. unit (d) 11 sq. unit

4. A linear equation $3x+4y=24$, intersects x–axis and y–axis respectively at points A and B. If $P(2, 0)$ and $Q\left(0, \frac{3}{2}\right)$ respectively lies on the straight line OA and OB, then area of the quadrilateral $PABQ$ is

(a) $\frac{5}{2}$ sq. unit (b) $\frac{15}{2}$ sq. unit (c) $\frac{35}{2}$ sq. unit (d) $\frac{45}{2}$ sq. unit

25. Area of triangle formed by straight lines $3x - 4y = 0$, $x = 4$ and x-axis
(a) 6 sq. unit (b) 8 sq. unit (c) 12 sq. unit (d) 16 sq. unit

26. If lines are drawn from the point $(-5, 3)$ to the coordinate axes then area of quadrilateral formed by these lines with coordinate axes is
(a) $\frac{15}{2}$ (b) 15 (c) 30 (d) $\frac{25}{2}$

27. If $b > a$, $d > c$ then area of quadrilateral formed by straight lines $x = a$, $x = b$, $y = c$ and $y = d$ is
(a) $(b - a)(d - c)$
(b) $\frac{1}{2}(b - a)(d - c)$
(c) $(b + a)(d + c)$
(d) $\frac{1}{2}(b + a)(d + c)$

28. Area of quadrilateral formed by straight lines $2x = -5$, $2y = 3$, $x + 1 = 0$ and $y + 2 = 0$ is
(a) $\frac{21}{2}$ sq. unit (b) $\frac{21}{4}$ sq. unit (c) $\frac{21}{8}$ sq. unit (d) $\frac{21}{16}$ sq. unit

29. Area of triangle formed by straight lines $3x + 4y = 24$, $x = 8$ and $y = 6$
(a) 12 sq. unit
(b) 24 sq. unit
(c) 48 sq. unit
(d) None of these

30. Area of quadrilateral formed by straight lines $x = 1$, $x = 3$, $y = 2$ and $y = x + 3$ is
(a) 6 sq. unit
(b) 12 sq. unit
(c) 3 sq. unit
(d) None of these

31. Area of triangle formed by straight lines $x - y = 0$, $x + 2y = 0$ and $y = 3$
(a) 27 sq. unit (b) 54 sq. unit (c) 9 sq. unit (d) 13.5 sq. unit

32. Area of triangle formed by straight lines $x - y = 0$, $x + y = 0$ and $2x = 5$
(a) 25
(b) $\frac{25}{2}$
(c) $\frac{25}{4}$
(d) None of these

33. Area enclosed by equation $y = |x| - 5$ with x-axis is
(a) 25 sq. unit
(b) 12.5 sq. unit
(c) 50 sq. unit
(d) None of these

34. Area enclosed by the equation $|x| + |y| = 4$ is
(a) 16 (b) 32 (c) 24 (d) 48

35. Area enclosed by equation $y = |x| - 1$ and $y = 1 - |x|$ is
(a) 2 (b) 4 (c) 8 (d) 16

36. Which of the following system of equations has unique solutions?
(a) $3x + 4y = 11, 6x + 8y = 15$
(b) $x + 2y = 3, 2x + 4y = 7$
(c) [illegible]
(d) $4x + 3y = 5, 4x - 3y = 5$

37. Which of the following system of equations has infinitely many solutions?

(a) $2x + 3y = 12, 4x + 6y = 12$
(b) $x - 3y = 10, 2x - 6y = 20$
(c) $x = 4, y = 3$
(d) $3x - 4y = 0, 3x + 4y = 0$

38. Which of the following system of equations doesnot have a solution ?

(a) $2x = 3y, 4x = 5y$
(b) $2x + y = 7, 4x + 2y = 14$
(c) $3x - 4y = 8, 3x - 4y = 12$
(d) $4x + y = 7, 4y + x = 7$

39. For what value of k system of equations $3x + 4y = 19, y - x = 3$ and $2x + 3y = k$ has a solution ?

(a) 11
(b) −11
(c) 14
(d) −14

40. Which of the following pair represent equation of parallel straight lines.

(a) $2x + 3y = 4, 4x + 6y = 9$
(b) $x + 2y = 4, 2x + y = 4$
(c) $y = 3x + 5, x = 3y + 5$
(d) None of these

41. Which of the following pair of straight lines donot represent intersecting lines ?

(a) $y = \frac{x}{3} + \frac{5}{4}, y = \frac{x}{2} + \frac{7}{3}$
(b) $3x - 4y = 0, x = 0$
(c) $4x + 3y = 1, y = 0$
(d) $2x + 3y = 7, 4x + 6y = 15$

42. The value of K for which system of equation $5x + 2y = K$ and $10x + 4y - 3 = 0$ has infinitely many solution is

(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) 6
(d) $\frac{1}{6}$

43. For what value of K system of equation $x + 3y = K$ and $2x + 6y = 2K$ has infinitely many solution ?

(a) $K = 1$
(b) $K = 2$
(c) for all real values of K
(d) for no real value of K

44. Values of a and b so that system of equations $2x + 3y = 7$ and $2ax + (a + b)\,y = 28$ has infinitely many solutions are

(a) $a = 4, \ b = 8$
(b) $a = 8, \ b = 4$
(c) $a = -4, b = -8$
(d) $a = -8, b = -4$

45. For what values of k straight lines $2x - ky + 3 = 0$ and $3x + 2y - 1 = 0$ are parallel ?

(a) $\frac{4}{3}$
(b) $\frac{-4}{3}$
(c) $\frac{2}{3}$
(d) $\frac{-2}{3}$

46. Value of k for which system of equations $kx + 2y = 5, 3x + y = 1$ has unique solution is

(a) $k = 1$
(b) $k = 2$
(c) $k = 3$
(d) all are true

Answers—3A

1. (b)	2. (a)	3. (d)	4. (b)	5. (c)	6. (c)	7. (a)	8. [illegible]
9. (a)	10. (b)	11. (a)	12. (a)	13. (b)	14. (c)	15. (d)	16. [illegible]
17. (b)	18. (d)	19. (b)	20. (b)	21. (b)	22. (d)	23. (a)	24. [illegible]
25. (a)	26. (b)	27. (a)	28. (c)	29. (c)	30. (a)	31. (d)	32. [illegible]
33. (a)	34. (b)	35. (a)	36. (d)	37. (b)	38. (c)	39. (c)	40. [illegible]
41. (d)	42. (c)	43. (c)	44. (a)	45. (b)	46. (d)		

Explanation

2. (a) $\because$ $(a, a+2)$, passes through line $y = 3x + 5$

$\therefore$ $a + 2 = 3a + 5$

or, $a = \frac{-3}{2}$

$\therefore$ distance from y–axis $= |a| = \left|\frac{-3}{2}\right| = \frac{3}{2}$

3. (d) solving $2x + 3y = 6$

and $y = x + 7$ we get $(x, y) = (-3, 4)$

$\therefore$ distance from origin $= \sqrt{(-3)^2 + 4^2} = 5$

4. (b) Mid point of $(-3, 5)$ and $(7, -6) = \left(\frac{-3+7}{2}, \frac{5-6}{2}\right) = \left(2, \frac{-1}{2}\right)$

distance of point $\left(2, \frac{-1}{2}\right)$ from x–axis $= \left|-\frac{1}{2}\right| = \frac{1}{2}$

distance of point $\left(2, \frac{-1}{2}\right)$ from y–axis $= 2$

Required difference $= \left|\frac{1}{2} - 2\right| = \frac{3}{2}$

5. (c) Equation of line is $3x - y - 14 = 0$

$\therefore$ Required area $= \frac{1}{2}\left|\frac{c^2}{ab}\right| = \frac{1}{2}\left|\frac{(-14)^2}{3(-1)}\right| = \frac{98}{3}$ square unit.

6. (c) Required intercept $= \sqrt{\left(\frac{c}{a}\right)^2 + \left(\frac{c}{b}\right)^2} = \sqrt{\left(\frac{108}{12}\right)^2 + \left(\frac{108}{9}\right)^2}$

$= \sqrt{81 + 144} = \sqrt{225} = 15$

7. (a) Required length $= \sqrt{\left(\frac{c}{m}\right)^2 + \left(\frac{c}{1}\right)^2} = \frac{c}{m}\sqrt{1 + m^2}$

9. (c) Length of intercept made by line $ax + by + c = 0$

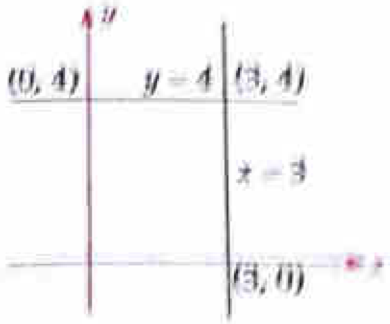
between axes $= \sqrt{\frac{c^2}{a^2} + \frac{c^2}{b^2}}$

$\therefore$ Required length $= \sqrt{\left(\frac{60}{8}\right)^2 + \left(\frac{60}{15}\right)^2}$

$= 60\sqrt{\left(\frac{1}{8^2} + \frac{1}{15^2}\right)}$

$= 60\sqrt{\left(\frac{15^2 + 8^2}{8^2 \times 15^2}\right)} = 60 \times \frac{17}{8 \times 15} = \frac{17}{2}$ sq. unit

10. (b) See the figure, solution is obvious



11. (a) If a line cuts x-axis at $(a, 0)$ and cuts y-axis at $(0, b)$ then its equation is $\frac{x}{a} + \frac{y}{b} = 1$

here $a = 2$ and $b = -3$

12. (a) Required Area $= \frac{1}{2}\left|\frac{c^2}{ab}\right| = \frac{1}{2}\left|\frac{6^2}{3 \times 2}\right| = \frac{36}{2 \times 3 \times 2} = 3$ square unit

13. (b) Required Area $= \frac{1}{2}\left|\frac{c^2}{ab}\right| = \frac{1}{2}\left|\frac{24 \times 24}{8(-3)}\right| = 12$ square unit

14. (c) Required Area $= \frac{1}{2}\left|\frac{c^2}{1 \times m}\right| = \frac{c^2}{2m}$

15. (d) Intercept made by $2x + 3y = 5$ on x-axis $= \frac{5}{2}$ (put $y = 0$)

Intercept made by $3x - y = 13$ on x-axis $= \frac{13}{3}$ (put $y = 0$)

(Solving both equations, point of intersection is $(4, -1)$)

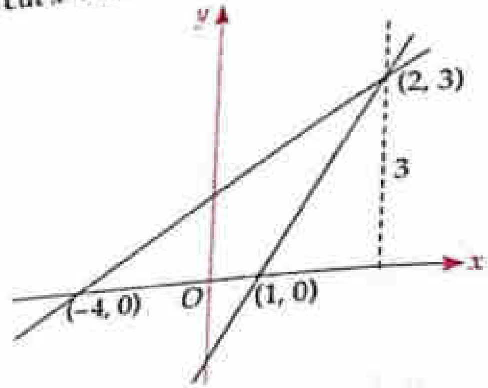
$\therefore$ Required Area $= \frac{1}{2}$ (difference between x-intercept) $\times$ (y-coordinate of point of intersection)

$= \frac{1}{2}\left|\left(\frac{13}{3} - \frac{5}{2}\right)(-1)\right| = \frac{11}{12}$

16. (c) Required Area $= \frac{1}{2}\ |5-(-1)| \times 4 = 12$ square unit

17. (b) Solving two given lines $x = 2, y = 3$

Both lines cut x-intercept respectively at $(-4, 0)$ and $(1, 0)$



$\therefore$ Required Area $= \frac{1}{2}$ (difference between x-intercept)

$\times$ (y-coordinate of point of intersection)

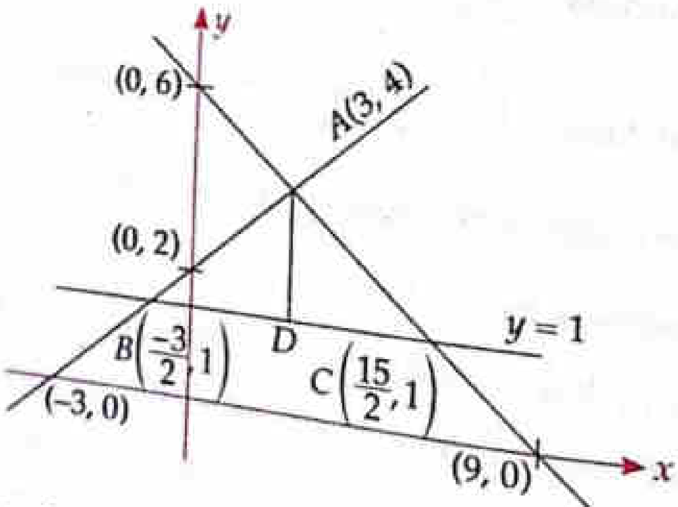
$$= \frac{1}{2}\left|\left(1-(-4)\right)\times 3\right| = \frac{15}{2} \text{ square unit}$$

18. (d) Required Area $= \frac{1}{2}\ |-4-7| \times 2 = 11$ square unit

(b) Required ratio $= \dfrac{\frac{1}{2}\left|\left(2-\left(\frac{-5}{3}\right)\right)2\right|}{\frac{1}{2}\left|\left(5-\frac{4}{3}\right)(-1)\right|} = \dfrac{\frac{22}{3}}{\frac{11}{3}} = \frac{22}{11} = \frac{2}{1}$

21. (b) Lines $2x - 3y + 6 = 0$ and $2x + 3y - 18 = 0$

intersect at $A\,(3, 4)$



Lines $y - 1 = 0$ and $2x - 3y + 6 = 0$ cuts at $B\left(-\frac{3}{2}, 1\right)$

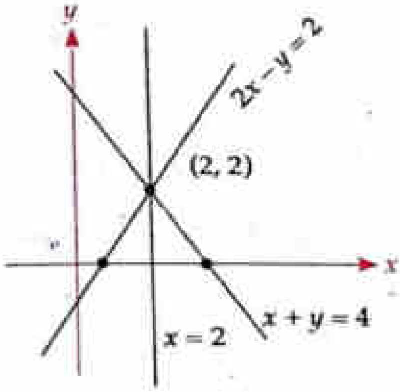
0)

Lines $y-1=0$ and $2x+3y=18$ cuts at $C\left(\frac{15}{2}, 1\right)$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$$= \frac{1}{2}\left|\frac{15}{2}-\left(\frac{-3}{2}\right)\right| \times |4-1| = \frac{1}{2} \times 9 \times 3 = \frac{27}{2} \text{ square unit}$$

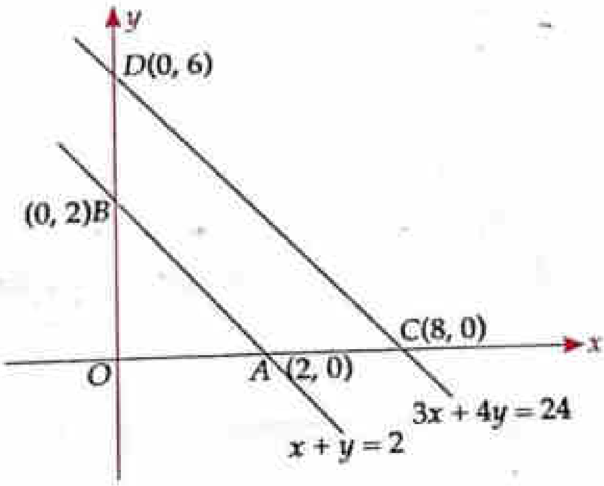
22. (d) Solve the equations taking two at a time.



:ersection)

In each case $x = 2$, $y = 2$ i.e. lines are concurrent, so donot make a triangle.

23. (a) Required Area = Area of = $\Delta OCD \sim$ area of ΔOAB (see figure)



$$= \frac{1}{2} \times 8 \times 6 - \frac{1}{2} \times 2 \times 2 = 24 - 2 = 22 \text{ square unit}$$

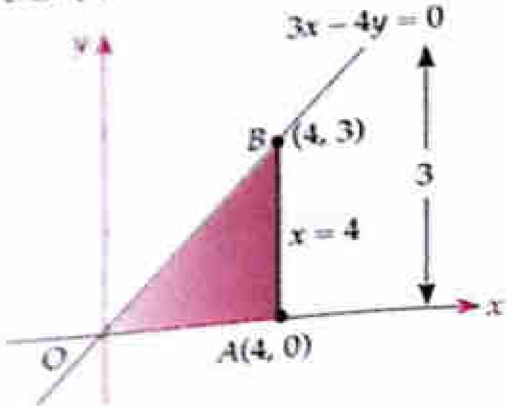
24. (d) Area of quadrilateral $PABQ$ = Area of ΔOAB – area of ΔOPQ

$$= \frac{1}{2} \cdot \left|\frac{c^2}{ab}\right| - \frac{1}{2} \cdot OP \cdot OQ$$

$$= \frac{1}{2} \cdot \left|\frac{24^2}{3 \times 4}\right| - \frac{1}{2} \cdot 2 \cdot \frac{3}{2}$$

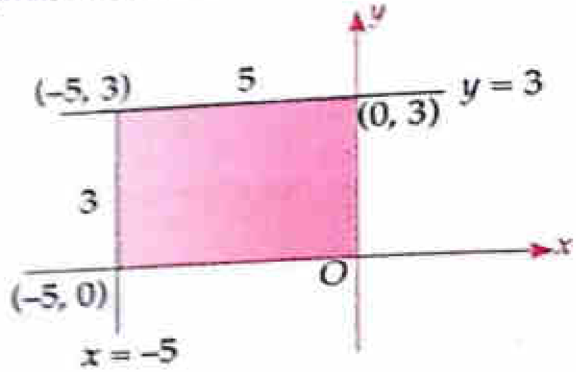
$$= 24 - \frac{3}{2} = \frac{45}{2} \text{ square unit}$$

25. (a) Required Area is show in figure. Point of intersection of line $x = 4$ and $3x - 4y = 0$ is (4, 3)



$$\text{Required Area} = \frac{1}{2} \times OA \times AB = \frac{1}{2} \times 4 \times 3 = 6 \text{ square unit}$$

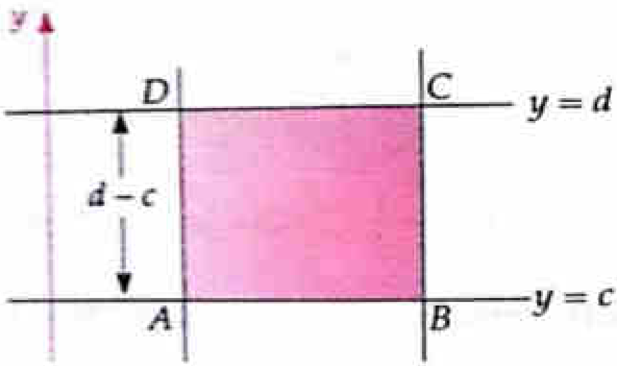
26. (b) See the figure, Area enclosed is a rectangle



$$\text{Required Area} = \left|(-5)3\right| = 15 \text{ square unit}$$

27. (a) $x = a$ and $x = b$, y are straight lines parallel to y–axis.

$y = c$ and $y = d$, x are straight lines parallel to x–axis. Point of intersection of these lines form a rectangle (see figure)

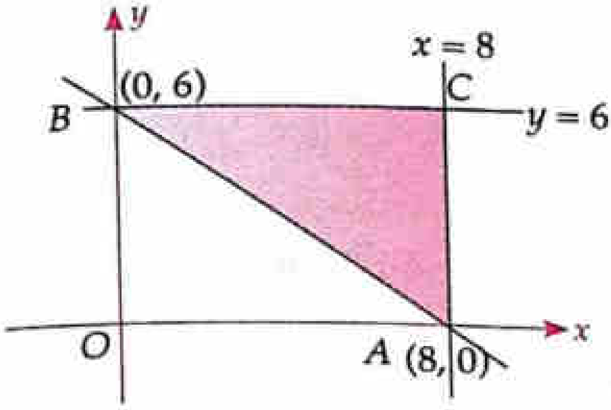


28. (c) Given lines are $x = \frac{-5}{2}$, $y = \frac{3}{2}$, $x = -1$ and $y = -2$

According to above question Required Area $= \frac{1}{2}(b-a)(c-d)$

$$= \frac{1}{2}\left|\left(-1+\frac{5}{2}\right)\left(\frac{3}{2}+2\right)\right| = \frac{1}{2}\times\frac{3}{2}\times\frac{7}{2} = \frac{21}{8} \text{ square unit}$$

29. (c) Line is $3x + 4y = 24$



or, $\frac{3x}{24} + \frac{4y}{24} = 1$

or, $\frac{x}{8} + \frac{y}{6} = 1$

cuts x–axis at (8, 0) and y–axis at (0, 6)

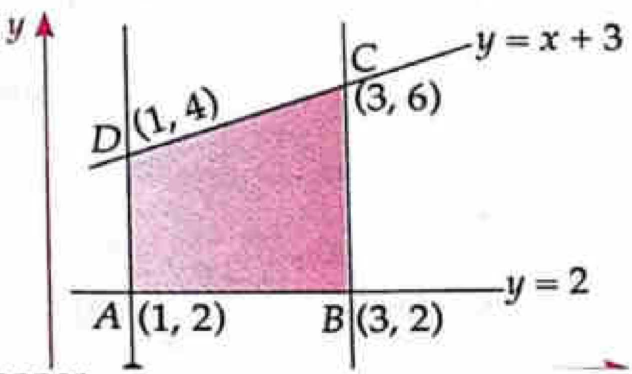
line $x = 8$ is parallel to y–axis while line $y = 6$ is parallal to x–axis.

These three lines formed a right angled ΔABC *(see figure)*

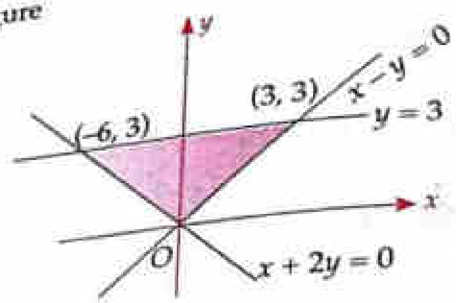
Its area $= \frac{1}{2}\times 8\times 6 = 24$ square unit

30. (a) Point of intersection of given lines are $A(1,2)$, $B(3, 2)$, $C(3, 6)$ and $D(1, 4)$ *(see figure)*

It is a trapezium.



31. (d) See the figure

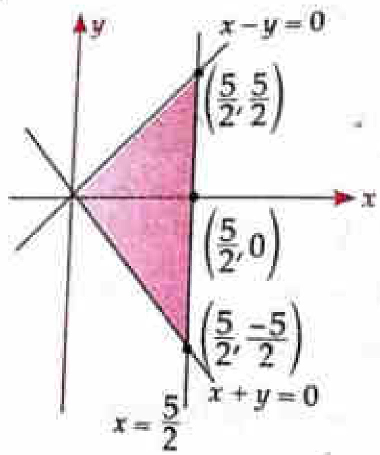


Required Area $= \frac{1}{2}\ |3-(-6)|\times 3$

$= \frac{1}{2}\times 9\times 3$

$= \frac{27}{2}$ square unit

32. (c) See the figure,



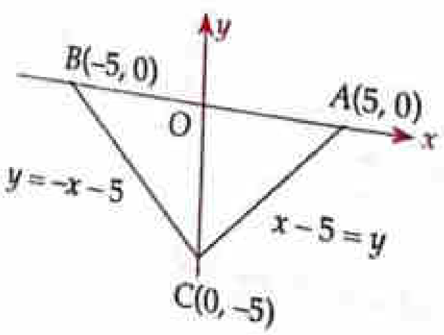
Required Area $= \frac{1}{2}\times\left(\frac{5}{2}-\left(-\frac{5}{2}\right)\right)\times\frac{5}{2}$

$= \frac{1}{2}\times 5\times\frac{5}{2} = \frac{25}{4}$ square unit.

33. (a) In the equation $y = |x| - 5$, there are two lines

$y = x - 5, x > 0$

and $y = -x - 5, x < 0$



these lines intersect each other at (0, –5) and respectively cut x–axis at (5, 0) and (–5, 0)

$\therefore$ Required Area $= \frac{1}{2} \times AB \times OC$

$= \frac{1}{2} \times 10 \times 5$

$= 25$ square unit.

34. (b) From solved example 15

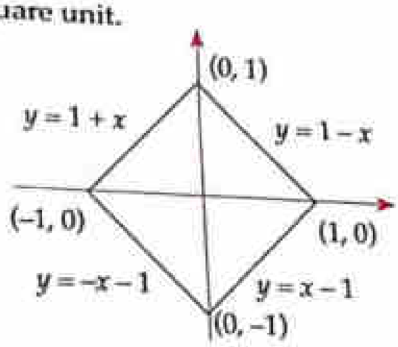
Required Area $= 2k^2 = 2 \times 4^2 = 32$ square unit.

35. (a) Here four lines are $y = x - 1, x \geq 0$

$y = -x - 1, x \leq 0$

$y = 1 - x, x \geq 0$

$y = 1 + x, x \leq 0$



These lines cut axes respectively at (1, 0), (0, 1), (–1, 0) and (0, –1) It is a square with each side $= \sqrt{1^2 + 1^2} = \sqrt{2}$

$\therefore$ Required Area $= \left(\sqrt{2}\right)^2 = 2$ square unit

36. (d) Checking each option one by one, in options (d)

$\frac{a_1}{a_2} = 1$ and $\frac{b_1}{b_2} = -1$ i.e. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So system of equation given in option (d) has unique solution.

37. (b) Option (b) follows (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$; rest are not.

38. (c) In option (c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = 1$ but $\frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

$\therefore$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ i.e. this system of equation has no solution.

39. (c) Solving $3x + 4y = 19$ and $y - x = 3$

we get $x = 1, y = 4$

putting $(x, y) = (1, 4)$ in $2x + 3y = k$

we have $2 \times 1 + 3 \times 4 = k \Rightarrow k = 14$

40. (a) In option (a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. Hence lines given in alternative (a) shows parallel lines.

41. (d) In option (d) $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$, which is condition for parallel lines.

43. (c) Here $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ is always true. It doesnot depend upon K.

44. (a) Required condition is $\frac{2a}{2}=\frac{a+b}{3}=\frac{28}{7}$

or, $a=\frac{a+b}{3}=4$

$\therefore\ a=4, a+b=12$

or, $a=4,\ \ b=8$

46. (d) System has unique solution if $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

$$\Rightarrow \frac{k}{3}\neq\frac{2}{1}$$

$$\Rightarrow k\neq 6$$

so options (a), (b), (c) are correct.

Exercise—3B

1. In the xy–coordinate system, if (a, b) and $(a+3, b+k)$ are two points on the line defined by the equation $x=3y-7$, then $k=$

(a) 9 (b) 3 (c) $\frac{7}{3}$ (d) 1

[SSC Tier-I 2012]

2. The x–intercept of the graph of $5x-4y=20$ is

(a) 4 units (b) 5 units (c) 9 units (d) 1 units

[SSC Tier-I 2012]

3. A triangle is formed by the x–axis and the lines $2x+y=4$ and $x-y+1=0$ as three sides. Taking the side along x–axis as its base, the corresponding altitude of the triangle is

(a) 2 unit (b) 3 unit (c) $\sqrt{5}$ unit (d) 1 unit

[SSC Tier-I 2012]

4. The length of the portion of the straight line $8x+15y=120$ intercepted between the axes is

(a) 14 units (b) 15 units (c) 16 units (d) 17 units

[SSC Tier-I 2012]

5. The area of the triangle formed by the lines $5x+7y=35$ and $4x+3y=12$ and x–axis is

(a) $\frac{160}{13}$ sq. units (b) $\frac{150}{13}$ sq. units (c) $\frac{140}{13}$ sq. units (d) 10 sq. units

[SSC Tier-I 2012]

or parallel lines.

end upon K.

re two points on

) 1

[SSC Tier-I 2012]

1 units

[SSC Tier-I 2012]

and $x-y+1=0$

corresponding

1 unit

SSC Tier-I 2012]

20 intercepted

17 units

SSC Tier-I 2012]

$7y = 35$ and

0 sq. units

SC Tier-I 2012]

6. Area of the triangle formed by the graph of the line $2x-3y+6=0$ along with the coordinate axes is
(a) $\frac{3}{2}$ sq. units (b) 3 sq. units (c) 6 sq. units (d) $\frac{1}{2}$ sq. units
[SSC Tier-I 2012]

7. Area of the trapezium formed by x–axis, y–axis and the lines $3x+4y=12$ and $6x+8y=60$ is
(a) 31.5 sq. units (b) 48 sq. units (c) 36.5 sq. units (d) 37.5 sq. units
[SSC Tier-I 2012]

8. For what value of k system of equation $x+2y=5, 3x+ky+15=0$ doesnot have any solution ?
(a) 2 (b) -2 (c) 6 (d) -6
[SSC CPO 2012]

Answers—3B

1. (d) 2. (a) 3. (a) 4. (d) 5. (a) 6. (b) 7. (a) 8. (c)

Explanation

1. (d) $\because (a, b)$ lies on straight line $x = 3y-7$

$\therefore a = 3b-7$ ... (i)

$\therefore (a+3, b+k)$ lies on straight line $x=3y-7$

$\therefore a+3 = 3(b+k)-7$ ... (ii)

Subtracting (ii) from, (i) $a+3-a = 3(b+k)-7-(3b-7)$

or, $3 = 3k$ $\therefore k=1$

Second Method

Slope of line joining the points (a, b) and $(a+3, b+k) = \frac{b+k-b}{a+3-a} = \frac{k}{3}$

For the line $x = 3y-7$

$$\text{or, } y = \frac{x}{3}+\frac{7}{3}$$

$$\text{slope} = \frac{1}{3}$$

$\therefore \frac{k}{3} = \frac{1}{3}$

$\Rightarrow k = 1$

2. (a) Putting $y=0$ in $5x-4y=20$

$5x = 20$ $\therefore x = 4$

3. (a) Solving $2x+y=4$ and $x-y+1=0$, we get

$x=1, y=2$ i.e. Vertex of triangle is (1, 2)

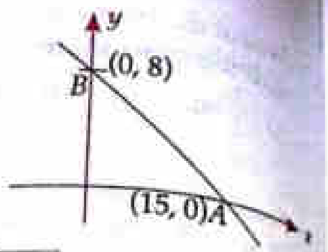
It is at a height of 2 unit from the x–axis

4. **(d)** In the equation $8x + 15y = 120$

Putting $x = 0$, $y = 8$

Putting $y = 0$, $x = 15$

Thus line cuts x-axis at A (15, 0) and y-axis at $B(0, 8)$



$\therefore$ length intercepted between $= AB = \sqrt{15^2 + 8^2} = \sqrt{289} = 17$

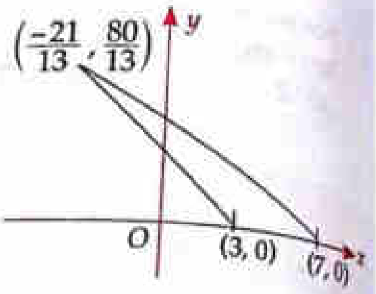
5. **(a)** In $5x + 7y = 35$ putting $y = 0$, $x = 7$

This line cuts x-axis at (7, 0)

Similarly $4x + 3y = 12$, cuts x- axis at (3, 0)

Solving $5x + 7y = 35$ and $4x + 3y = 12$

$$(x, y) = \left(\frac{-21}{13}, \frac{80}{13}\right)$$



From figure it is clear that base of triangle $= 7 - 3 = 4$

and height $= \frac{80}{13}$

$\therefore$ Area $= \frac{1}{2} \times 4 \times \frac{80}{13} = \frac{160}{13}$ square unit

(b) Required Area $= \frac{1}{2}\left|\frac{c^2}{ab}\right| = \frac{1}{2}\left|\frac{6^2}{2(-3)}\right| = \frac{36}{12} = 3$ square unit

$x + 4y = 12 \Rightarrow \frac{3x + 4y}{12} \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$

$6x + 8y = 60 \Rightarrow \frac{6x + 8y}{60} = 1$

$$\Rightarrow \frac{x}{10} + \frac{y}{\frac{15}{2}} = 1$$

see figure, Area of trapezium $ABCD$ = area of ΔOCD – area of ΔOAB

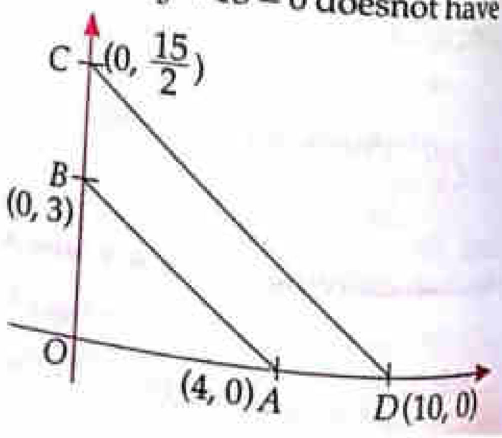
$= \frac{1}{2} \times 10 \times \frac{15}{2} - \frac{1}{2} \times 3 \times 4 = \frac{150}{4} - 6 = 37{\cdot}5 - 6 = 31{\cdot}5$ square unit

8. **(c)** System of equation $x + 2y - 5 = 0$ and $3x + ky + 15 = 0$ doesnot have Solution if $\frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15}$

$\Rightarrow \frac{1}{3} = \frac{2}{k}$ and $\frac{2}{k} \neq \frac{-5}{11}$

$\therefore$ $k = 6$ and $k \neq -6$

Hence, $k = 6$



★★★

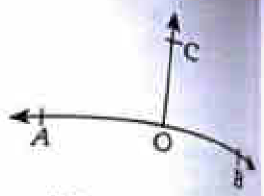
04

Lines and Angles

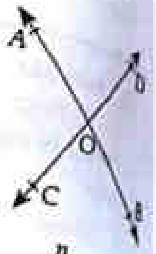
Fundamental terms and Definitions :

1. Line segment and ray : The part of a straight line whose both ends are fixed is called a line segment. If one point of a line is fixed, it is called a ray.
2. Collinear points and Non-collinear points : If three or more points lie on a straight line, they are called collinear point. If three or more points donot lie on a straight line, they are called non-collinear points.
3. Types of angles : According to measurement, angles are of following types.
   - 3.1. Acute angle : If an angle lies between 0° and 90°, it is called acute angle.
   - 3.2. Right angle : An angle whose measurement is 90° is called a right angle.
   - 3.3. Obtuse angle : If an angle lies between 90° and 180°, it is called obtuse angle.
   - 3.4. Straight angle : An angle whose measurement is 180° is called a straight angle.
   - 3.5. Reflex angle : If an angle lies between 180° and 360°, it is called Reflex angle.
4. Complementary angles and Supplementary angles : If sum of two angles

6. **Linear pair of angles** : In the adjacent figure $\angle AOC$ and $\angle COB$ are adjacent angles and AOB is a straight line i.e., uncommon sides of adjacent sides form a straight line. Such angles are called linear pair of angles.

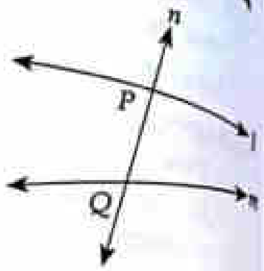


7. **Vertically opposite angles** : If two straight lines AB and CD intersect each other at point O, then angles facing each other is called vertically opposite angles. In the adjacent figure, $\angle AOD$ and $\angle BOC$ are one pair of vertically opposite angles, while $\angle AOC$ and $\angle BOD$ are another pair of vertically opposite angles.

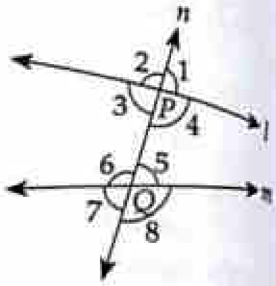


8. **Transversal line** : A straight line intersecting two or more lines at different points is called a transversal line.

In the given figure straight line n intersects two different lines l and m respectively at point P and Q, so line n is a transversal line.



9. **Exterior angles and Interior angles** : In the figure given below, a transversal line n intersects two straight lines l and m respectively at P and Q. Around each point P and Q, four angles are formed, among these angles $\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$ are called exterior angles while $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$ are called interior angles.



10. **Corresponding angles and Alternate angles** : In the figure drawn above

10.1. $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$ and $\angle 4$ and $\angle 8$ are called pair of corresponding angles.

10.2. "$\angle 4$ and $\angle 6$" and "$\angle 3$ and $\angle 5$" are called pair of alternate interior angles.

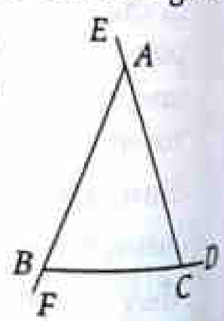
10.3. "$\angle 1$ and $\angle 7$" and "$\angle 2$ and $\angle 8$" are called alternate exterior angles.

10.4. "$\angle 4$ and $\angle 5$" and "$\angle 3$ and $\angle 6$" are called consecutive interior angles or Alternate interior/exterior allied angles or co-interior angle.

All type of alternate angles are commonly known as alternate angles.

11. **Exterior angle and Interior opposite angle of a triangle** : In the adjacent figure sides BC, CA and AB of triangle ABC are respectively produced to points D, E and F. $\angle ACD$, $\angle BAE$ and $\angle CBF$ thus formed are called exterior angles of the triangle.

Interior angles $\angle A$ and $\angle B$ are called interior opposite



angles to the exterior angle $\angle ACD$. Similarly $\angle B$ and $\angle C$ are interior opposite angles to the exterior angle BAE etc.

12. Types of triangles according to sides :

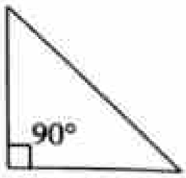
12.1. Equilateral triangle : When all the sides of a triangle are equal, it is called an equilateral triangle.

12.2. Isosceles triangle : If any two sides of a triangle are equal, it is called an isosceles triangle.

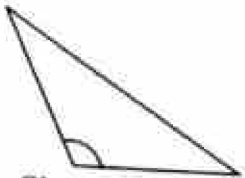
12.3. Scalene triangle : If sides of a triangle are unequal, it is called a scalene triangle.

13. Types of triangles according to their angles :


Acute angle triangle



Right angle triangle


Obtuse angle triangle

13.1. Acute angle triangle : If all the three angles of a triangle are acute, then the triangle is called an acute angle triangle.

13.2. Right angle triangle : If one of the angle of a triangle is right angled ($= 90^\circ$) then it is called a right angle triangle.

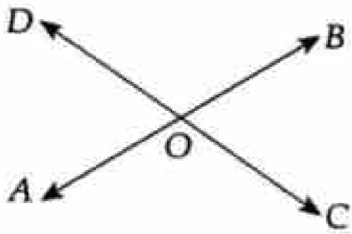
A triangle has at most one right angle.

13.3 Obtuse angle triangle : If one of the angle of a triangle is obtuse (lies between 90° and 180°) then it is called an obtuse angle triangle.

A triangle has at most one obtuse angle.

Some Theorems (Results) based on angles and straight line

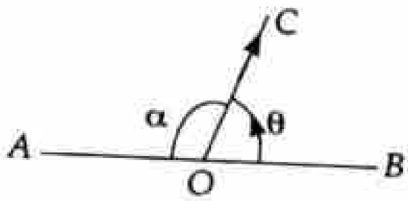
1. If two straight lines intersect each other then vertically opposite angles are equal. In the given figure,



$\angle BOC = \angle AOD$ (Vertically opposite angles)

$\angle AOC = \angle BOD$ (Vertically opposite angles)

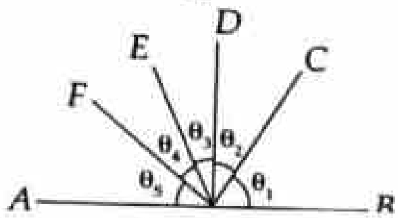
2. If a ray is inclined on a line then the sum of linear pair of angles thus formed is equal to 180° and its converse is also true.



In the given figure ray OC is standing (inclined) on the line AB,

Therefore $\theta + \alpha = 180^\circ$

Conversely, if $\theta + \alpha = 180^\circ$ then AOB will be a straight line.

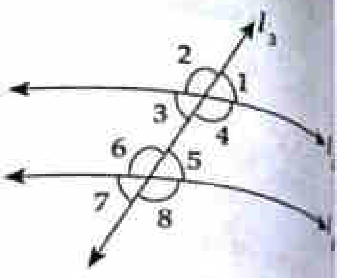


In general, in the adjacent figure many rays are coming out from a point O on the straight line AB,

$\therefore \quad \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 = 180°$

Converse of this statement is also true.

3. If a transversal line intersects two parallel lines then the pair of corresponding angles thus formed are equal and its converse is also true.



In the adjacent figure, two parallel lines l_1 and l_2 are intersected by a transversal line l_3 and thus the following pair of corresponding angles are equal—

$\angle 1 = \angle 5, \angle 2 = \angle 6, \angle 3 = \angle 7$ and $\angle 4 = \angle 8$

Conversely, it at least one of the pair of corresponding angles are equal (say $\angle 1 = \angle 5$) the lines l_1 and l_2 are parallel. If $\angle 2 = \angle 6$ then the two lines are parallel etc.

4. If a transverse line intersects two parallel lines then pair of alternate angles are equal and its converse is also true. In above figure,

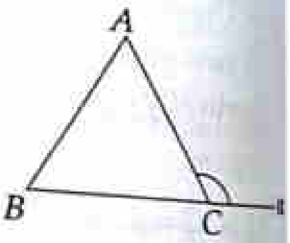
$\angle 3 = \angle 5$ and $\angle 4 = \angle 6$ (Alternate interior angles)

$\angle 1 = \angle 7$ and $\angle 2 = \angle 8$ (Alternate exterior angles)

When a transversal line intersects two parallel lines, sum of consecutive interior angles (or allied angles or co-interior angles is equal to 180° (i.e., consecutive interior angles are supplementary), its converse is also true. In the above figure,

$\angle 4 + \angle 5 = 180°$ and $\angle 3 + \angle 6 = 180°$

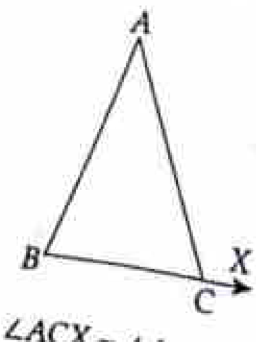
6. The sum of all the three angles of a triangle is equal to 180° (i.e., two right angle)



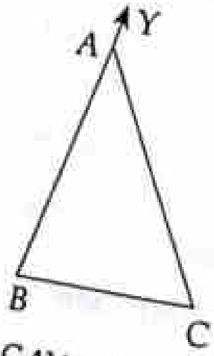
$\angle A + \angle B + C = 180°$

or, $\angle BAC + \angle ABC + \angle BCA = 180°$

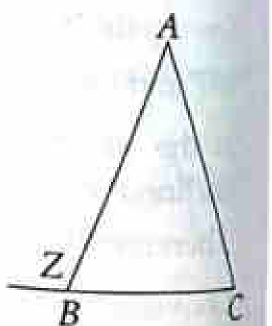
7. If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of two interior opposite angles.



In fig. $\angle ACX = \angle A + \angle B$

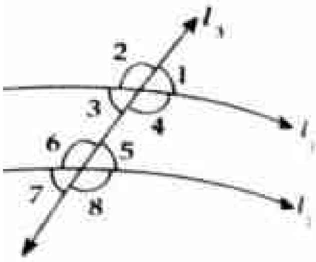


$\angle CAY = \angle B + \angle C$



$\angle ABZ = \angle A + \angle C$

...re coming out from a point



...nding angles are equal
... = ∠6 then the two lines

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... above figure,

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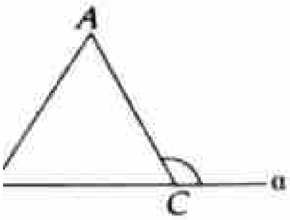
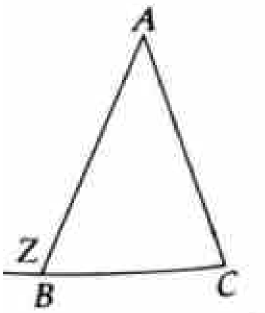
...s converse is also true



... angle so formed is

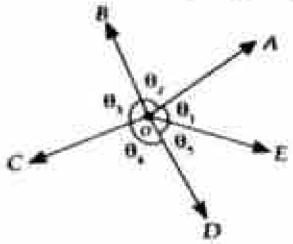


$\angle ABZ = \angle A + \angle C$

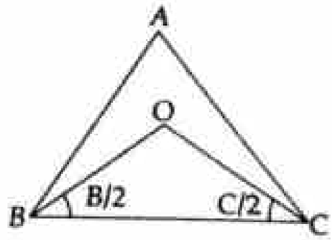
...me Important Points to Solve Objective Questions.

1. Sum of all angles around a point is 360°

In the figure given below $\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 = 360°$



2. If OB and OC are angle bisector of base angles $\angle B$ and $\angle C$ of ΔABC, then $\angle BOC = 90° + \frac{\angle A}{2}$



Proof : In the adjacent figure,

$\angle OBC = \frac{\angle B}{2}$ and $\angle OCB = \frac{\angle C}{2}$

$\therefore \angle BOC = 180° - \frac{\angle B}{2} - \frac{\angle C}{2}$

$= 180° - \left(\frac{B+C}{2}\right)$

$= 180° - \left(\frac{180° - A}{2}\right)$

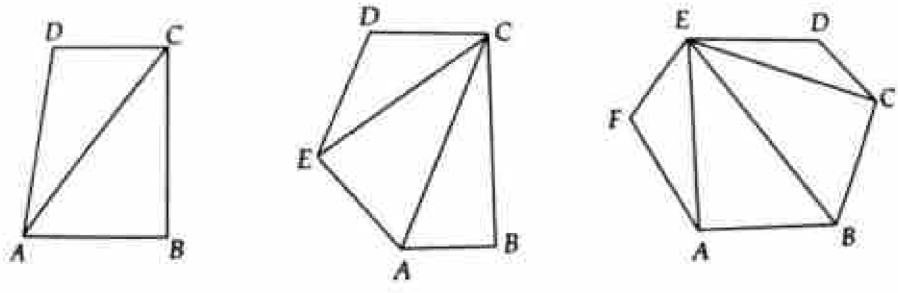
$(\because \angle A + \angle B + \angle C = 180° \Rightarrow \angle B + \angle C = 180° - \angle A)$

$= 90° + \frac{\angle A}{2}$

3. (i) Sum of all the internal angles of a Quadrilateral is 360°

(ii) Sum of all the internal angles of a pentagon (five sides) is 540°.

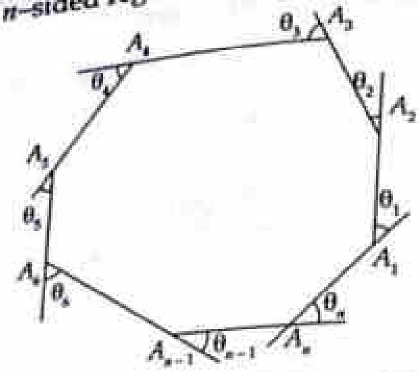
(iii) Sum of all the internal angles of a hexagon (six sides) is 720° etc.



It is due to the fact that a quadrilateral can be divided into two triangles, a pentagon can be divided into three triangles, a hexagon can be divided into four triangles etc.

4. From the above result we can conclude that sum of all the internal angles of a n sided polygon is $(n-2) \times 180°$.

5. Each angle of a n–sided regular polygon is $\frac{(n-2)180^\circ}{n}$.



6. In the above figure, side of a polygon are produced in the same order. Angle $\theta_1 + \theta_2 + \theta_3 + \dots + \theta_n$ thus formed are called exterior angles.

The sum of all the exterior angles of a polygon is 360°.

i.e., $\theta_1 + \theta_2 + \theta_3 + \dots + \theta_n = 360^\circ$

If polygon is a regular polygon then each angle $= \frac{360^\circ}{n}$.

From the above discussion we can conclude that if side of a triangle or quadrilateral or a pentagon or a hexagon etc. are produced in the same order, sum of exterior angles in all cases = 360°

7. $\boxed{n \text{ sided polygon has } \frac{n(n-3)}{2} \text{ diagonals.}}$

sided polygon has $\frac{n(n-3)}{2}$ diagonals.

at $n = 4$, Quadrilateral has $\frac{4(4-3)}{2} = 2$ diagonals.

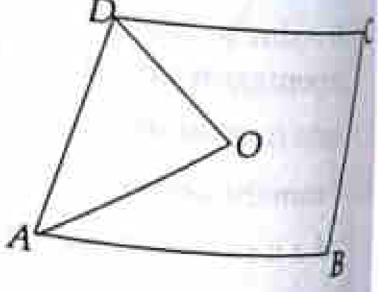
at $n = 5$, Pentagon has $\frac{5(5-3)}{2} = 5$ diagonals.

at $n = 6$, Hexagon has $\frac{6(6-3)}{2} = 9$ diagonals etc.

8. The angle between angle bisector of two adjacent angles of a quadrilateral is equal to half the sum of remaining angles.

In the given figure OD and OA are internal bisector of $\angle D$ and $\angle A$ respectively.

$$\angle AOD = 180^\circ - \frac{\angle A}{2} - \frac{\angle D}{2}$$
$$= 180^\circ - \frac{1}{2}(\angle A + \angle D)$$
$$= 180^\circ - \frac{1}{2}(360^\circ - \angle B - \angle C)$$
$$= \frac{1}{2}(\angle B + \angle C)$$



In the ... intersects at O. If $\angle$... $\angle BOD = 40^\circ$ then find $\angle BOE$ and reflexive $\angle COE$.

Solution : $\angle AOC = \angle BOD$ (Vertically opposite angle)

$\therefore$ $\angle AOC = 40^\circ$

According to question, $\angle AOC + \angle BOE = 70^\circ$

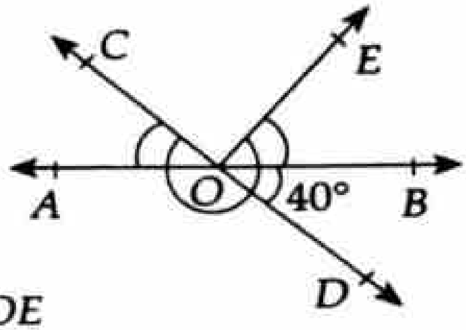
or, $40^\circ + \angle BOE = 70^\circ$

$\therefore$ $\angle BOE = 30^\circ$

Now, $\angle COD + \angle DOB + \angle BOE =$ reflexive $\angle COE$

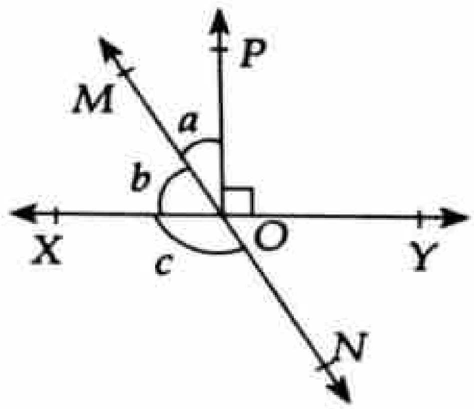
$\therefore$ $180^\circ + 40^\circ + 30^\circ =$ reflexive $\angle COE$

$\therefore$ reflexive $\angle COE = 250^\circ$



2. In the figure given below lines XY and MN intersect at O.

If $\angle POY = 90^\circ$ and $a : b = 2 : 3$ then find the measure of c.



Solution : Given, $\frac{a}{b} = \frac{2}{3}$

Let $a = 2k$ and $b = 3k$

$\because$ $\angle POY = 90^\circ$ $\quad \therefore$ $\angle POX = 90^\circ$

or, $\angle a + \angle b = 90^\circ$ $\quad$ or, $2k + 3k = 90^\circ$

or, $5k = 90^\circ$ $\quad$ or, $k = 18^\circ$

$\therefore$ $\angle b = 3k = 3 \times 18^\circ = 54^\circ$

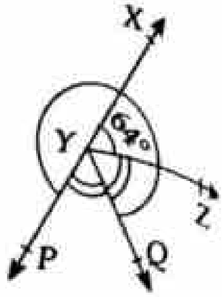
Now, $\angle XOM = \angle YON$ (Vertically opposite angle)

$\therefore$ $\angle YON = 54^\circ$

Again, $\angle XON + \angle YON = 180^\circ$ (Linear pair of angles axiom)

or, $c + 54^\circ = 180^\circ$ $\quad \therefore$ $c = 126^\circ$

3. Given that $\angle XYZ = 64^\circ$ and line XY is produced to point P. Draw a diagram from the given inf...



or, $2\angle ZYQ = 116^\circ$

$\therefore\quad \angle ZYQ = 58^\circ$

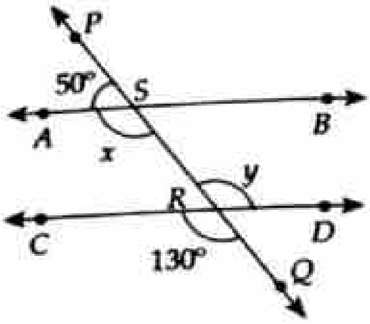
$\because\quad \angle XYQ = \angle XYZ + \angle ZYQ$

$\therefore\quad \angle XYQ = 64^\circ + 58^\circ = 122^\circ$

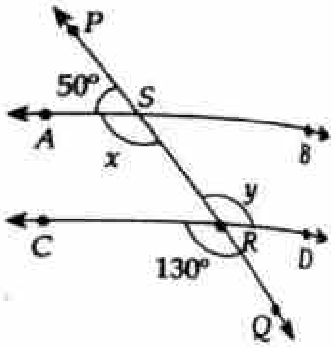
Now reflex, $\angle QYP = \angle PYX + \angle XYQ$

$= 180^\circ + 122^\circ = 302^\circ$

4. In the figure given below find x and y and hence prove that $AB \parallel CD$.



Solution : $\angle DRS = \angle CRQ$ *(Vertically opposite angle)*



$\therefore\quad y = 130^\circ$

Also, $\angle ASP + \angle ASR = 180^\circ$

(Linear pair of angles)

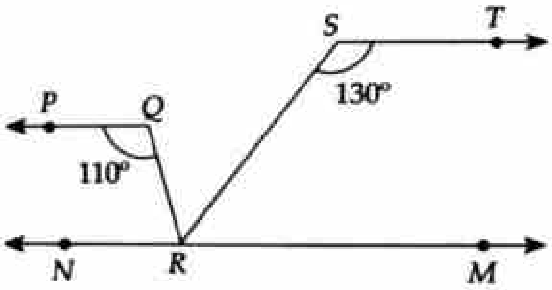
or, $50^\circ + x = 180^\circ$

or, $x = 130^\circ$

$\because\quad x = y = 130^\circ$

$\therefore\quad AB \parallel CD$ *(Alternate angle)*

5. In the given figure if $PQ \parallel ST$, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, Find $\angle QRS$.



Solution : From point R draw a line RM parallel ST.

$\angle RST + \angle SRM = 180^\circ$ *(Consecutive interior angle*

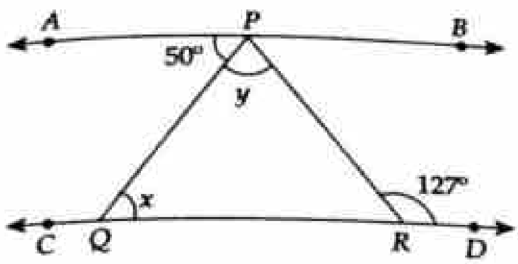
or, $130^\circ + \angle SRM = 180^\circ \quad \therefore\quad \angle SRM = 50^\circ$ ...(i)

Now, $\angle QRM = \angle PQR$ *(Alternate angle*

or, $\angle QRM = 110^\circ$ or, $\angle QRS + \angle SRM = 110^\circ$

or, $\angle QRS + 50^\circ = 110^\circ$ (from (i)) $\therefore$ $\angle QRS = 60^\circ$

6. In the given figure if $AB \parallel CD$, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find x and y.



Solution : $\because$ $AB \parallel CD$ $\therefore$ $\angle APR = \angle PRD$ (Alternate angle)

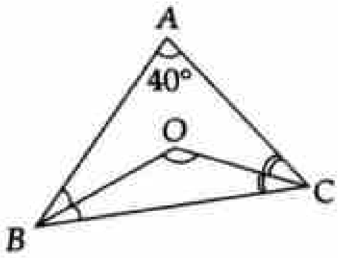
or, $\angle APQ + \angle QPR = \angle PRD$

or, $50^\circ + y = 127^\circ$ or, $y = 77^\circ$

and $\angle APQ = \angle PQR$ (Alternate angle)

or, $50^\circ = x$, $\therefore$ $x = 50^\circ$

7. In ΔABC, $\angle A = 40^\circ$. If bisector of $\angle B$ and $\angle C$ meets at O then prove that $\angle BOC = 110^\circ$



Solution : In ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$

or, $40^\circ + \angle B + \angle C = 180^\circ$

or, $\angle B + \angle C = 140^\circ$

or, $\frac{\angle B}{2} + \frac{\angle C}{2} = 70^\circ$ ... (i)

Now, In ΔBOC, $\angle OBC + \angle OCB + \angle BOC = 180^\circ$

or, $\frac{\angle B}{2} + \frac{\angle C}{2} + \angle BOC = 180^\circ$ or, $70^\circ + \angle BOC = 180^\circ$ (from (i))

$\therefore$ $\angle BOC = 110^\circ$ Proved.

[Shortcut : $\angle BOC = 90^\circ + \frac{A}{2} = 90^\circ + 20^\circ = 110^\circ$ Proved.]

8. If angles of a triangle are is the ratio 2 : 3 : 4, then find the least and greatest angle.

Solution : Let angle be $2x^\circ$, $3x^\circ$ and $4x^\circ$.

$\therefore$ $2x^\circ + 3x^\circ + 4x^\circ = 180^\circ$

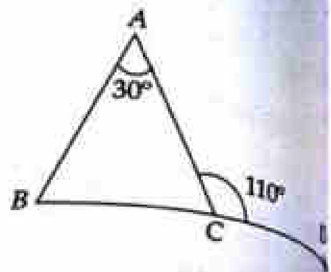
or, $9x^\circ = 180^\circ$ or, $x^\circ = \frac{180^\circ}{9} = 20^\circ$

$\therefore$ least angle $= 2x^\circ = 2 \times 20^\circ = 40^\circ$

greatest angle $= 4x^\circ = 4 \times 20^\circ = 80^\circ$

9. The exterior angle of a triangle is 110° and one of its interior opp... angle is 30°, find other angles.

Solution : Consider the triangle ABC in which exterior $\angle ACD = 110°$ and $\angle A = 30°$, we have to find $\angle B$ and $\angle C$.



$\because \quad \angle ABC + \angle BAC = \angle ACD$

or, $\angle ABC + 30° = 110°$

or, $\angle ABC = 80°$

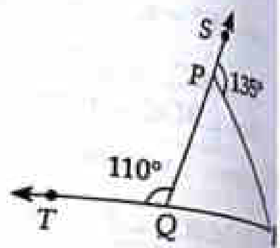
or, $\angle ACB + \angle ACD = 180°$

or, $\angle ACB + 110° = 180°$

$\therefore \quad \angle ACB = 70°$

10. In triangle PQR, sides QP and RQ respectively produced to point S and T. If $\angle SPR = 135°$ and $\angle PQT = 110°$ find $\angle PRQ$.

Solution : $\angle SPR + \angle QPR = 180°$ (linear pair of angles)



or, $135° + \angle QPR = 180°$

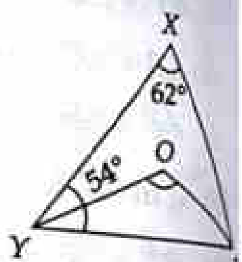
or, $\angle QPR = 45°$ ... (i)

Now, $\angle QPR + \angle PRQ = \angle PQT$

or, $45° + \angle PRQ = 110°$ (from (i))

$\therefore \quad \angle PRQ = 65°$

...n the adjacent figure $\angle X = 62°$ and $\angle XYZ = 54°$. If YO and ZO respectively bisects $\angle XYZ$ and $\angle XZY$ then find $\angle OZY$ and $\angle YOZ$.



Solution : In ΔXYZ, $\angle YXZ + \angle XYZ + \angle XZY = 180°$

or, $62° + 54° + \angle XZY = 180°$

$\therefore \quad \angle XZY = 64°$

From question,

$$\angle OYZ = \frac{1}{2}\angle XYZ = \frac{1}{2} \times 54° = 27°$$

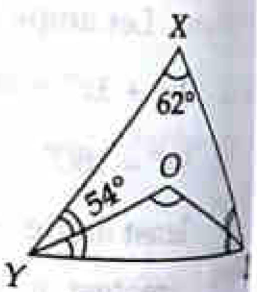
and $\angle OZY = \frac{1}{2}\angle XZY = \frac{1}{2} \times 64° = 32°$

Now, In ΔOYZ,

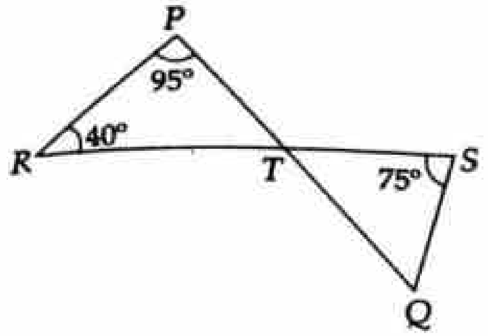
$$\angle OYZ + \angle OZY + \angle YOZ = 180°$$

or, $27° + 32° + \angle YOZ = 180°$

$\therefore \quad \angle YOZ = 121°$



In the given figure lines PQ and RS intersect at point T such that $\angle PRT = 40°$; $\angle RPT = 95°$ and $\angle TSQ = 75°$, Find $\angle SQT$.



Solution : In ΔPRT, $\angle RPT + \angle PRT = \angle PTS$

or, $95° + 40° = \angle PTS$

$\therefore$ $\angle PTS = 135°$ ... (i)

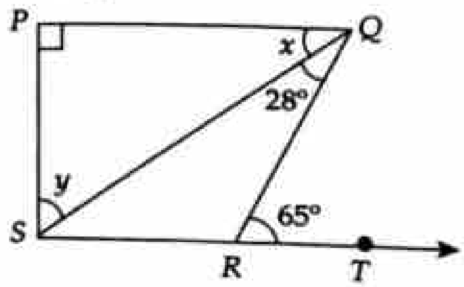
Again in,

ΔTSQ, $\angle TSQ + \angle SQT = \angle PTS$

or, $75° + \angle SQT = 135°$ ($\because \angle TSQ = 75°$ and $\because \angle PTS = 135°$)

$\therefore$ $\angle SQT = 135° - 75° = 60°$.

In the figure given below if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$ find x and y.



Solution : $\because$ $PQ \parallel SR$

$\therefore$ $\angle PQR = \angle QRT$ (Alternate angle)

or, $x + 28° = 65°$

or, $x = 65° - 28° = 37°$ ... (i)

Now, In

ΔPQS, $\angle SPQ + \angle PQS + \angle QSP = 180°$

or, $90° + x + y = 180°$

or, $90° + 37° + y = 180°$ (from (i))

or, $y = 180° - 127° = 53°$

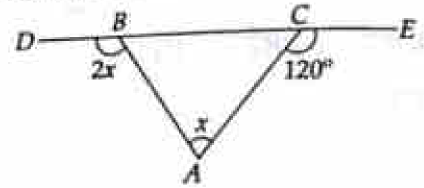
Hence, $x = 37°$ and $y = 53°$.

Exercise–4A

1. The angle between two legs of a compass is 60° and length of each leg is 10 cm. The distance between end points of the leg is
(a) 5 cm (b) 10 cm (c) $5\sqrt{3}$ cm (d) $10\sqrt{3}$ cm

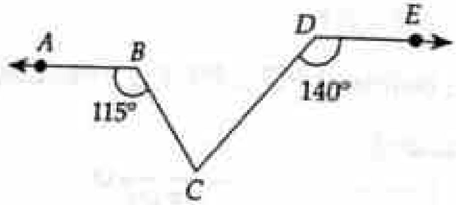
2. Angle at vertex of an isosceles triangle is 15° more than one of the angles at base. Angle at vertex is
(a) 35° (b) 55° (c) 65° (d) 70°

3. In the given figure, value of x is



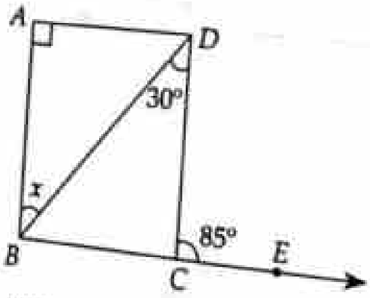
(a) 30° (b) 40° (c) 45° (d) 60°

4. Given that $AB \parallel DE$, $\angle ABC = 115°$, $\angle CDE = 140°$, What is the value of $\angle BCD$?



(a) 45° (b) 55° (c) 65° (d) 75°

5. ... that $AD \parallel BE$, $AB \perp r AD$ $\angle DEC = 85°$, $\angle BDC = 30°$, What is the ... of x?



(a) 30° (b) 35° (c) 45° (d) 55°

6. O is a point on the line LM. A line ON is drawn which is neither coincident with OL nor with OM. If $\angle MON$ is one third of $\angle LON$, then $\angle MON$ is equal to
(a) 45° (b) 60° (c) 75° (d) 80°

7. In the given figure $AB \parallel CD$, $\angle PTB = 55°$ and $\angle DVS = 45°$. Sum of $\angle CUQ$ and $\angle RTP$ is

length of each leg is

(d) $10\sqrt{3}$ cm

one of the angle

(d) 70°

60°

is the value of

75°

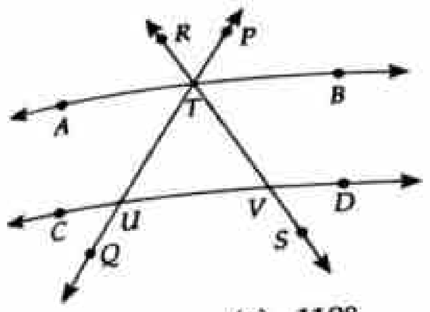
What is the

°

r coincident

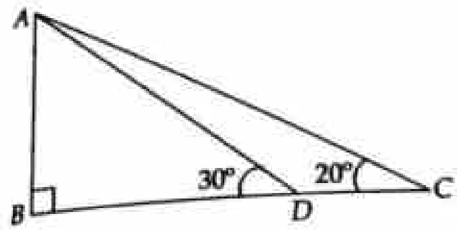
n $\angle MON$ is

5°. Sum of



(a) 180° (b) 135° (c) 110° (d) 100°

In the given figure, $\angle ABD = 90°$, $\angle BDA = 30°$ and $\angle BCA = 20°$. What is the value of $\angle CAD$?



(a) 10° (b) 20° (c) 30° (d) 15°

Length of line segment AB is 2 unit. It is divided by point C such that $AC^2 = AB \times CB$, the length of CB is

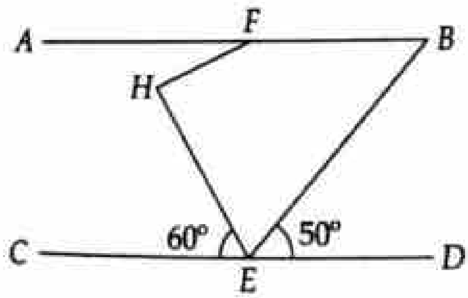
(a) $3+\sqrt{5}$ unit (b) $3-\sqrt{5}$ unit

(c) $2-\sqrt{5}$ unit (d) $\sqrt{3}$ unit

Sides AB and AC of a triangle ABC are equal. Side BC is produced to point D. From a point E on AC, line EF is drawn parallel to AB. Consider the quadrilateral $ECDF$ thus formed. If $\angle ABC = 65°$ and $\angle EFD = 80°$, then what is the value of $\angle FDC$?

(a) 43° (b) 41° (c) 37° (d) 35°

In the given figure AB is parallel to CD and BE is parallel to FH. Measure of $\angle FHE$ is

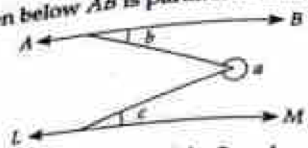


(a) 110° (b) 120° (c) 125° (d) 130°

Which of the following cannot be number of diagonals of a polygon ?

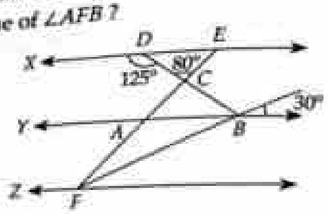
(a) 14 (b) 20 (c) 28 (d) 35

13. In the figure given below AB is parallel to LM. Angle a is equal to



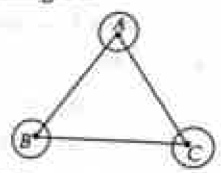
(a) $\pi + b + c$ (b) $2\pi - b + c$ (c) $2\pi - b - c$ (d) $2\pi + b - c$

14. Three lines X, Y and Z are parallel and angles are as shown in the figure. What is the value of $\angle AFB$?



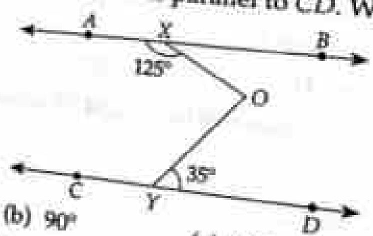
(a) 20° (b) 15° (c) 30° (d) 10°

15. In the figure given below sum of angles around the point A, B, C, excluded angles of the triangle is



(a) 60° (b) 720° (c) 900° (d) 1000°

16. In the figure given below AB is parallel to CD. What is $\angle XOY$?



(a) 80° (b) 90° (c) 95° (d) 100°

17. Side BC of the triangle ABC is produced to point D. If $\angle ACD = 12$ $\angle ABC = \frac{2}{3}\angle CAB$, then what is the measure of $\angle BAC$?

(a) 60° (b) 45° (c) 30° (d) 72°

18. Line segments AB and CD intersects at O. OF is the internal bisector of obtuse angle BOC and OE is the internal bisector of acute angle AOC. If $\angle BOC = 130°$ then measure of $\angle FOE$ is

(a) 90° (b) 110° (c) 115° (d) 120°

gle a is equal to

(d) $2\pi + b - c$

nown in the figur

i) 10°

point A, B, C a

1000°

$\angle XOY$?

100°

$\angle ACD = 120°$,

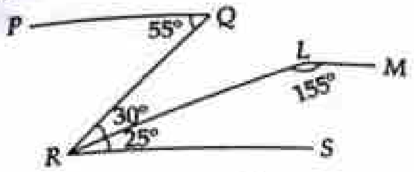
72°

al bisector of

e angle AOC

20°

19. In the figure given below RS is parallel to PQ. What is the angle between lines PQ and LM ?

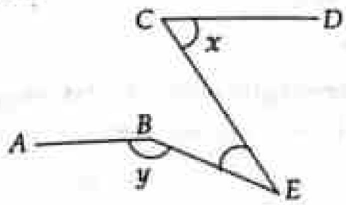


(a) 175° (b) 177° (c) 179° (d) 180°

20. Which angle is two third of its complementary angle ?

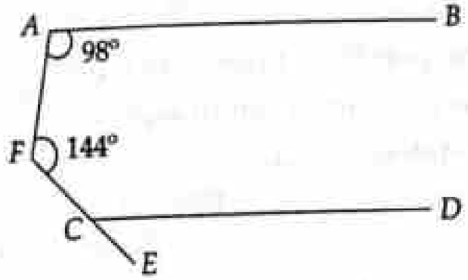
(a) 36° (b) 45° (c) 48° (d) 60°

21. In the figure given below AB is parallel to CD. If $\angle DCE = x$ and $\angle ABE = y$, then $\angle CEB$ is equal to



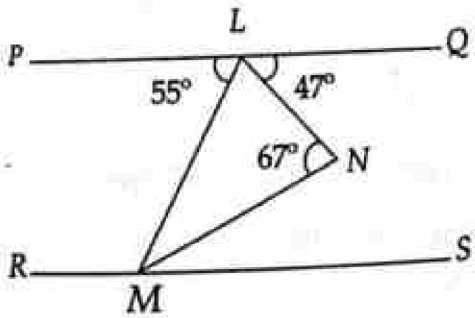
(a) $y - x$ (b) $\frac{(x+y)}{2}$ (c) $x + y - \left(\frac{\pi}{2}\right)$ (d) $x + y - \pi$

22. In the figure given below AB and CD are parallel. If $\angle BAF = 98°$ and $\angle AFC = 144°$, then $\angle ECD$ equals



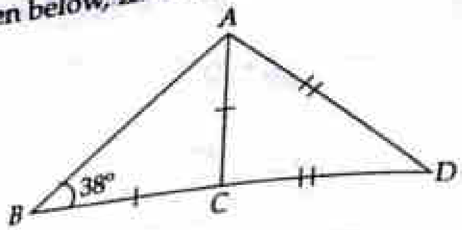
(a) 62° (b) 64° (c) 82° (d) 84°

23. In the figure given below PQ is parallel to RS. What is the measure of $\angle NMS$?



(a) 20° (b) 23° (c) 27° (d) 47°

24. In the figure given below, $\angle B = 38°$, AC ... CD. $\angle D$ is equal



(a) 26° (b) 28° (c) 38° (d) 52°

25. If two angles are complementary to each other then each angle is
(a) Obtuse angle (b) Right angle
(c) Acute angle (d) Supplementary angle

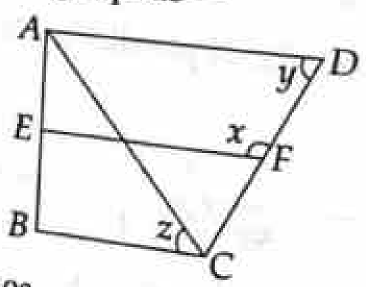
26. If each interior angle of a regular polygon is $\frac{7}{6}$ times each angle of a regular hexagon, then what is the number of sides in the polygon?
(a) 7 (b) 8 (c) 9 (d) 10

27. If ratio of angles of a triangle is 5 : 3 : 10, then what is the difference between its largest and smallest angle ?
(a) 20° (b) 30° (c) 50° (d) 70°

28. What is the measure of the angle which is one fifth of its supplementary part ?
(a) 15° (b) 30° (c) 36° (d) 75°

29. Consider the following statements :
If a transversal line cuts two parallel lines then
1. Each pair of corresponding angles are equal.
2. Each pair of alternate angles are unequal.
Among these, true statements are—
(a) only 1 (b) only 2
(c) both 1 and 2 (d) Neither 1 nor 2

30. $ABCD$ is a trapezium such that $AD \parallel BC$. If EF is parallel to BC, $\angle x =$ 120° and $\angle z = 50°$, then $\angle y$ equals—



(a) 50° (b) 60° (c) 70° (d) 80°

31. If each interior angle of a regular polygon is 144°, then what is the number of sides in the polygon ?
(a) 10 (b) 20 (c) 24 (d) 36

(d) 30

33. ... of a regular polygon is 1080°, then number of sides in the polygon is [SSC Tier-I 2014]

(a) 6 (b) 8 (c) 10 (d) 12

34. The ratio of sides of two regular polygon is 1 : 2 and ratio of their internal angle is 2 : 3. What is the number of sides of polygon having more sides?

(a) 4 (b) 8 (c) 6 (d) 12

35. In the two regular polygon number of sides are in the ratio 5 : 4. If difference between their internal angles is 6°, then number of sides in the polygon is

(a) 15, 12 (b) 5, 4 (c) 10,8 (d) 20, 16

36. If each of interior angle of a polygon is double its each exterior angle, then number of sides in the polygon is

(a) 8 (b) 6 (c) 5 (d) 7

37. Which the following cannot be measure of an interior angle of a regular polygon ?

(a) 150° (b) 105° (c) 108° (d) 144°

38. Number of diagonals in a polygon having 10 sides is

(a) 20 (b) 40 (c) 35 (d) 32

39. If one internal angle of a regular polygon is 135°, then number of diagonals in the polygon is

(a) 16 (b) 18 (c) 24 (d) 20

Answers –4A

1. (b)	2. (b)	3. (d)	4. (d)	5. (b)	6. (a)	7. (b)	8. (a)
9. (b)	10. (d)	11. (a)	12. (c)	13. (c)	14. (b)	15. (c)	16. (b)
17. (d)	18. (a)	19. (d)	20. (a)	21. (d)	22. (a)	23. (a)	24. (b)
25. (c)	26. (c)	27. (d)	28. (b)	29. (a)	30. (b)	31. (a)	32. (c)
33. (b)	34. (b)	35. (a)	36. (b)	37. (b)	38. (c)	39. (d)	

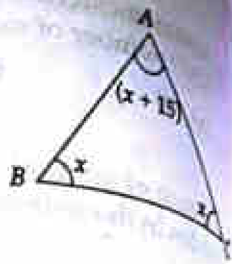
2. **(b)** Let each base angle of isosceles triangle $= x$

$\therefore$ Angle at vertex $= (x + 15°)$

We know that

$\therefore$ $x + 15° + x + x = 180°$

$\Rightarrow$ $x = \frac{165}{3} = 55°$



3. **(d)** Let $\angle ABC = \theta$ and $\angle ACB = \alpha$

then, $\alpha + 120° = 180°$

$\Rightarrow$ $\alpha = 60°$

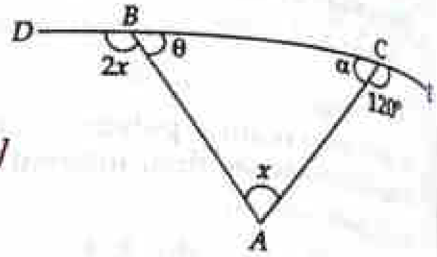
$\Rightarrow$ $\alpha + x = 2x$

[Sum of interior opposite angle]

$\Rightarrow$ $60° + x = 2x$

$\Rightarrow$ $2x - x = 60°$

$\therefore$ $x = 60°$



4. **(d)** Draw a line through point C, which is parallel to each of AB and DE

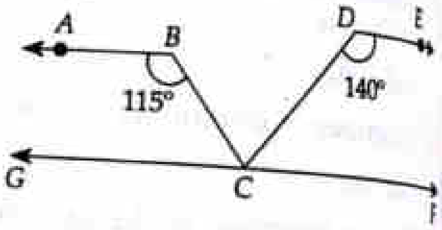
$\because$ $AB \parallel GF \parallel DE$

$\therefore$ $\angle BCG = 180° - \angle ABC$

$= 180° - 115° = 65°$

and $\angle DCF = 180° - \angle CDE$

$= 180° - 140° = 40°$



Now $\angle BCG + \angle BCD + \angle DCF = 180°$ *(Linear pair of angles)*

$\Rightarrow$ $65° + \angle BCD + 40° = 180°$

$\therefore$ $\angle BCD = 180° - 105° = 75°$

5. **(b)** Let $\angle ADB = \theta$

$\because$ $AD \parallel BE$

$\therefore$ $\angle CBD = \theta$ *(alternate angle)*

$\therefore$ $\theta + 30° = 85°$

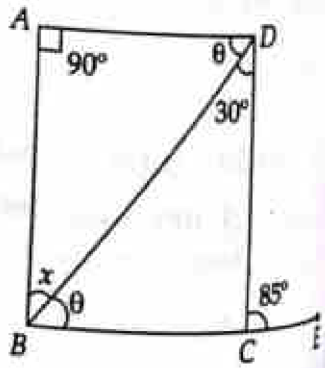
$\Rightarrow$ $\theta = 55°$ *[sum of interior opposite angle]*

In ΔABD,

$90° + \theta + x = 180°$

or, $90° + 55° + x = 180°$

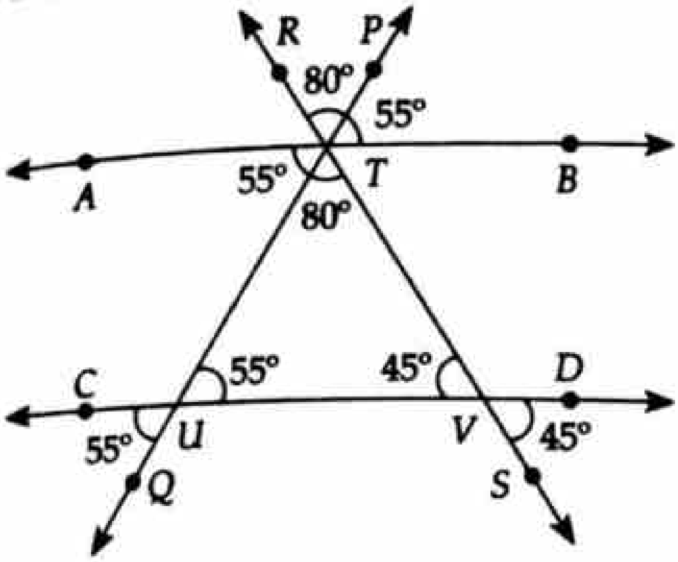
$\therefore$ $x = 35°$



$3x + x = 180°$

$\Rightarrow x = 45°$

Since $\angle PTB = 55°$



then $\angle TUV = 55°$ (Corresponding angle)

and $\angle CUQ = \angle TUV = 55°$ (Vertically opposite angle)

Given, $\angle DVS = 45°$

$\therefore \angle UVT = 45°$ (Alternate angle)

In ΔUTV,

$\angle T = 180° - (55° + 45°) = 80°$

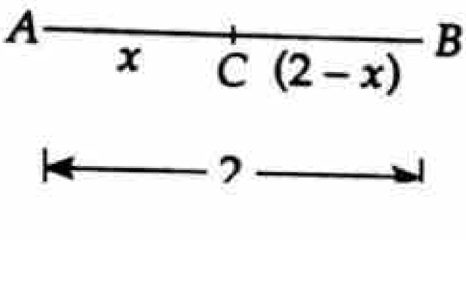
$\Rightarrow \angle T = \angle PTR = 80°$ (Vertically opposite angles)

$\therefore \angle CUQ + \angle RTP = 55° + 80° = 135°$

$\angle CAD = \angle CAB - \angle DAB$

$= (90° - 20°) - (90° - 30°) = 10°$

Given, $AC^2 = AB \times CB$



$\Rightarrow x^2 = 2 \times (2 - x)$

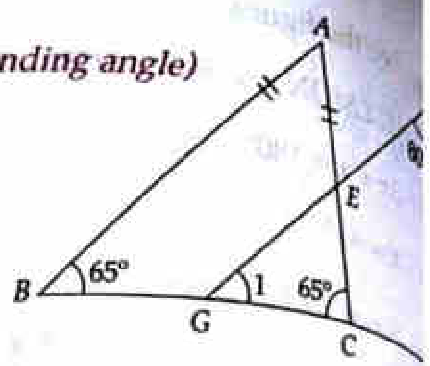
$\Rightarrow x^2 = 4 - 2x$

$\Rightarrow x^2 + 2x - 4 = 0$

$\Rightarrow x = \frac{-2 \pm \sqrt{4 + 16}}{}$

10. (d) Here, $\angle B = \angle C = 65°$

$\angle 1 = \angle B = 65°$ *(Corresponding angle)*



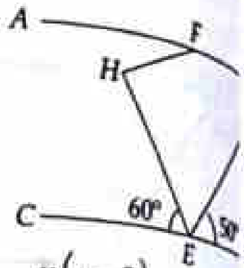
In ΔFGD,

$\angle 1 + \angle F + \angle D = 180°$

$\Rightarrow$ $65° + 80° + \angle D = 180°$

$\Rightarrow$ $\angle D = 35°$

11. (a) $\angle HEB = 180° - 60° - 50° = 70°$



Since $HF \parallel BE$ and HE is a transversal line

$\therefore$ $\angle FHE + \angle HEB = 180°$ *(co-interior angle)*

$\Rightarrow$ $\angle FHE + 70° = 180°$

$\Rightarrow$ $\angle FHE = 110°$

12. (c) Number of diagonals is a n sided polygon $= \dfrac{n(n-3)}{2}$

No. of Sides (n)	4	5	6	7	8	9	1
No. of diagonals	2	5	9	14	20	27	3

Clearly 28 doesnot occur is the list.

For, $n = 4$, $\dfrac{n(n-3)}{2} = 2$

$= 5$, $\dfrac{n(n-3)}{2} = 5$

$= 6$, $\dfrac{n(n-3)}{2} = \dfrac{6 \times 3}{2} = 9$

$= 7$, $\dfrac{n(n-3)}{2} = \dfrac{7 \times 4}{2} = 14$

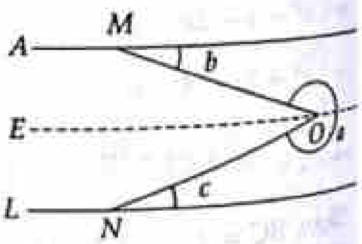
$= 8$, $\dfrac{n(n-3)}{2} = \dfrac{8 \times 5}{2} = 20$

$= 9$, $\dfrac{n(n-3)}{2} = \dfrac{9 \times 6}{2} = 27$

$= 10$, $\dfrac{n(n-3)}{2} = \dfrac{10 \times 7}{2} = 35$

So, number of diagonals cannot be 28.

13. (c) Let EF is drawn parallel to AB



$\therefore$ $\angle EOM = \angle OMB$ *(Alternate angle)*

$\Rightarrow$ $\angle EOM = b$

and $\angle EON = \angle ONM$ *(Alternate angle)*

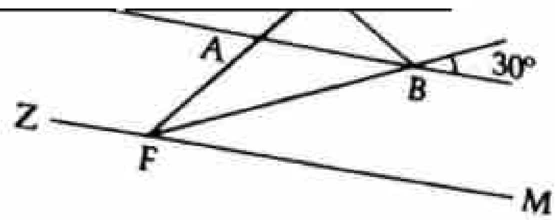
$\Rightarrow$ $\angle EON = c$ $\quad\therefore$ $\angle MON = b + c$

$\because$ $\angle MON + a = 2\pi$ $\quad\therefore$ $a = 2\pi - (b + c)$

In ...

and $\angle B = \angle E$

(Corresponding angle)



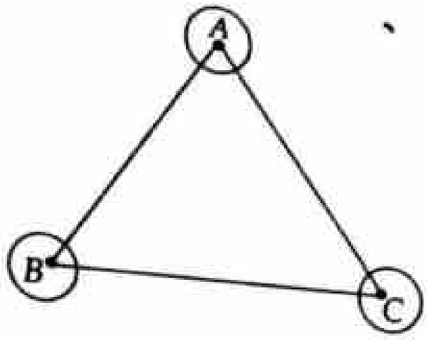
From figure,

$\angle DEF = \angle EFM$ (Alternate angle)

$\Rightarrow \angle EFM = 45^\circ \quad \Rightarrow \angle EFB + \angle BFM = 45^\circ$

$\Rightarrow \angle EFB = 45^\circ - 30^\circ \Rightarrow \angle AFB = 15^\circ$

... $\because \quad \angle A = 360^\circ -$ External $\angle A$



Similarly, $\angle B = 360^\circ -$ External $\angle B$

and $\quad \angle C = 360^\circ -$ External $\angle C$

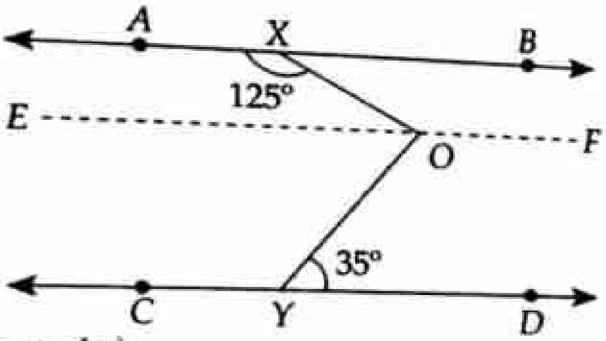
We know that, $\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 360^\circ -$ External $\angle A + 360^\circ -$ External $\angle B + 360^\circ -$ External $\angle C = 180^\circ$

$\Rightarrow$ External $\angle A +$ External $\angle B +$ External $\angle C = 1080^\circ - 180^\circ = 900^\circ$

(b) Draw a line EF through O such that

$EF \parallel AB \parallel CD$



Now, $AB \parallel EF$

$\therefore \angle AXO + \angle XOE = 180^\circ$

$\Rightarrow \angle XOE = 180^\circ - 125^\circ = 55^\circ$

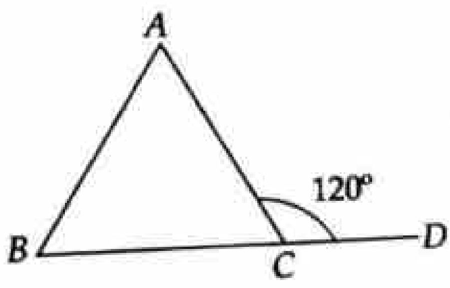
But $EF \parallel CD$

$\Rightarrow \angle EOY = \angle OYD = 35^\circ$ (Alternate angle)

Hence, $\angle XOY = \angle XOE + \angle EOY = 55^\circ + 35^\circ = 90^\circ$

(d) $\therefore \quad \angle ACD = 120^\circ$

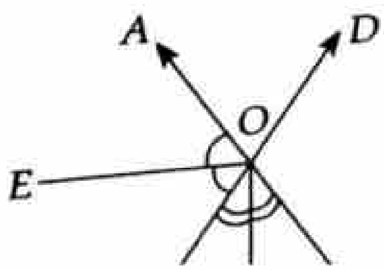
$\Rightarrow \angle CAB + \angle ABC = 120^\circ$



(Since exterior angle of triangle is equal to sum of co-interior angles)

$\Rightarrow \angle CAB + \frac{2}{3} \angle CAB = 120^\circ$

$\Rightarrow \angle CAB = \frac{120^\circ \times 3}{5} = 72^\circ$



(a) $\because \quad \angle BOC = 130^\circ$

$\therefore \quad \angle BOC + \angle AOC = 180^\circ$

Now, $\angle BOC = 130°$

$\Rightarrow \angle BOF + \angle FOC = 130°$

$\Rightarrow \angle FOC + FOC = 130°$ ($\because$ OF is bisector of $\angle BOC$)

$\Rightarrow \angle FOC = 65°$

and $\angle AOC = 50°$

$\Rightarrow \angle AOE + \angle EOC = 50°$

$\Rightarrow \angle EOC + \angle EOC = 50°$ ($\because$ OE is bisector of $\angle AOC$)

$\Rightarrow \angle EOC = 25°$

$\Rightarrow \angle EOF = \angle EOC + \angle FOC$

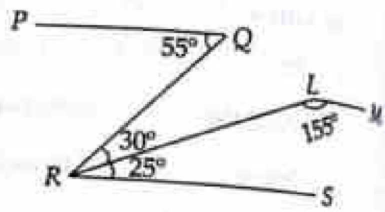
$= 65° + 25° = 90°$

19. (d) $\because \angle PQR = \angle QRS$ (Alternate angle)

$\therefore PQ \parallel RS$ ... (i)

and $\angle SRL + \angle RLM = 180°$ ... (ii)

$\Rightarrow RS \parallel LM$



From (i) and (ii),

$PQ \parallel LM$

$\therefore$ Angle between PQ and LM is $180°$.

20. (a) We know that if α and β. are complementary then

$\because \alpha + \beta = 90° \quad \therefore \alpha = 90 - \beta$ ...(i)

According to question, β tis two third of its complementary angle α.

$\therefore \beta = \frac{2}{3}\alpha$

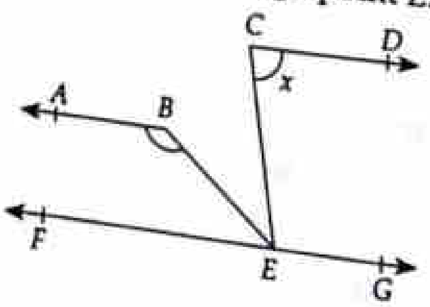
$\Rightarrow \beta = \frac{2}{3}(90° - \beta)$

$\Rightarrow \beta = 60° - \frac{2}{3}\beta$ [from (i)]

$\Rightarrow \frac{5\beta}{3} = 60° \Rightarrow \beta = 36°$

Thus, Required angle = 36°

21. (d) $FG \parallel AB \parallel CD$ is drawn through point E.



$\angle BEF = \pi - y$ [co-interior angles]

$\angle CEG = (\pi - x)$ [co-interior angles]

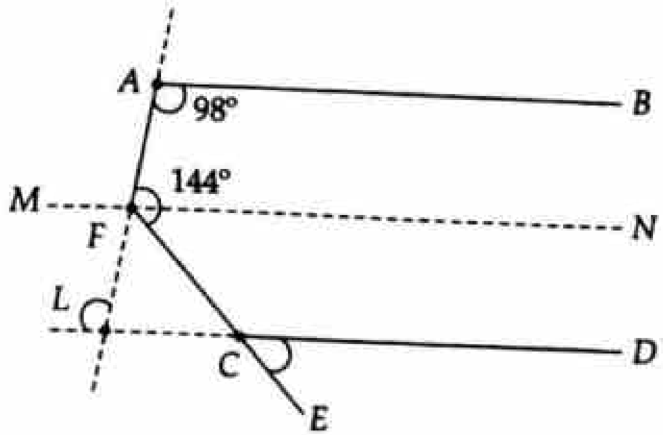
$\angle BEF + \angle BEC + \angle CEG = \pi$

$\pi - y + \angle BEC + \pi - x = \pi$

$2\pi - x - y + \angle BEC = \pi$

$\therefore \angle BEC = x + y - \pi$

22. (a) In figure, $\angle A = \angle L = 98^\circ$



In ΔFLC,

$\angle FLC = 180^\circ - 98^\circ = 82^\circ$

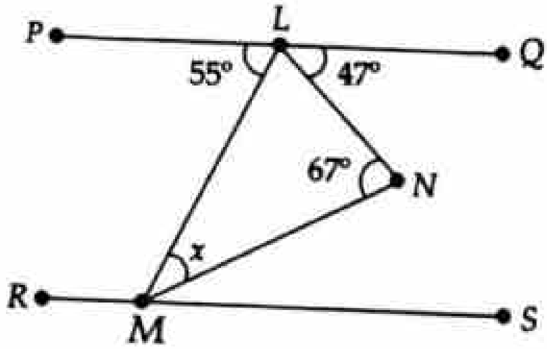
and $\angle F = 180^\circ - 144^\circ = 36^\circ$

$\Rightarrow \angle FCL = 180^\circ - (36^\circ + 82^\circ)$

$= 180^\circ - 118^\circ = 62^\circ$

Hence, $\angle ECD = \angle FCL = 62^\circ$

23. (a) In figure,



$\angle PLM = \angle LMS = 55^\circ$

$\angle LMS = \angle LMN + \angle NMS = 55^\circ$

$\Rightarrow x + \angle NMS = 55^\circ$

$\Rightarrow \angle NMS = 55^\circ - x$

$\angle MLN = 180^\circ - (\angle PLM + \angle QLN)$

$= 180^\circ - (55^\circ + 47^\circ)$

$= 180^\circ - 102^\circ = 78^\circ$

In ΔMLN, $\angle LMN + \angle MNL + \angle MLN = 180^\circ$

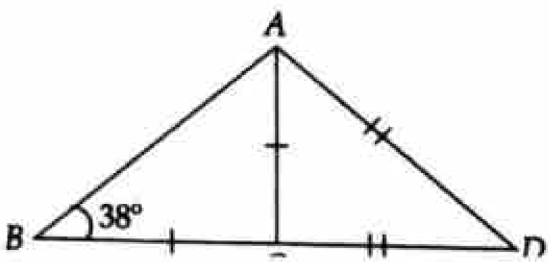
$\Rightarrow x + 67^\circ + 78^\circ = 180^\circ$

$\Rightarrow x = 180^\circ - 145^\circ = 35^\circ$

$\therefore$ From equation (i),

$\Rightarrow \angle NMS = 55^\circ - 35^\circ = 20^\circ$

24. (b) Given that $AC = BC$



$\therefore$ In ΔABC,

$\angle A = \angle B = 38^\circ$

$\therefore \angle ACB = 180^\circ - (\angle A + \angle B)$

25. (c) If two angles are complementary (sum = 90°) then each of them ... acute angle.

26. (c) Since each interior angle fo regular hexagon = 120°

$\therefore$ Each interior angle of polygon $= \frac{7}{6} \times 120° = 140°$

Let number of sides in polygon be n.

$$\frac{(n-2)\times 180°}{n} = 140°$$

$\Rightarrow 18n - 36 = 14n \Rightarrow 4n = 36 \Rightarrow n = 9$

27. (d) Required difference $= \frac{10-3}{5+3+10} \times 180° = \frac{7}{18} \times 180° = 70°$

28. (b) Let required angle be x then its supplementary angle is $(180° - x)$

According to question,

$x = \frac{1}{5}(180° - x)$

$\Rightarrow 5x = 180° - x$

$\therefore x = \frac{180°}{6} = 30°$

29. (a) Statement (1) is true. Statement (2) is wrong.

30. (b) $\because ABCD$ is a trapezium

$\therefore AD \parallel BC$

$EF \parallel BC$ (Given)

Hence $EF \parallel AD$

$\therefore \angle x + \angle y = 180°$ (Linear pair of angles)

$\therefore \angle y = 180° - 120° = 60°$

31. (a) $\because$ Let number of sides be n

According to question, $\frac{(n-2)180}{n} = 144$

$\Rightarrow (n-2)5 = 4n$

$\Rightarrow 5n - 10 = 4n \therefore n = 10$

32. (c) If number of sides in regular polygon be n then

$\frac{(2n-4)}{n} \times 90° - \frac{360°}{n} = 150°$

$\Rightarrow \frac{(2n-4)\times 3}{n} - \frac{12}{n} = 5$

$\Rightarrow \frac{6n-12-12}{n} = 5$

$\Rightarrow 6n - 24 = 5n \therefore n = 24$

33. (b) Sum of interior angles of a regular polygon of n sides $= (2n-4)\times 9$...

$\therefore$ $(2n-4)\times 90^\circ = 1080^\circ$

$\Rightarrow$ $2n-4 = 1080 \div 90 = 12$

$\Rightarrow$ $2n = 12+4 = 16$

$\therefore$ $n = \frac{16}{2} = 8$

4. (b) Let number of sides in two regular polygon are respectively n and $2n$, Then their each internal angle are respectively $\frac{n\pi-2\pi}{n}$ and $\frac{2n\pi-2\pi}{2n}$

According to question, $\dfrac{\left(\frac{n\pi-2\pi}{n}\right)}{\left(\frac{2n\pi-2\pi}{2n}\right)} = \frac{2}{3}$

or $\frac{(n-2)\pi}{(n-1)2\pi}\times 2 = \frac{2}{3}$

or $\frac{n-2}{n-1} = \frac{2}{3}$

or $3n-6 = 2n-2$

$\Rightarrow$ $n = 4$

$\therefore$ $2n = 8$

35. (a) Let number of sides be respectively $5x$ and $4x$.

$\therefore$ $\frac{(2\times 5x-4)90^\circ}{5x} - \frac{(2\times 4x-4)\times 90^\circ}{4x} = 6^\circ$

$\left[\text{each interior angle} = \left(\frac{2n-4}{n}\right)\times 90^\circ\right]$

$\Rightarrow$ $(10x-4)\times 360^\circ - (8x-4)\times 450^\circ = 20x \times 6^\circ$

$\Rightarrow$ $(10x-4)\times 12 - (8x-4)15 = 4x$

$\Rightarrow$ $120x - 48 - 120x + 60 = 4x$

$\Rightarrow$ $4x = 12$

$\Rightarrow$ $x = 3$

$\therefore$ Number of sides are respectively 15 and 12.

36. (b) Each internal angle of polygon $= \left[\frac{(n-2)180}{n}\right]^\circ$

Each exterior angle of polygon $= \left[\frac{360}{n}\right]^\circ$

According to question, $\frac{(n-2)180}{n} = 2\times\frac{360}{n}$

$\Rightarrow$ $n-2 = 4$ $\quad\therefore n = 6$

37. (b) Each interior angle of polygon $= \frac{n-2}{n} \times 180^\circ$.

$= 60^\circ$, when $n = 3$ $\quad 90^\circ$, when $n = 4$ $\quad 108^\circ$, when $n = 5$

120°, when $n = 6$ $\quad 135^\circ$, when $n = 8$ $\quad 140^\circ$, when $n = 9$

144° when $n = 10$ $\quad 150^\circ$, when $n = 12$

38. (c) Since number of diagonals in n sided polygon $= \frac{n(n-3)}{2}$

For, $n = 3$, Number of diagonals $= \frac{10 \times 7}{2} = 35$

39. (d) Each interior angle of a regular polygon $= \frac{n-2}{n} \times 180^\circ$

Given $\frac{n-2}{n} \times 180^\circ = 135^\circ \quad \Rightarrow \quad 4(n-2) = 3n \quad \Rightarrow \quad n = 8$

$\therefore$ Number of diagonals $= \frac{n(n-3)}{2} = \frac{8(8-3)}{2} = 20$

Exercise–4B

1. A, O, B are three points on a line segment and C is a point not lying on AOB. If $\angle AOC = 40^\circ$ and OX, OY, are the internal and external bisectors of $\angle AOC$ respectively, then $\angle BOY$ is

(a) 72° (b) 68° (c) 70° (d) 80°

[SSC Tier-I 2012]

2. If each interior angle is double of each exterior angle of a regular polygon with n sides, then the value of n is

(a) 5 (b) 6 (c) 8 (d) 10

[SSC Tier-I 2012]

3. Side BC of ΔABC produces to D. If $\angle ACD = 108^\circ$ and $\angle B = \frac{1}{2} \angle A$ then $\angle A$ is

(a) 108° (b) 59° (c) 36° (d) 72°

[SSC Tier-I 2012]

4. The external bisectors of $\angle B$ and $\angle C$ of ΔABC meet at point P. If $\angle BAC = 80^\circ$, then $\angle BPC$ is

(a) 50° (b) 40° (c) 80° (d) 100°

[SSC Tier-I 2012]

5. By decreasing 15° of each angle of a triangle, the ratios of their angles are $2 : 3 : 5$. The radian measure of greatest angle is :

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{24}$ (c) $\frac{5\pi}{24}$ (d) $\frac{11\pi}{24}$

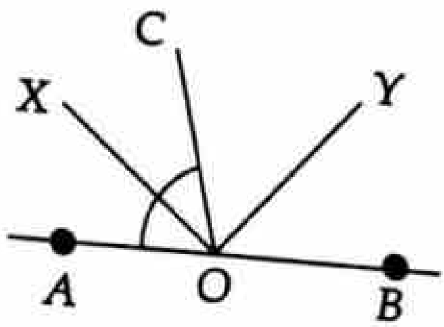
[SSC Tier-I 2012]

Explanation

1. (c)

$\because \angle BOC = 180° - 40° = 140°$

$\therefore \angle BOY = \frac{140°}{2} = 70°$



2. (b) Each internal angle of polygon $= \frac{(n-2)\pi}{n}$

Each exterior angle of polygon $= \frac{2\pi}{n}$

According to question $= \frac{(n-2)\pi}{n} = 2 \cdot \frac{2\pi}{n}$

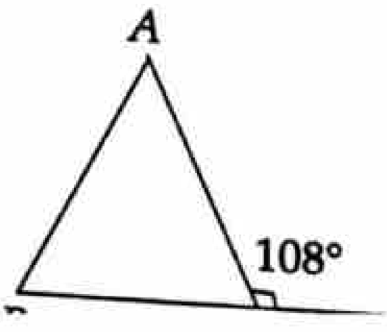
$\Rightarrow n - 2 = 4 \Rightarrow n = 6$

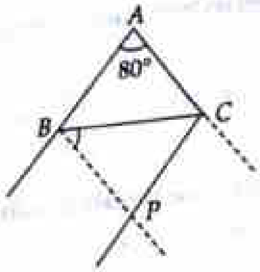
3. (d) Clearly, $\angle C = 180° - 108° = 72°$

or, $\angle A + \angle B = 180° - 72° = 108°$

$\therefore \angle A + \frac{1}{2}\angle A = 108°$

$\Rightarrow \frac{3\angle A}{2} = 108°$





$$\therefore \quad \angle CBP + \angle BCP = 180° - \frac{B+C}{2}$$

$$= 180° - 50° = 130° \qquad (\because B + C = 180° - 80° = 100°)$$

Hence, $\angle BPC = 180° - 130° = 50°$

5. **(d)** $2k + 3k + 5k = 180° - 3 \times 15°$

or, $10k = 135°$

$\Rightarrow \quad k = \frac{135°}{10}$

$\therefore$ Greatest angles $= 5k + 15°$

$$= 5 \times \frac{135°}{10} + 15°$$

$$= \left(\frac{135°}{2} + 15°\right)$$

$$= \frac{165°}{2} = \left(\frac{165}{2} \times \frac{\pi}{180}\right) \text{ rad}$$

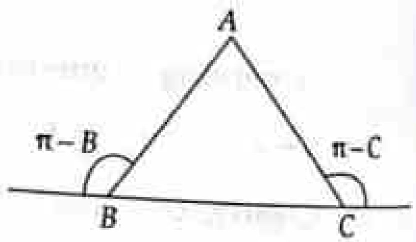
$$= \left(\frac{11}{2} \times \frac{\pi}{12}\right) = \frac{11\pi}{24} \text{ rad}$$

) Required sums $= (\pi - B) + (\pi - C)$

$$= 2\pi - (B + C)$$

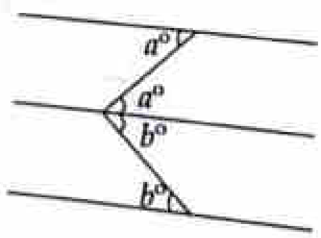
$$= 2\pi - (\pi - A)$$

$$= \pi + A$$



7. **(c)** From alternate angle,

$a° + b° = 45°$

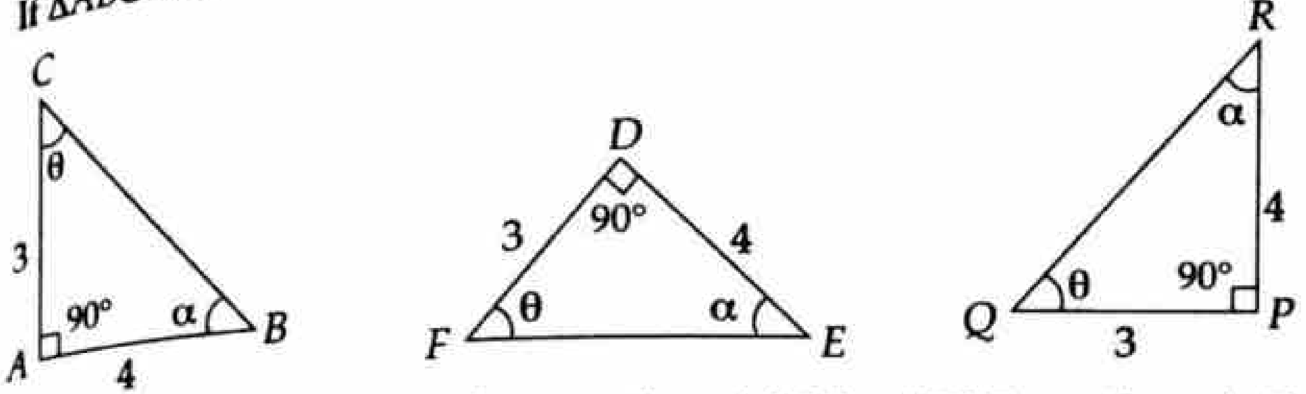


★★★

1. Meaning ...

If three sides of a triangle to three sides of another triangle are equal and their corresponding angles are also equal, then the two triangles are said to be congruent.

If ΔABC and ΔDEF are congruent then it is denoted by $\Delta ABC \cong \Delta DEF$.



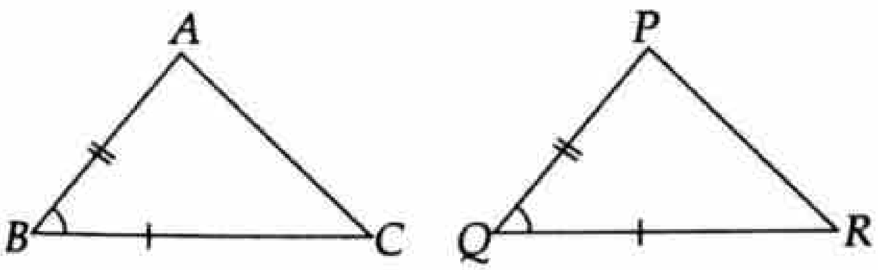
If must be noted that in the notation $\Delta ABC \cong \Delta DEF$, vertices A, B, C respectively correspond to vertices D, E, F

In the above figure ΔABC and ΔPQR are congruent. It is written as $\Delta ABC \cong \Delta PRQ$ as $A \longleftrightarrow P$, $B \longleftrightarrow R$ and $C \longleftrightarrow Q$ are equal angles. Hence while writing the congruity of two triangles, vertices are written in the order in which they are equal.

2. Condition for congruence of two triangles :

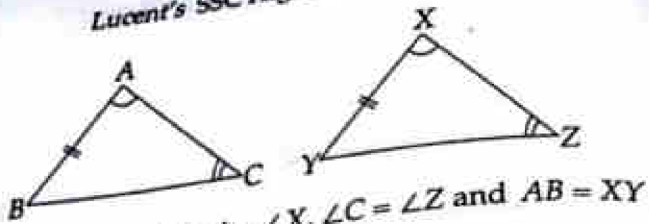
In the following conditions two triangles are congruent.

2.1. SAS condition : Two triangle are congruent if two sides and included angle of one triangle is equal to two sides and included angle of the other triangle. This is called side-angle-side (SAS) condition



In the given figure if $AB = PQ$, $BC = QR$ and $\angle ABC = \angle PQR$ then $\Delta ABC \cong \Delta PQR$

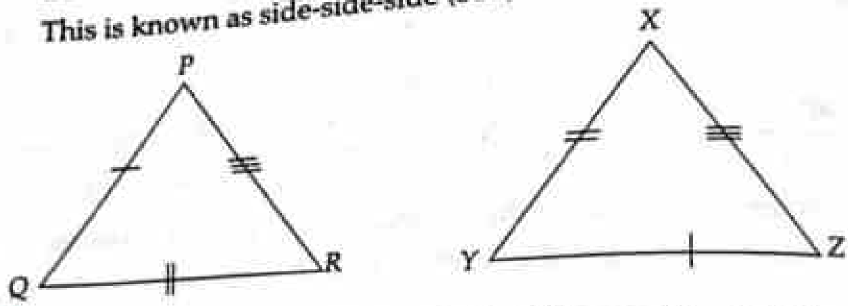
2.2. ASA condition : Two triangles are said to be congruent if any two angles of first triangle is equal to any two angles of second triangle



In the given figure if $\angle A = \angle X$, $\angle C = \angle Z$ and $AB = XY$ then $\Delta ABC \cong \Delta XYZ$

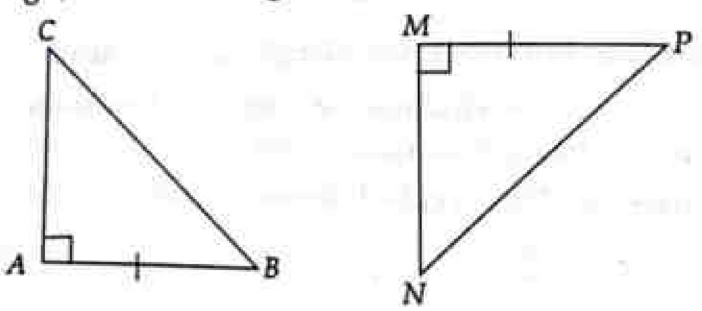
Note : In SAS condition angle must be included between two sides; However in ASA condition, any two angle and one side can be taken.

2.3. SSS condition : If three sides of a triangle are equal to corresponding three sides of another triangle then the two triangles are congruent. This is known as side-side-side (SSS) condition.



[illegible] the given figure if $PQ = YZ$, $QR = XY$ and $PR = XZ$ then ΔPQR [illegible] ZYX

[illegible] condition : If hypotenuse and one side of a right angle triangle [illegible] equal to hypotenuse and one side of another right angled triangle, then the two right angle triangle are said to be equal.

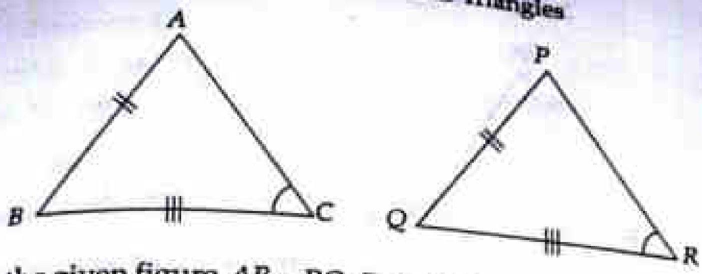


In the given figure if $BC = PN$ and $AB = MP$ then $\Delta ABC \cong \Delta MPN$

3. In the following conditions two triangles are not congruent

3.1. If each angle of one triangles is equal to corresponding angle of another triangle. Thus AAA doesnot follow congruence of triangles.

3.2. If two sides of one triangle is equal to two sides of another triangle but angle included between sides are not equal but any other angle of the two triangles are equal.



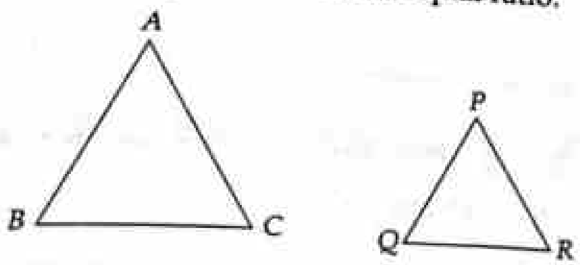
In the given figure $AB = PQ$, $BC = QR$ and $\angle C = \angle R$, But ΔABC and ΔPQR are not congruent as $\angle C$ and $\angle R$ are not the angle between given sides

4. Condition for two triangles to be similar :

Two triangles are said to be similar if

4.1. their corresponding angles are equal and

4.2. their corresponding sides are in the equal ratio.



In the given figure ΔABC and ΔPQR are said to be similar if

$\angle A = \angle P$,

$\angle B = \angle Q$,

$\angle C = \angle R$ and

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

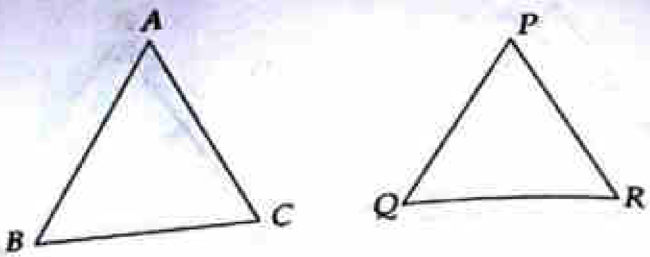
This is denoted by $\Delta ABC \sim \Delta PQR$

i.e. $\sim$ is the sign of similarity.

5. Relation between similarity and congruity : If two triangles are congruent then they must be similar but its converse is not true i.e. if two triangles are similar then it is not necessary that they are congruent.

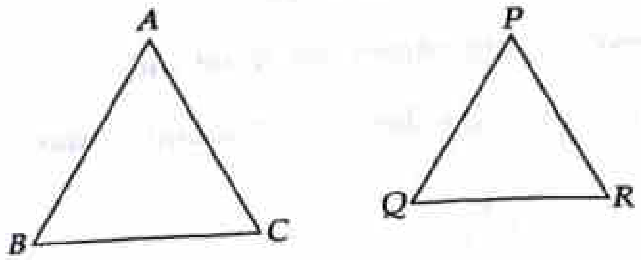
6. Condition for two triangles to be similar :

6.1. *AAA* condition : If corresponding angles of two triangles are equal then the two triangles are similar.



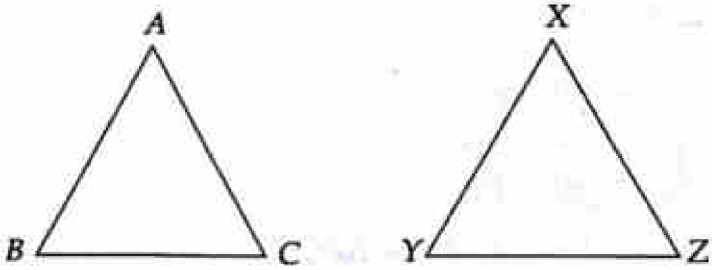
In the given figure if $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$ then $\Delta ABC \sim \Delta PQR$ and from it we can conclude that $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

6.2. S–S–S ratio condition : If corresponding sides of two triangle are in the same ratio, then the two triangles are similar. In the given figure



if $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ then $\Delta ABC \sim \Delta PQR$, and we can conclude that $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$.

6.3. S–A–S ratio condition : If one angle of a triangle is equal to one angle of another triangle and sides supporting these angles are in the same ratio then the two triangles are similar.



In the given figure if $\frac{AB}{XY} = \frac{BC}{YZ}$ and $\angle B = \angle Y$ then $\Delta ABC \sim \Delta XYZ$ and from this we conclude that $\angle A = \angle X$, $\angle C = \angle Z$

and $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

6.4. *AA* condition : If any two angles of the two triangles are equal then their third angle are also equal. So equality of any two angles of two given triangles are sufficient condition for similarities of two triangles.

∠C = ∠R then
$\frac{B}{Q} = \frac{AC}{PR} = \frac{BC}{QR}$
triangle are in
he given figure

onclude that

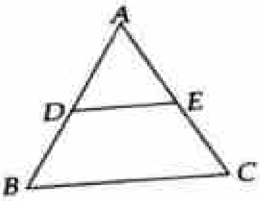
equal to one
ingles are in

3C ~ ΔXYZ

equal then
angles of
ies of two

... Theorem

In ΔABC if points D and E respectively lie on side AB and AC such that



DE ∥ BC then $\frac{AD}{DB} = \frac{AE}{EC}$

Converse of this theorem is also true.

Thale's theorem is very useful in solving similarity of two triangles.

... important conclusion based on mid points of sides of a triangle

If mid point of sides BC, CA and AB of a ΔABC are respectively D, E and F then following theorems always hold true.

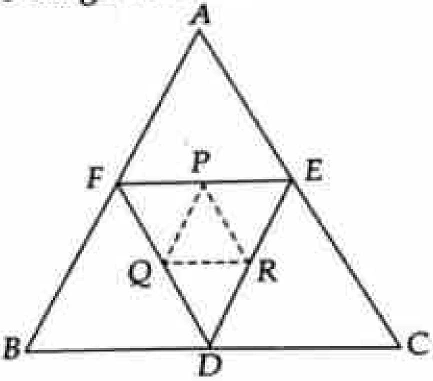
$EF \parallel BC$ and $EF = \frac{1}{2}BC$; similarly $ED \parallel AB$ and $ED = \frac{1}{2}AB$

$\square BDEF$, $\square DCEF$, $\square DEAF$ are parallelogram

$\Delta DBF \cong \Delta FED \cong \Delta EFA \cong \Delta CDE$ (be careful about order of the angles).

Since they are congruent they are also similar

Since ΔDBF, ΔFED, ΔEFA and ΔCDE are congruent therefore area of each of them $= \frac{1}{4}$ (area of ΔABC)



Since each of parallelogram $BDEF$, $DCEF$, $DEAF$ contains two of the similar triangles (as mentioned in 8.4), So area of each of parallelogram

$= 2 \times \frac{1}{4}$ (area of ΔABC)

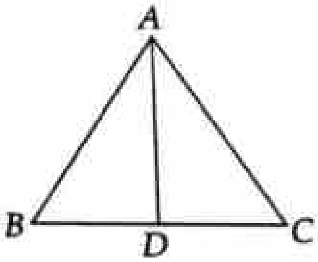
$= \frac{1}{2} \times$ area of ΔABC

If P, Q, R be respectively mid points of sides EF, FD and DE, then

area of $PQR = \frac{1}{16} \times$ area of ΔABC,

area of parallelogram $QRPF = \frac{1}{8} \times$ area of ΔABC etc.

...s dividing triangle in equal area



If point D is mid point of side BC of a triangle ABC then area of ΔABD = area of ΔACD

Note that these two triangles are not congruent.

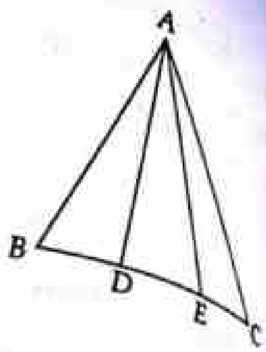
However if $\angle B = \angle C$ then the two triangles will be congruent

If points D and E divide side BC of a ΔABC in three equal parts (i.e. $BD = DE = EC$) then

area of ΔABD = area of ΔADE

= area of ΔAEC

$= \frac{1}{3}$ (area of ΔABC)

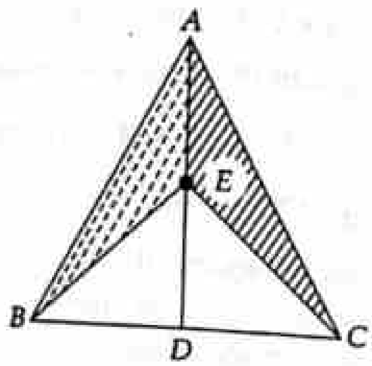


Here all the three triangles are not congruent too. If $\angle B = \angle C$ then triangles at two ends are congruent i.e. $\Delta ABD \cong \Delta ACE$ but they are not congruent to ΔADE

Note : property (9.1) and (9.2) can be extended for points 3, 4, 5, ... also.

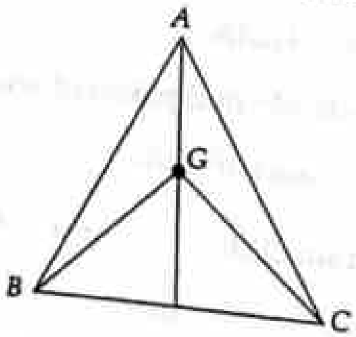
9.3. If E is any point on median AD of triangle ABC (i.e. D is mid point of BC) then

area of ΔABE = area of ΔACE



Here the two triangles are not congruent too.

9.4. In the above property if E is centroid (denoted by G) then

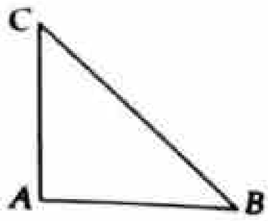


area of ΔABG = area of ΔBCG = area of ΔACG

$= \frac{1}{3}$ (area of ΔABC)

10. Important properties of right angled triangle :

10.1. If ABC is a right angled triangle with $\angle A = 90°$ then $BC^2 = AB^2 + AC^2$. Converse of the statement is also true.



10.2 **Suppose ABC is a right angled triangle with $\angle A = 90°$ if $AD \perp r\ BC$ then**

In ΔABD, $\angle BAD = 180° - 90° - B$

$= 90° - B = C$ $\quad (\because \angle B + \angle C = 90°)$

In ΔACD, $\angle CAD = 180° - 90° - C$

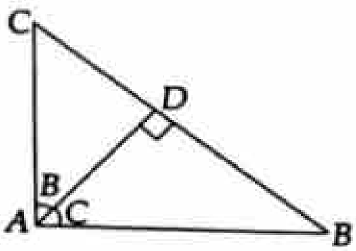
$= 90° - C = B$

Thus $\Delta ABC \sim \Delta DBA \sim \Delta DAC$ *(note)*

Again from $\Delta DBA \sim \Delta DAC$

$$\frac{DB}{DA} = \frac{DA}{DC}$$

or, $DA^2 = DB \cdot DC$ *(Note)*



11. Other important properties of triangles.

11.1. **If ΔABC is an acute angle triangle then**

$AB^2 + BC^2 > CA^2$

$BC^2 + CA^2 > AB^2$

and $AB^2 + CA^2 > BC^2$

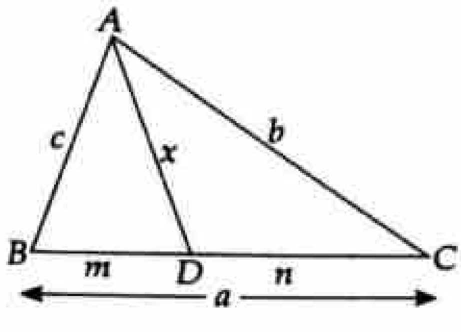
11.2. **For an obtuse angle triangle ABC,**

$AB^2 + BC^2 < CA^2$ (if $\angle B$ = obtuse angle)

or, $BC^2 + CA^2 < AB^2$ (if $\angle C$ = obtuse angle)

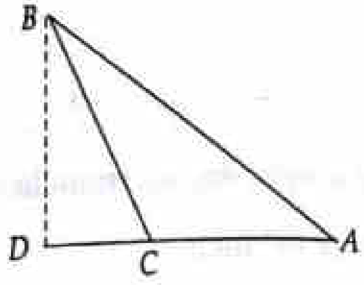
or, $CA^2 + AB^2 < BC^2$ (if $\angle A$ = obtuse angle)

11.3. (Stewart Theorem) **If AD (length= x) divides side BC of ΔABC such that $BD = m$ and $DC = n$ then**



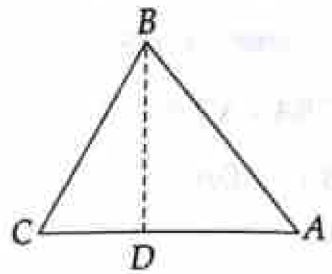
$a(x^2 + mn) = b^2m + c^2n$ where $BC = a$, $CA = b$, $AB = c$

11.4. If ΔABC is an obtuse angle triangle with $\angle C > 90°$ and $BD \perp r\ AC$ (produced part) then



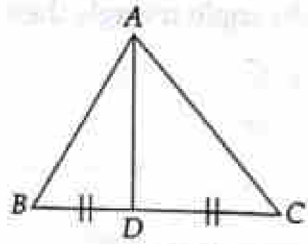
$$AB^2 = BC^2 + CA^2 + 2AC \cdot CD$$

11.5. If ABC is an acute angle triangle and $BD \perp r\ AC$ then



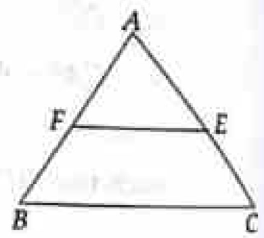
$$AB^2 = BC^2 + CA^2 - 2AC \cdot CD$$

11.6. (Appolonius Theorem) If D is mid point of side BC of ΔABC then



$$AB^2 + AC^2 = 2\,(AD^2 + BD^2)$$

11.7. If a line divides two sides of a triangle in the same ratio then it is parallel to the third side. Converse of the statement is also true.



Hence in ΔABC, $BC \parallel EF$

$$\Rightarrow \frac{AF}{FB} = \frac{AE}{EC}$$

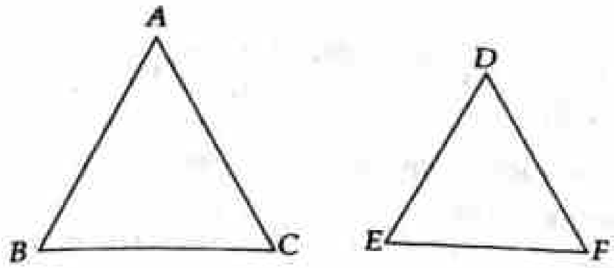
and $\frac{AF}{FB} = \frac{AE}{EC}$

$\Rightarrow BC \parallel EF$

(here $\Delta AEF \sim \Delta ACB$)

12. Relation among areas, sides, altitudes and perimeters of similar triangles

If $\Delta ABC \sim \Delta DEF$ then

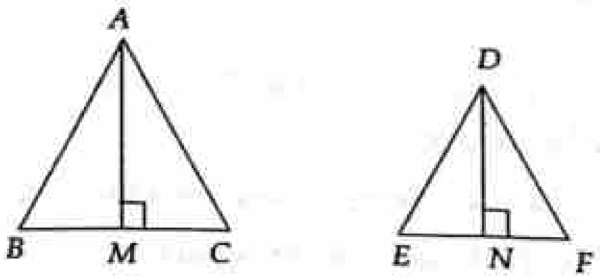


12.1. $\dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \left(\dfrac{AB}{DE}\right)^2 = \left(\dfrac{BC}{EF}\right)^2 = \left(\dfrac{AC}{DF}\right)^2$

i.e. In two similar triangles, the ratios of their areas is the square of the ratio of their sides.

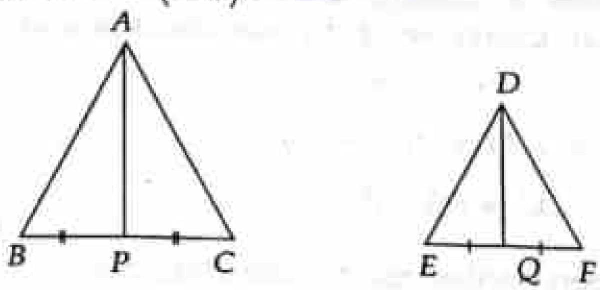
12.2. If AM and DN are respectively altitudes of two similar triangles then

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \left(\frac{AM}{DN}\right)^2$$



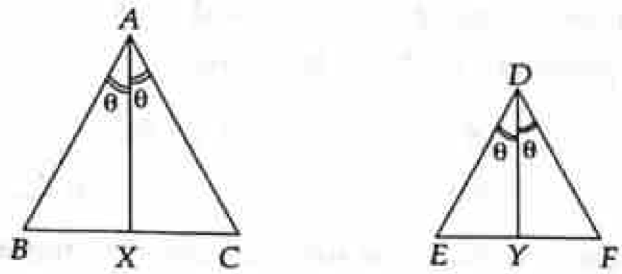
12.3. If AP and DQ are medians of two triangles then

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \left(\frac{AP}{DQ}\right)^2$$



12.4. If AX and DY are respectively bisectors of $\angle A$ and $\angle D$, then

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \left(\frac{AX}{DY}\right)^2$$



12.5. $\dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \left(\dfrac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta DEF}\right)^2$

12.6. We can conclude from the above result that
$$\frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta DEF} = \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{AM}{DN} = \frac{AP}{DQ} = \frac{AX}{DY}$$

12.7. If area of two similar triangles are equal then they are congru

13. Relations regarding areas of congruence and similarities of triangl

13.1. If two triangles are congruent then their area are equal.

13.2. If areas of two triangles are equal then it is not necessary that th are congruent.

13.3. If areas of two similar triangles are equal then they are congrue

13.4. Two congruent triangles are always similar but converse is n true.

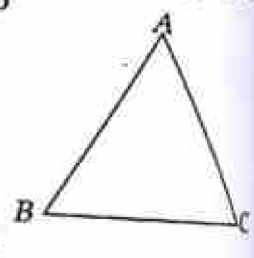
13.5. To prove that area of two triangles are equal, we commonly us the following two methods.

(a) If two triangles are congruent then their areas are equal.

(b) If base of two triangles are equal then draw perpendicular t the base so that length of two perpendicular are equal.

14. Some properties of isosceles and equilateral triangle

14.1. If two sides of a triangle are equal then their opposite angles are also equal. Converse of the statement is also true.

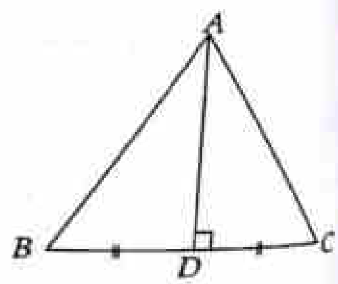


i.e. In ΔABC, $AB = AC \Rightarrow \angle B = \angle C$

and $\angle B = \angle C \Rightarrow AB = AC$

This property is also true for equilateral triangle

14.2. Line joining the vertex to the mid point of base of an isosceles triangle divides the angle at vertex and perpendicular to the base. Converse of the statement is also true.



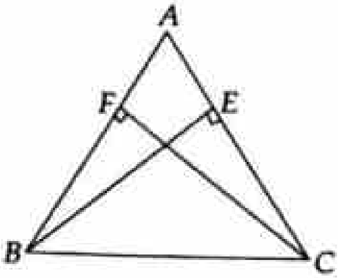
In the given figure, if $AB = AC$ and D is the mid point of BC then $AD \perp r\ BC$

Conversely, if $AB = AC$ and $AD \perp r\ BC$ then D is mid point of BC

This property is also true for equilateral triangle.

14.3. From each vertex of equal angle of an isosceles triangle, perpendiculars drawn to the oppsite sides are equal.

In the given figure, In ΔABC if $AB = AC$ and $BE \perp r\ AC$, $CF \perp r\ AB$ then $BE = CF$. Converse of the statement is also true

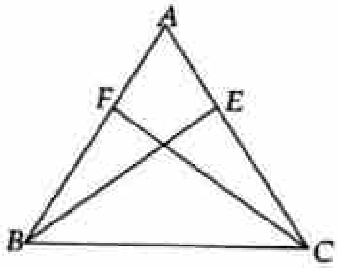


This property is also true for equilateral triangle.

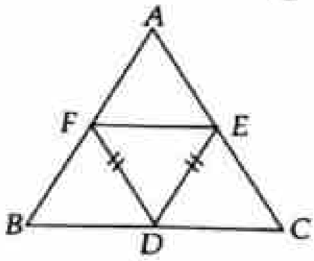
14.4. Medians drawn from the vertices of equal angles of an isosceles triangle to sides are equal.

Hence in ΔABC if $AB = AC$ (i.e. $\angle B = \angle C$) and E, F are respectively mid point of AC and AB then $BE = CF$

This property is also true for equilateral triangle.



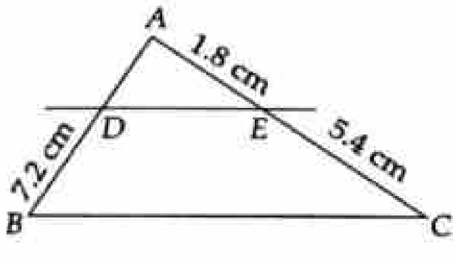
14.5. Triangle formed by joining midpoint of sides of an isosceles triangle is also an isosceles triangle. Suppose ΔABC is an isosceles triangle with $AB = AC$ and D, E, F are respectively mid point of BC, AC and AD then ΔDEF is an isosceles triangle with $DE = DF$



14.6. Triangle formed by joining midpoint of sides of an equilateral triangle is also an equilateral triangle.

Solved Examples

1. In the given figure if $DE \parallel BC$ then find length AD



Solution : $\because\ DE \parallel BC$

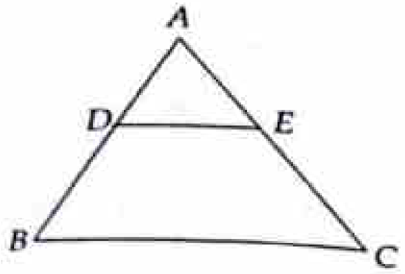
$$\therefore\ \frac{AD}{DB} = \frac{AE}{EC}$$

$$\text{or, } \frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$\text{or, } AD = 7.2 \times \frac{1.8}{5.4}$$

$$= 7.2 \times \frac{1}{3} = 2.4 \text{ cm}$$

2. In the figure given below $DE \parallel BC$. If $\frac{AD}{DB} = \frac{3}{5}$ and $AC = 4$[illegible] then find AE.



Solution : $DE \parallel BC$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} = \frac{3}{5}$$

Let $AE = x$

$\therefore \quad AC = AE + EC$

or, $EC = AC - AE = 4.8 - x$

Thus from (i), $\frac{x}{4.8 - x} = \frac{3}{5}$

or, $5x = 3 \times (4.8 - x)$

or, $5x = 14.4 - 3x$

or, $8x = 14.4$

$\therefore \quad x = \frac{14.4}{8} = 1.8$

Hence, $AE = 1.8$ cm

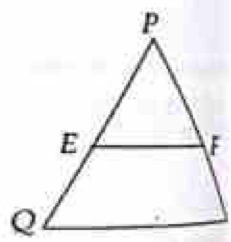
3. Points E and F respectively lie on sides PQ and PR of ΔPQR. For condition given below state whether $EF \parallel QR$

(i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm

(ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm

Solution : (i) Given $PE = 3.9$ cm, $EQ = 3$ cm,

$PF = 3.6$ cm and $FR = 2.4$ cm



Now, $\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$

and, $\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = 1.5$

Clearly, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

$\because$ EF doesnot divide PQ and PR in the same ratio

$\therefore$ EF and QR are not parallel.

(ii) Given $PE = 4$ cm, $QE = 4.5$ cm

$PF = 8$ cm and $RF = 9$ cm

Now, $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$ and $\frac{PF}{RF} = \frac{8}{9}$

$\therefore \frac{PE}{EQ} = \frac{PF}{RF}$

$\because$ EF divides sides PQ and PR in the same ratio

$\therefore$ $EF \parallel QR$

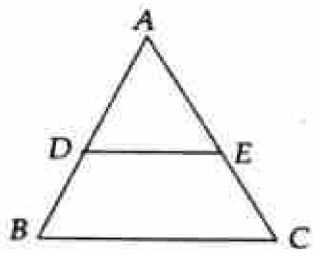
4. In the given figure if $DE \parallel BC$ then prove that $\frac{AB}{DB} = \frac{AC}{EC}$ and $\frac{AD}{AB} = \frac{AE}{AC}$

[learn as property]

Solution : If $DE \parallel BC$ then

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \textit{(Thale's Theorem)} \qquad \dots \text{(i)}$$

adding '1' to each side



$$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

or, $\frac{AD + DB}{DB} = \frac{AE + EC}{EC}$

or, $\frac{AB}{DB} = \frac{AC}{EC}$; first part is proved

Again from (i) $\frac{AD}{DB} = \frac{AE}{EC}$

$$\Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

adding 1 to each side

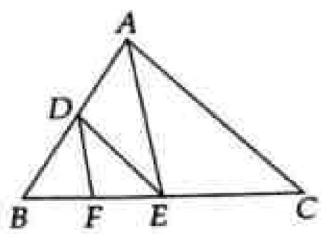
$$\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$$

or, $\frac{DB + AD}{AD} = \frac{EC + AE}{AE}$

or, $\frac{AB}{AD} = \frac{AC}{AE}$

or, $\frac{AD}{AB} = \frac{AE}{AC}$, ; 2^{nd} part is proved.

5. In the given figure $DE \parallel AC$ and $DF \parallel AE$ prove that $\frac{BF}{FE} = \frac{BE}{EC}$



Solution : In ΔABC, $DE \parallel AC$ *(given)*

$\therefore \quad \frac{BE}{EC} = \frac{BD}{DA}$ *(Thale's Theorem)*

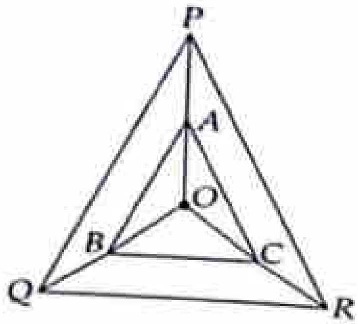
and In ΔBEA, $DF \parallel AE$ *(given)*

$\frac{BF}{FE} = \frac{BD}{DA}$

From (i) and (ii)

$\frac{BF}{FE} = \frac{BE}{EC}$; Proved

6. In the given figure points A, B, C respectively lie on sides OP, OQ, such that $AB \parallel PQ$ and $AC \parallel PR$ show that $BC \parallel QR$



Solution : In ΔOPQ, $AB \parallel PQ$ *(given)*

$\therefore \quad \frac{OA}{AP} = \frac{OB}{BQ}$

In ΔOPR, $AC \parallel PR$ *(given)*

$\therefore \quad \frac{OA}{AP} = \frac{OC}{CR}$

from (i) and (ii)

$\frac{OB}{BQ} = \frac{OC}{CR}$

clearly in ΔOQR, BC divides sides OQ and OR in the same ratio

$\therefore BC \parallel QR$

7. Area of two similar triangles are respectively 81 cm^2 and 121 cm^2. If altitude of first triangle is 4.5 cm, find the corresponding altitude of the second triangle.

Solution : Since ratio of area of two similar triangles is equal to ratio of square of their corresponding altitudes

$$\therefore \quad \frac{\text{Area of first triangle}}{\text{Area of second triangle}} = \frac{(\text{altitude of first triangle})^2}{(\text{altitude of second triangle})^2}$$

$$\therefore \quad \frac{81}{121} = \frac{(4.5)^2}{h^2}$$

(where 'h' is altitude of second triangle)

$$\text{or,} \quad \frac{9^2}{11^2} = \frac{(4.5)^2}{h^2}$$

$$\text{or,} \quad \frac{9}{11} = \frac{4.5}{h} \qquad \therefore \quad h = \frac{11 \times 4.5}{9} = 5.5 \text{ cm}$$

8. Area of two similar triangles are respectively 196 cm^2 and 256 cm^2. If shortest side of the shorter triangle is 7 cm then find the shortest side of the larger triangle.

Solution : In two similar triangles the ratio of their area is the square of the ratio of their sides. Assuming required side to be x cm,

we have $\frac{(x)^2}{(7)^2} = \frac{256}{196}$

$\therefore \quad x^2 = \frac{256 \times 7^2}{196} = \frac{16^2 \times 7^2}{14^2}$

$\Rightarrow \quad x = \frac{16 \times 7}{14} = 8$ cm

9. The sides of triangles are given below. Classify them as types of triangle.
(i) 9, 12, 15 (ii) 6, 7, 10 (iii) 6, 12, 13

Solution : (i) Let $a = 9, b = 12, c = 15,$ here c is the largest side

$\because \quad a^2 + b^2 = 9^2 + 12^2 = 81 + 144 = 225$

and $\quad c^2 = 15^2 = 225$

$\therefore \quad a^2 + b^2 = c^2$, so given triangle is a right angled triangle

(ii) Let $a = 6, b = 7, c = 10$; 'c' is the largest side

here $a^2 + b^2 = 6^2 + 7^2 = 36 + 49 = 85$

and $c^2 = 10^2 = 100$

$\because \quad c^2 > a^2 + b^2$

$\therefore$ Given triangle is an obtuse angled triangle

(iii) Let $p = 6, q = 12, r = 13$; r is the largest side

here $p^2 + q^2 = 6^2 + 12^2 = 36 + 144 = 180$

and $r^2 = 13^2 = 169$

$\because \quad r^2 < p^2 + q^2$

$\therefore$ given triangle is acute angled triangle

10. If diagonals of a rhombus are respectively 10 cm and 24 cm, find the length of its sides.

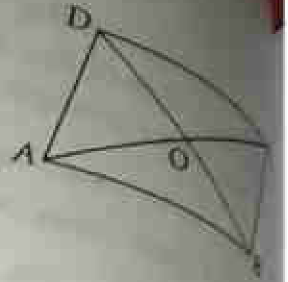
Solution : Let $ABCD$ is a rhombus whose diagonals are $BD = 10$ and $AC = 24$ cm. Since diagonals of rhombus intersect each other at right angle

$\therefore \quad BO = 5$ cm, $AO = 12$ cm and $\angle AOB = 90°$

(Here O is the point of intersection of diagonals)

In right angle ΔAOB

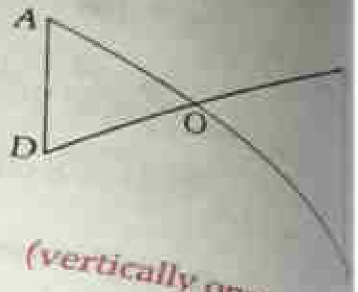
$$AB^2 = AO^2 + OB^2$$
$$= 5^2 + 12^2 = 25 + 144 = 169$$
$$\therefore \quad AB = \sqrt{169} = 13$$



11. In the given figure if $OA \cdot OB = OC \cdot OD$ then

(i) Prove that $\Delta OAD \sim \Delta OCD$

(ii) Does $AD \parallel BC$?



Solution : (i) Given $OA \cdot OB = OC \cdot OD$

or, $\dfrac{OA}{OD} = \dfrac{OC}{OB}$

also, $\angle AOD = \angle COB$ (vertically opposite angles)

$\therefore$ from S-A-S similarity criteria

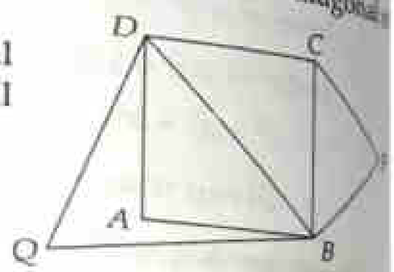
$\Delta AOD \sim \Delta COB$

Hence, $\angle A = \angle C$ and $\angle D = \angle B$

(ii) It is not necessary that AD and BC are parallel as $\angle A = \angle C$ or $\angle D = \angle B$ doesnot justify either alternate angle or corresponding angle.

12. Prove that area of an equilateral triangle formed on one side of a given square is one half of area of equilateral triangle formed on diagonal of the same square

Solution : Let $ABCD$ be a square. Equilateral ΔBPC is formed on side BC and equilateral ΔBQD is formed on diagonal BD.



To prove : ar $(\Delta BPC) = \dfrac{1}{2}$ ar (ΔBDQ)

In ΔBCD

$$BD^2 = BC^2 + CD^2 \quad [\angle C \text{ is right angle}]$$
$$= BC^2 + BC^2 = 2BC^2 \quad [\because BC = CD = \text{side of a square}]$$

Now, Since ΔBCD is an equilateral triangle, its each angle is 60°

similarly, ΔBQD is an equilateral triangle, its each angle is 60°

Now, $\Delta BCP \sim \Delta BDQ$

But in two similar triangles, the ratio of their area is the square of ratio of its sides

$$\therefore \quad \frac{\text{ar}(\Delta BCP)}{\text{ar}(\Delta BDQ)} = \frac{BC^2}{BD^2} = \frac{BC^2}{2BC^2} = \frac{1}{2}$$

$\therefore$ ar $(\Delta BCP) = \dfrac{1}{2}$ ar ΔBDQ; Proved

13. In the adjacent figure $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Prove that $BC = DE$

Solution : Given that

$AC = AE$

$AB = AD$

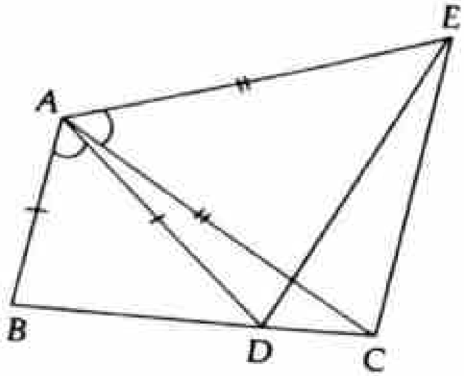
and $\angle BAD = \angle EAC$

or, $\angle DAC + \angle BAD = \angle DAC + \angle EAC$

or, $\angle BAC = \angle EAD$

$\therefore$ from S-A-S condition

$\Delta ABC \cong \Delta ADE$



$\because$ Corresponding sides of two congruent triangles are equal

$\therefore BC = DE$

14. AB is a line and P is its midpoint. D and E are two points on the same side of line segment AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$ (see the figure). Prove that

(i) $\Delta DAP \cong \Delta EBP$

(ii) $AD = BE$

Solution : (i) Given

$\angle BAD = \angle ABE$

and $\angle EPA = \angle DPB$

or, $\angle EPD + \angle EPA = \angle EPD + \angle DPB$

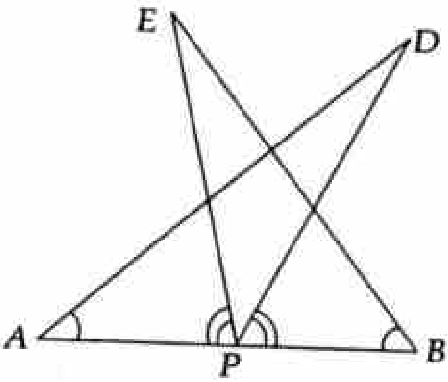
or, $\angle APD = \angle BPE$

and $AP = BP$ (given)

$\therefore$ from A-S-A , $\Delta DAP \cong \Delta EBP$

(ii) $\because$ $\Delta DAP \cong \Delta EBP$

$\therefore$ $AD = BE$



($\because$ corresponding sides of two congruent triangle are equal)

15. Triangle ABC is an isosceles triangle with $AB = AC$. Bisectors of $\angle B$ and $\angle C$ intersect each other at O. join $A - O$ and prove that

(i) $OB = OC$

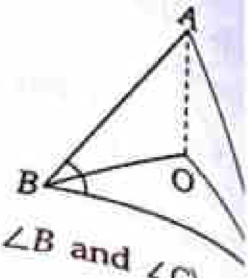
(ii) AO is bisector of $\angle A$

Solution : (i) In ΔABC, $AB = AC$ *(given)*

$\therefore$ $\angle B = \angle C$

or, $\frac{1}{2}\angle B = \frac{1}{2}\angle C$

or, $\angle OBC = \angle OCB$



($\because$ OB and OC are respectively bisector of $\angle B$ and $\angle C$)

$\therefore$ In ΔOBC

$OB = OC$ proved (i)

(ii) Now, in ΔABO and ΔACO

$AB = AC$

$AO = AO$ (common

and $OB = OC$ (from

$\therefore$ from S–S–S condition

$\Delta ABO \cong \Delta ACO$

$\therefore$ $\angle BAO = \angle CAO$

($\because$ corresponding part of congruent triangle are eq

16. In the given figure ABC and DBC are two isosceles triangle common base BC. prove that $\angle ABD = \angle ACD$

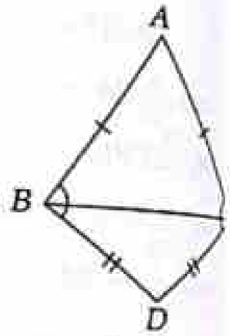
Solution : In ΔABC, $AB = AC$ *(given)*

$\therefore$ $\angle ABC = \angle ACB$ ... (i)

Again in ΔBDC

$BD = CD$ *(given)*

$\therefore$ $\angle DBC = \angle DCB$ ... (ii)



Adding (i) and (ii)

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

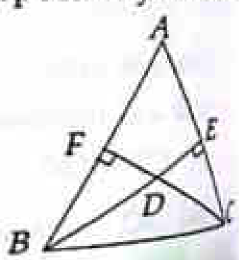
or, $\angle ABD = \angle ACD$; Proved.

17. ABC is a triangle in which altitude BE and CF respectively drawn AC and AB are equal. Prove that $\Delta BEC \cong \Delta CFB$

Solution : In ΔCFB and ΔBEC

$CF = BE$ *(given)*

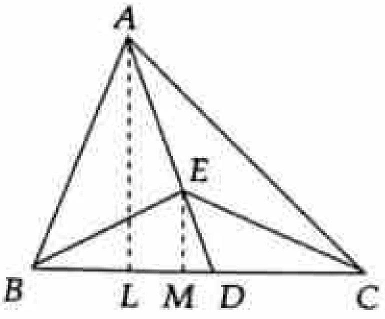
$\angle BEC = \angle CFB = 90°$ *(given)*



and $BC = BC$ (common)

Hence, $\Delta BEC \cong \Delta CFB$ (from RHS)

18. If E is any point on median AD of ΔABC prove that ar (ΔABE) = ar (ΔACE)



Solution : D is midpoint of side BC

$\therefore \quad BD = DC$

From, A draw $AL \perp r\ BC$ and from E draw $EM \perp r\ BC$

Now ar $(\Delta ABD) = \frac{1}{2} BD \times AL$

ar $(\Delta ACD) = \frac{1}{2} CD \times AL = \frac{1}{2} BD \times AL$ $[\because BD = CD]$

$\therefore$ ar (ΔABD) = ar (ΔACD) ... (i)

similarly ar $(\Delta EBD) = \frac{1}{2} BD \times EM$

ar $(\Delta ECD) = \frac{1}{2} CD \times EM = \frac{1}{2} \times BD \times EM$

$\therefore$ ar (ΔEBD) = ar (ΔECD) ... (ii)

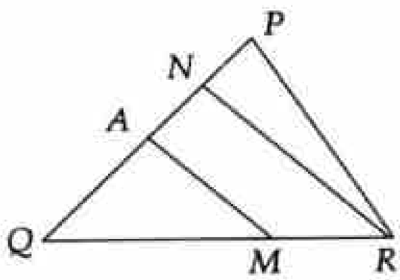
subtracting equation (ii) from equation (i)

ar (ΔABD) – ar (ΔEBD) = ar (ΔACD) – ar (ΔECD)

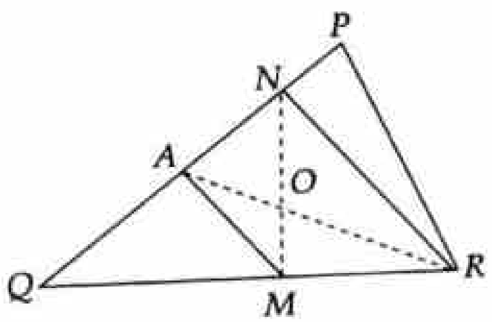
or, ar (ΔABE) = ar (ΔACE); Proved.

19. In the adjacent figure, point A is midpoint of side PQ of ΔPQR. M is any point on QR RN is drawn parallel to AM which intersects PQ at N

Prove that ar $(\Delta NQM) = \frac{1}{2}$ ar (ΔPQR)



Solution : Join NM and AR. Let they intersect at O



ar $(\Delta APR) = \frac{1}{2}$ $(AP) \times$ (height)

or, ar $(\Delta AQR) = \frac{1}{2} \times QA \times$ (height)

($\therefore$ It is given that $QA = AP$, and perpendicular drawn from R to side PQ is altitude for the two triangle)

$\therefore$ ar (ΔAPR) = ar (ΔAQR)

$= \frac{1}{2}$ ar (ΔPQR) ... (i)

Again $\because$ $AM \parallel NR$ [$\because$ base $PQ = 2 \times AP$]

$\therefore$ ar (ΔAMN) = ar (ΔAMR) (here base is AM) ... (ii)

Now, ar (ΔAQR) = ar (ΔAQM) + ar (ΔAMR)

or, ar (ΔAQR) = ar (ΔAQM) + ar (ΔAMN) (from (ii))

or, ar (ΔAQR) = ar (ΔNQM)

or, $\frac{1}{2}$ ar (ΔPQR) = ar (ΔNQM) (from (i))

$\therefore$ ar (ΔNQM) = $\frac{1}{2}$ ar (ΔPQR); Proved.

20. Two triangles BAC and BDC have same base BC and are situated at the same side of BC. If $\angle A = \angle D = 90°$ and AC and BD intersects at P, then prove that $AP \times PC = DP \times PB$

Solution : In ΔAPB and ΔDPC

$\angle A = \angle D = 90°$

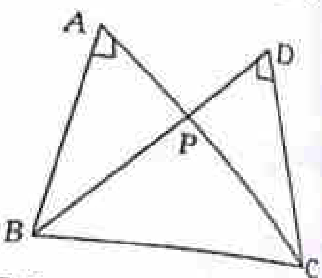
$\angle APB = \angle DPC$ (vertically opposite angle)

$\therefore$ from $A - A$ condition

$\Delta APB \sim \Delta DPC$

$\Rightarrow \frac{AP}{DP} = \frac{BP}{PC}$

or, $AP \times PC = DP \times PB$; Proved



21. In a ΔABC, altitude drawn from vertex A to side BC meets BC at D such that $DB = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$

Solution : Given $DB = 3CD$

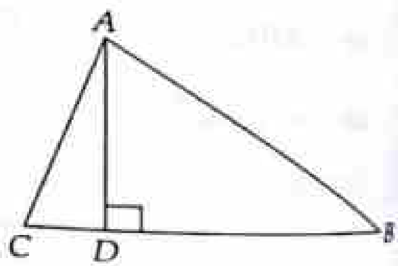
Now $BC = DB + CD$

$\Rightarrow BC = 3CD + CD$ [$\because BD = 3CD$]

$\Rightarrow BC = 4CD$

$\Rightarrow CD = \frac{1}{4} BC$ and

$DB = 3CD = \frac{3}{4} BC$ ... (i)



Applying Pythagorous theorem in ΔABD

We have $AB^2 = AD^2 + DB^2$ ... (ii)

Similarly in ΔACD

$AC^2 = AD^2 + CD^2$ ... (iii)

Subtracting equation (iii) from equation (ii)

$$AB^2 - AC^2 = DB^2 - CD^2$$

$$AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 \qquad \text{(From (i))}$$

$$AB^2 - AC^2 = \left(\frac{9}{16} - \frac{1}{16}\right)BC^2$$

$$AB^2 - AC^2 = \frac{1}{2}BC^2$$

$$\Rightarrow 2AB^2 - 2AC^2 = BC^2$$

$$2AB^2 = 2AC^2 + BC^2; \text{ Proved}$$

22. In a triangle ABC, D is midpoint of side BC. If $AC > AB$ and $AE \perp BC$, then prove that $AB^2 = AD^2 - BC \cdot ED + \frac{1}{4}BC^2$

Solution : In the given figure D, is midpoint of BC, $AE \perp BC$

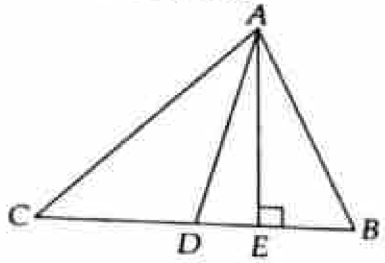
$\because$ $AC > AB$

$\therefore$ ΔABD is an acute angled triangle in which $\angle ADB$ is acute

$\therefore$ $AB^2 = AD^2 + BD^2 - 2BD \cdot DE$

or, $AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 - 2\cdot\frac{BC}{2}\cdot DE$ (see 11.5)

or, $AB^2 = AD^2 - BC \cdot DE + \frac{1}{4}BC^2$; Proved.



Exercise—5A

1. Assertion (A) : If two triangles are congruent their corresponding angles are equal.

   Reason (R) : Area of two congruent triangles are equal.

   (a) Both A and R are correct and Statement R is true explanation of Statement A

   (b) Both A and R are correct but Statement R is not a true explanation of Statement A

   (c) Statement A is correct, Statement R is wrong.

   (d) Statement A is wrong, Statement R is correct.

2. Consider the following statements :

   A unique triangle can be formed if

   1. its two sides and included angles are given.
   2. its three angles are given
   3. its two angles and included sides are given

   Among the given statements, true statements are

   (a) 1 and 2 only (b) 1 and 3 only (c) 2 and 3 only (d) all 1, 2 and 3.

3. The length of hypotenuse of a right angled triangle is $3\sqrt{10}$. If among two perpendicular lines, smallest one is tripled and bigger one is doubled, the hypotenuse of new right angled triangle thus formed is $9\sqrt{5}$ unit. The length of smallest and the bigger sides are respectively.
   (a) 5 unit, 9 unit (b) 5 unit, 6 unit
   (c) 3 unit, 9 unit (d) 3 unit, 6 unit

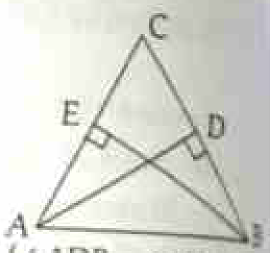
4. In a right angled triangle ABC, $\angle C$ is right angle. If sides of triangles are respectively a, b, c and length of perpendicular from C to AB is p, then which one of the following is true ?
   (a) $(a^2 + b^2)p^2 = a^2b^2$ (b) $a^2 + b^2 = a^2b^2p^2$
   (c) $p^2 = a^2 + b^2$ (d) $p^2 = a^2 - b^2$

5. In ΔABC, $\angle B = 90°$. If M and N are respectively midpoint of sides AB and BC then $4\,(AN^2 + CM^2)$ is equal to
   (a) $3AC^2$ (b) $4AC^2$ (c) $5AC^2$ (d) $6AC^2$

6. Area of a right angled triangle is A. If its one of the perpendicular side is b then length of altitude from right vertex to hypotenuse is
   (a) $\frac{2Ab}{\sqrt{b^4+4A^2}}$ (b) $\frac{2A^2b}{\sqrt{b^4+4A^2}}$ (c) $\frac{2Ab^2}{\sqrt{b^4+4A^2}}$ (d) $\frac{2A^2b^2}{\sqrt{b^4+A^2}}$

Triangle ABC is right angled at A. If $AB = 3$ unit, $AC = 4$ unit and AD is perpendicular to side BC, then what is the area of the triangle ADB ?
   (a) $\frac{9}{25}$ (unit)^2 (b) $\frac{54}{25}$ (unit)^2 (c) $\frac{72}{25}$ (unit)^2 (d) $\frac{96}{25}$ (unit)^2

8. Consider the following statements regarding the given figure.



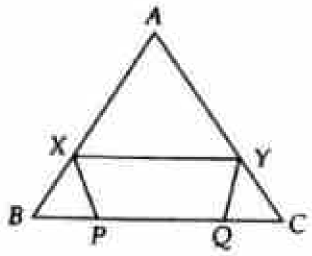
$(\angle ADB = \angle AEB = 90°)$

   1. $\Delta DAC \sim \Delta EBC$ 2. $CA/CB = CD/CE$
   3. $AD/BE = CD/CE$

   which of the following are true ?
   (a) 1, 2, 3 (b) 1, 2
   (c) 1, 3 (d) 2, 3

9. Suppose that $WXYZ$ is a square. Suppose points P, Q, R are respectively midpoint of WX, XY and ZW. K, L are respectively midpoint of PQ and PR. what is the value of $\frac{\text{area of triangle PKL}}{\text{area of triangle WXYZ}}$?
   (a) $\frac{1}{32}$ (b) $\frac{1}{16}$
   (c) $\frac{1}{8}$ (d) $\frac{1}{64}$

10. In the adjacent figure ABC is an equilateral triangle with each side equals to 30 cm. XY is parallel to BC; XP is parallel to AC and YQ is parallel to AB. If $XY + XP + YQ = 40$ cm, then what is the length of PQ ?

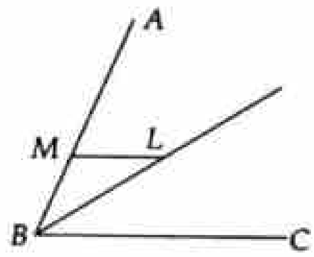


(a) 5 cm (b) 12 cm
(c) 15 cm (d) None of these

11. If corresponding sides of two similar triangles are in the ratio 9 : 4, then what is the ratio of its area ?
(a) 9 : 4 (b) 3 : 2 (c) 81 : 16 (d) 27 : 8

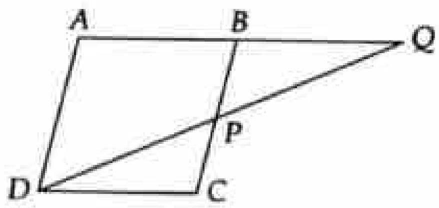
12. If sum of two angles of a triangle is equal to the third angle, then the triangle is
(a) right angled (b) acute angled
(c) equilateral (d) obtuse angled

13. In the adjacent figure, L is a point on the internal bisector of acute angle ABC and line ML is parallel to BC. Which one of the following is true ?



(a) ΔBML is equilateral
(b) ΔBML is isosceles and right angled equiangular
(c) ΔBML is isosceles but not right angled
(d) ΔBML is not isosceles.

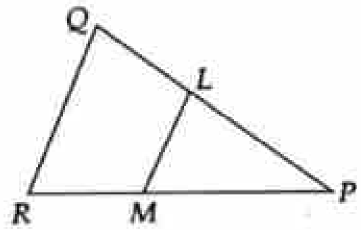
14. In the adjacent figure $ABCD$ is a parallelogram. P is a point on BC such that $PB : PC = 1 : 2$. Produced parts of side AB and DP meet at Q. If area of ΔBPQ is 20 (unit)^2, then what is the area of ΔDPC ?



(a) 20 (unit)^2 (b) 30 (unit)^2
(c) 40 (unit)^2 (d) None of these

15. ABC is an isosceles triangle such that $AB = BC = 8$ cm and $\angle ABC = 90°$ what is the length of altitude drawn from B to AC ?
(a) 4 cm (b) $4\sqrt{2}$ cm (c) $2\sqrt{2}$ cm (d) 2 cm

16. In the adjacent figure LM is parallel to QR. If LM divides ΔPQR in such a way so that area of trapezium $LMRQ$ is double the area of ΔPLM, what is the ratio $PL : PQ$?



(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

17. Points D and E respectively lie on sides AB and AC of triangle ABC such that DE is parallel to BC. If $AD = 2$ cm, $DB = 1$ cm, $AE = 3$ cm, then the length of EC is

(a) 1.5 cm (b) 1.6 cm (c) 1.8 cm (d) 2.1 cm

18. In ΔABC line PQ is drawn parallel to side BC where P and Q are respectively lie on side AB and AC. If $AB = 3AP$, what is the ratio of area of ΔAPQ to area of ΔABC ?

(a) 1 : 3 (b) 1 : 5 (c) 1 : 7 (d) 1 : 9

19. Consider a triangle ABC right angled at B. Side BC is produced to $D(BD>BC)$ such that $BD = 2DC$. Which one of the following is true?

(a) $AC^2 = AD^2 - 3CD^2$ (b) $AC^2 = AD^2 - 2CD^2$

(c) $AC^2 = AD^2 - 4CD^2$ (d) $AC^2 = AD^2 - 5CD^2$

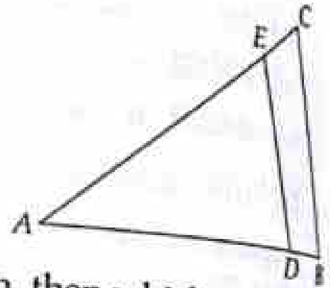
20. Angle Q of ΔPQR is right angled. If midpoint of sides PQ and QR are respectively X and Y, then which of the following is not true ?

(a) $RX^2 + PY^2 = 5\,XY^2$ (b) $RX^2 + PY^2 = XY^2 + PR^2$

(c) $4(RX^2 + PY)^2 = 5\,PR^2$ (d) $RX^2 + PY^2 = 3(PQ^2 + QR^2)$

21. In the figure given below, what is the value of x. It is given

that $DE \parallel BC$, $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$



(a) 3 (b) 4

(c) 5 (d) 6

22. If sides of a triangle are 20 cm, 16 cm and 13 cm, then which one of the following is true ?

(a) triangle is obtuse angled (b) triangle is equiangular

(c) triangle is acute angled (d) connot known

23. ΔABC and ΔDEF are similar such that $\frac{AB}{DE} = \frac{BC}{EF}$. Area of the two triangles are respectively 16 cm^2 and 49 cm^2. If $BC = 2\sqrt{2}$ cm, then what is length of EF ?

(a) 3.5 cm (b) $(3.5)\sqrt{2}$ cm (c) $(3.5)\sqrt{3}$ cm (d) 7.0 cm

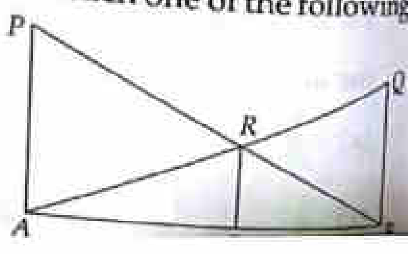
24. In the adjacent figure $PA = x$, $RC = y$, $QB = z$. Which one of the following is true ?

(a) $2y = x + z$

(b) $4y = x + z$

(c) $xy + yz = xz$

(d) $xy + xz = yz$



25. In ΔABC, $\angle A = 90°$. From point A, perpendicular AD is drawn to side BC. Which one of the following is true ?
(a) $\Delta ABC \sim \Delta DAC$ only (b) $\Delta DAC \sim \Delta DBA$ only
(c) $\Delta ABC \sim \Delta DAC \sim \Delta DBA$ only (d) $\Delta ABC \sim \Delta DBA$ only

26. In ΔABC, $DE \parallel BC$ where D and E are respectively lie on AB and AC and $DE : BC = 3 : 5$. What is the ratio of area of triangle ABC to area of triangle DAE ?
(a) $3 : 1$ (b) $5 : 3$ (c) $9 : 2$ (d) $25 : 9$

27. Two points D and E are respectively taken on sides AB and AC of a triangle in such a way that $AD = \frac{1}{3} AB$ and $AE = \frac{1}{3} AC$. If length of BC is 15 cm, what is the length of DE ?
(a) 10 cm (b) 8 cm (c) 6 cm (d) 5 cm

28. AD is the internal bisector of $\angle A$ of triangle ABC and meets side BC at D. If $BD = 5$ cm, $BC = 7.5$ cm than $AB : AC$ is
(a) $2 : 1$ (b) $1 : 2$ (c) $4 : 5$ (d) $3 : 5$

29. A line parallel to side BC of the triangle ABC meets side AB at D and side AC at E. If area of ΔABE is 36 square cm, then what is the area of ΔACD ?
(a) 18 sq. cm (b) 36 sq. cm (c) 9 sq. cm (d) 72 sq. cm

30. Consider a point D on the side AC of ΔABC. If P, Q, X, Y are respectively midpoints of AB, BC, AD and DC then what is the ratio of PX and QY ?
(a) $1 : 2$ (b) $1 : 1$ (c) $2 : 1$ (d) $2 : 3$

31. In ΔABC, PQ is parallel to BC. Accordingly if $AP : PB = 1 : 2$ and $AQ = 3$ cm then length of AC is equal to
(a) 6 cm (b) 9 cm (c) 12 cm (d) 8 cm

32. A line parallel to side BC of ΔABC meets AB and AC respectively at P and Q. If $AP = QC$, $PB = 4$ unit and $AQ = 9$ unit, then length of AP is
(a) 2.5 unit (b) 3 unit (c) 6 unit (d) 6.5 unit

33. ABC is an equilateral triangle. Points P and Q lie on $\overline{AB}$ and $\overline{AC}$ such that $\overline{PQ} \parallel \overline{BC}$. According if $\overline{PQ} = 5$ cm, then area of ΔAPQ is
(a) $\frac{25}{4}$ sq. cm (b) $\frac{25}{\sqrt{3}}$ sq. cm
(c) $\frac{25\sqrt{3}}{4}$ sq. cm (d) $25\sqrt{3}$ sq. cm

34. In ΔABC, XY is parallel to BC and it divides the triangle into two equal areas (X lies on AB and Y lies on AC) then $BX : AB$ equals lies on AC
(a) $\sqrt{2} : \sqrt{2} - 1$ (b) $1 : 1$ (c) $2 : 1$ (d) $\sqrt{2} - 1 : \sqrt{2}$

35. In triangle ΔABC, points E and F lie on sides AB and AC such that $EF || BC$. If $EF : BC = 3 : 7$ then what is the ratio of areas of ΔAEF and trapezium $EBCF$?
(a) 9 : 49 (b) 9 : 58 (c) 40 : 49 (d) 9 : 40

36. Points D and E respectively lie on the sides AB and AC of a triangle such that $DE || BC$. If area of ΔABC and trapezium $DECB$ are in the ratio 16 : 15, then $AE : AC$ is
(a) $\sqrt{5} : 4$ (b) 3 : 4 (c) 1 : 4 (d) 1 : 2

37. If ratio of area of two similar triangles are 16 : 9 then ratio of perimeter of squares formed on their corresponding sides are
(a) 256 : 81 (b) 16 : 9 (c) 4 : 3 (d) 5 : 4

38. If area of two similar triangles are equal then ratio of their corresponding altitude is.
(a) 1 : 1 (b) $1 : \sqrt{2}$
(c) 1 : 2 (d) Connot be determined

39. If ratio of area of two similar triangles are 64 : 81 and length of internal bisector of an angle of first triangle is 4, then what is the length of internal bisector of corresponding angle of the second triangle.
(a) 5 cm (b) $\frac{9}{2}$ cm (c) 10 cm (d) 20.25 cm

40. Diagonals AC and BD of a quadrilateral intersect at O. If $AO : OC = 1 : 2 = BO : OD$ and $AB = 16$ cm then DC is
(a) 16 cm (b) 8 cm (c) 4 cm (d) 32cm

41. In a triangle ABC, points D and E respectively lie on side AB and AC such that $DE || BC$. If $AD = 6$ cm, $DB = (12x - 6)$ cm, $AE = 2x$ cm and $CE = 16 - 2x$ cm then what is the value of x
(a) 2 (b) 4 (c) $\sqrt{2}$ (d) 8

42. Point D lies on side BC of a ΔABC such that $\angle ADC = \angle BAC$. If $CB = 8$ cm, $BD = 6$ cm then CA is
(a) $4\sqrt{3}$ cm (b) 4 cm (c) 2 cm (d) 16 cm

43. Among the following which group represents sides of an acute angled triangle
(a) 6, 9, 16 (b) 7, 8, 11 (c) 5, 12, 13 (d) both (a) & (b)

44. Among the following group which one represent the sides of an obtuse angled triangle
(a) 6, 7, 13 (b) 5, 6, 8 (c) 4, 5, 6 (d) None of these

45. If length of two sides of a triangle are 8 cm and 12 cm then what is the possible length of its third side ?
(a) between 8 cm and 12 cm (b) more than 20 cm
(c) less than 4 cm (d) between 4 cm and 20 cm

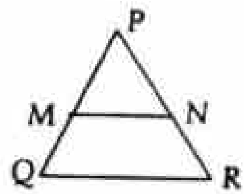
46. If two sides of an acute angled triangle is 8 cm and 15 cm and the length of third side is x, then
(a) $13 < x < 17$ (b) $7 < x < 17$
(c) $\sqrt{161} < x < 17$ (d) $7 < x < 23$

47. If two sides of an obtuse angled triangle are 8 cm and 15 cm and third sides is x, then
(a) $7 < x < 23$ (b) $7 < x < \sqrt{161}$
(c) $17 < x < 23$ (d) (b) or, (c)

48. In the ΔABC, points M and N respectively lie on side AB and AC such that area of triangle ABC is double than area of trapezium $BMNC$, The ratio $AM : MB$ is.
(a) $\sqrt{2} - 1$ (b) $2 - \sqrt{2}$ (c) $\sqrt{2} + 1$ (d) $2 + \sqrt{2}$

49. line PQ meets triangle ABC such that P lies on AB and Q lies on AC. If $AP = 1$ cm, $PB = 3$ cm, $AQ = 1.5$ cm and $QC = 4.5$ cm, then what is the ratio of area of ΔAPQ and quadrilateral $PBCQ$?
(a) 1 : 16 (b) 1 : 15 (c) 1 : 9 (d) 1 : 8

50. In the given figure $PM \cdot PR = PN \cdot PQ$ and is such that $4\,PM = 3\,PQ$. If area of ΔPQR is 32 cm^2, then area of quadrilateral $MNRQ$ is
(a) 18 cm^2 (b) 14 cm^2
(c) 20 cm^2 (d) 12 cm^2



51. ABC is a given triangle. A straight line EF is drawn parallel to BC. It cuts AB at E and AC at F. If area of AEF is one third area of quadrilateral $EBCF$ then $EB : AB$ is
(a) 1 : 2 (b) $1 : \sqrt{2}$ (c) $1 : \sqrt{3}$ (d) 1 : 9

Answers–5A

1. (b)	2. (b)	3. (c)	4. (a)	5. (c)	6. (a)	7. (b)	8. (a)
9. (b)	10. (d)	11. (c)	12. (a)	13. (c)	14. (d)	15. (b)	16. (b)
17. (a)	18. (d)	19. (a)	20. (c)	21. (b)	22. (c)	23. (b)	24. (c)
25. (c)	26. (d)	27. (d)	28. (a)	29. (b)	30. (b)	31. (b)	32. (c)
33. (c)	34. (d)	35. (d)	36. (c)	37. (c)	38. (a)	39. (b)	40. (d)
41. (a)	42. (b)	43. (b)	44. (b)	45. (d)	46. (c)	47. (d)	48. (c)
49. (b)	50. (b)	51. (a)					

Explanation

1. **(b)** both A and R are true but R is not the correct explanation of A.

2. **(b)** If three angles of a triangle are given then length of sides are not fixed. So unique triangle cannot be formed.

3. **(c)** Let two $\perp r$ sides of triangle be x and y

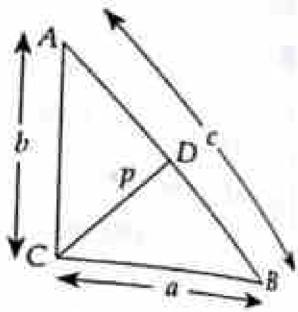
According to question, $x^2 + y^2 = (3\sqrt{10})^2$

$$\Rightarrow x^2 + y^2 = 90 \quad \cdots (i)$$

$$\text{and } 9x^2 + 4y^2 = 405 \quad \cdots (ii)$$

Solving equation (i) and (ii), $x = 3$ unit $y = 9$ unit

4. **(a)** In right angled ΔABC



area $= \frac{1}{2} \times a \times b \quad \ldots (i)$

Again, In right angle ΔABC,

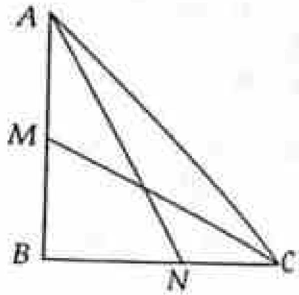
Area $= \frac{1}{2} \times AB \times DC$

$\Rightarrow \frac{1}{2} ab = \frac{1}{2} \times c \times p$

$\Rightarrow ab = p(\sqrt{a^2 + b^2}) \; (\because c^2 = a^2 + b^2)$

$\Rightarrow a^2b^2 = p^2 (a^2 + b^2)$

5. **(c)** In right angled ΔABC



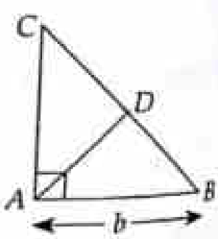
$4(AN^2 + CM^2) = 4\{AB^2 + BN^2 + BM^2 + BC^2\}$

$= 4\left\{(AB)^2 + \left(\frac{BC}{2}\right)^2 + \left(\frac{AB}{2}\right)^2 + (BC)^2\right\}$

$= 4AB^2 + BC^2 + AB^2 + 4BC^2$

$= 5(AB^2 + BC^2) = 5AC^2$

6. **(a)** In ΔABC, $A = \frac{1}{2} \times$ base $\times$ altitude



$$= \frac{1}{2} \times b \times AC$$

$$AC = \frac{2A}{b}$$

From Pythagoras theorem, $AC^2 + AB^2 = BC^2$

$\Rightarrow BC = \sqrt{\frac{4A^2}{b^2} + b^2}$

Again in ΔABC : $A = \frac{1}{2} \times BC \times AD$

$$\Rightarrow AD = \frac{2A}{\sqrt{\frac{4A^2 + b^4}{b^2}}}$$

$$= \frac{2Ab}{\sqrt{4A^2 + b^4}}$$

(b) From Pythagoras theorem,

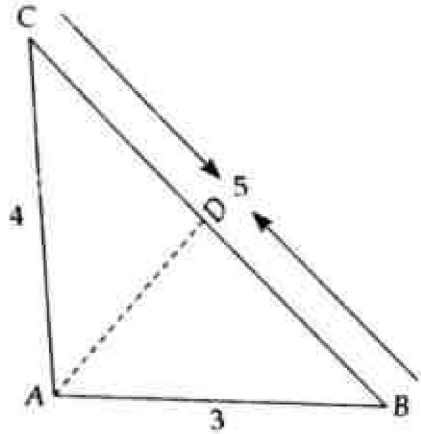
hypotenuse $BC = \sqrt{3^2 + 4^2} = 5$

Area of $\Delta ABC = \frac{1}{2} \times 3 \times 4 = 6$

But, Area $= \frac{1}{2} \times BC \times AD$

$$\Rightarrow \quad 6 = \frac{5}{2} AD$$

$$\Rightarrow AD = \frac{12}{5}$$



In right angled ΔADB

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow \quad (3)^2 = \left(\frac{12}{5}\right)^2 + BD^2$$

$$\Rightarrow \quad BD = \sqrt{9 - \frac{144}{25}} = \frac{9}{5}$$

$\therefore$ area of $\Delta ABD = \frac{1}{2} \times BD \times AD$

$$= \frac{1}{2} \times \frac{9}{5} \times \frac{12}{5} = \frac{54}{25}$$

8. (a) In ΔCAD and ΔCEB

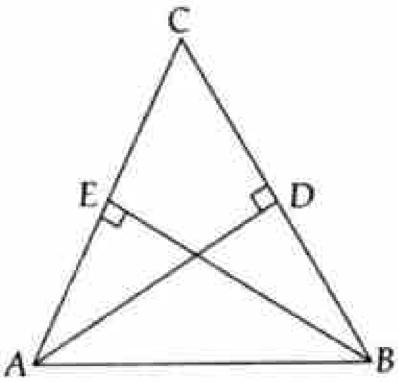
$\angle C = \angle C$ (common)

$\angle CEB = \angle ADC$ (each equals to 90°)

$\angle CAD = \angle CBE$ (Remaining angle)

$\Delta CAD \sim \Delta CEB$

so sides are proportional



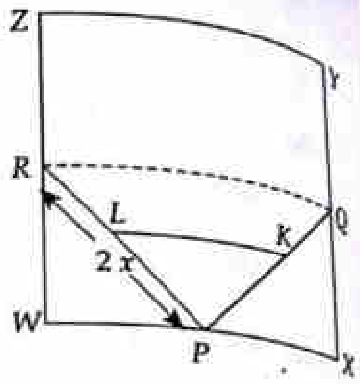
$$\therefore \quad \frac{CA}{CB} = \frac{CD}{CE} \quad \text{and} \quad \frac{AD}{BE} = \frac{CD}{CE}$$

9. (b) Area $(PRQ) = \frac{1}{2}$ Area $(WXQR)$

$$= \frac{1}{2}\left[\frac{1}{2} \text{area} (WXYZ)\right]$$

$= \frac{1}{4}$ area (WXYZ) ...(i)

$\therefore \frac{\text{area}(PRQ)}{\text{area}(PLK)} = \frac{RP^2}{LP^2}$ (from similar triangle property)



$\Rightarrow \frac{\text{area}(PRQ)}{\text{area}(PLK)} = \frac{(2x)^2}{x^2}$

$\Rightarrow$ area $(PRQ) = 4$ area (PLK)

$\Rightarrow \frac{1}{4}$ area $(WXYZ) = 4$ area (PLK)

(from equation (i))

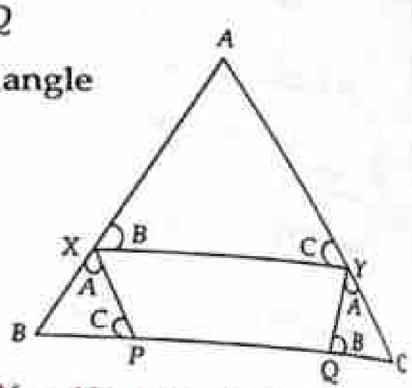
$\Rightarrow \frac{1}{16}$ area $(WXYZ) =$ area (PLK)

$\Rightarrow \frac{1}{16} = \frac{\text{area}(PLK)}{\text{area}(WXYZ)}$

10. (d) $\because$ $XP \parallel AC$, $YQ \parallel AB$

$\therefore$ $\angle XBP = \angle YQC$ and $\angle XPB = \angle YCQ$

$\therefore$ ΔXBP and ΔYCQ are equilateral triangle



Now, since $XY \parallel BC$

$\therefore \frac{AX}{AB} = \frac{XY}{BC}$

$\Rightarrow AX = XY$ $(\because AB = BC = 30 \text{ cm})$

Now, $XY + XP + YQ = 40$ cm

$\Rightarrow AX + XB + YQ = 40$ $(\because XY = AX, XP = XB)$

$\Rightarrow AB + YQ = 40$

$\Rightarrow YQ = 40 - 30 = 10$ cm

$\therefore$ $YQ = XP = 10$ cm

$\therefore$ $BP = CQ = 10$ cm

$\therefore$ $PQ = 30 - BP - CQ$

$= 30 - 10 - 10 = 10$ cm

11. (c) $\therefore$ ratio of sides $= 9 : 4$

$\therefore$ ratio of their areas $= 9^2 : 4^2 = 81 : 16$

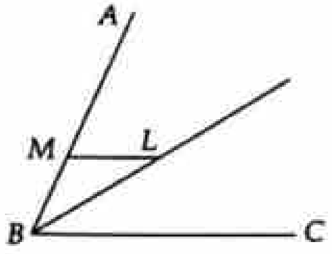
12. (a) Given $B + C = A$

$\therefore$ $A + B + C = 180$

$\Rightarrow A + A = 180°$

$\Rightarrow A = 90°$

13. (c) $\because$ BL is bisector of $\angle ABC$



Let $\angle MBL = \angle LBC = x$

Now, $ML \parallel BC$

$\Rightarrow$ $\angle LBC = \angle MLB = x$

$\Rightarrow$ $\angle MLB = \angle MBL = x$

$\Rightarrow$ ΔBLM is an isosceles triangle but it is not necessary that $\angle BML$ is 90°

So, ΔBML is isosceles but not right angled.

14. (d) In ΔBPQ and ΔCPD,

Let $\angle BQP = \angle CDP =$ *(alternate angle)*

$\angle BPQ = \angle CPD =$ *(vertically opposite)*

$\therefore$ $\Delta BPQ \sim \Delta CPD$,

$\therefore$ $\dfrac{\text{area of } \Delta BPQ}{\text{area of } \Delta CPD} = \dfrac{PB^2}{PC^2}$

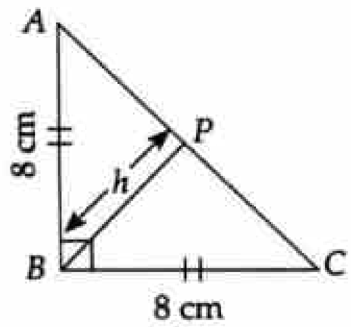
$\Rightarrow$ $\dfrac{20}{\text{area of } \Delta DPC} = \dfrac{1}{4}$

$\Rightarrow$ area of $\Delta DPC = 80$ sq. unit

15. (b) Given that ΔABC is an isosceles right angled triangle

$AC^2 = BC^2 + AB^2$ *(Pythagoras theorem)*

$AC^2 = 8^2 + 8^2 = 64 + 64$



$AC = 8\sqrt{2}$ cm

area of $\Delta ABC = \dfrac{1}{2} \times AB \times BC$

$= \dfrac{1}{2} \times 8 \times 8 = 32$ sq. unit

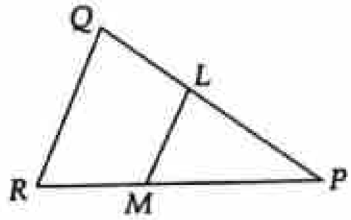
and area of $\Delta ABC = \dfrac{1}{2} \times AC \times BP$

$= \dfrac{1}{2} \times 8\sqrt{2} \times h = 4\sqrt{2}\,h$

$\Rightarrow$ $4\sqrt{2}\,h = 32$

$\Rightarrow$ $h = \dfrac{8}{\sqrt{2}} = 4\sqrt{2}$ cm

16. (b) In figure, ΔRQP and ΔMLP are similar



$\therefore$ $\dfrac{\text{area of } \Delta MLP}{\text{area of } \Delta RQP} = \dfrac{PL^2}{PQ^2}$ ... (i)

Let area of $\Delta MLP = x$

then area of $\Delta RQP =$ area of ΔMLP + area $RQLM$

Given, area of $RQLM = 2 \times$ area of $\Delta MLP = 2x$

from (i), $\frac{PL^2}{PQ^2} = \frac{x}{3x} = \frac{1}{3}$

or, $\frac{PL}{PQ} = \frac{1}{\sqrt{3}}$

17. **(a)** In ΔADE and ΔABC

$\angle A = \angle A$

$DE \parallel BC$

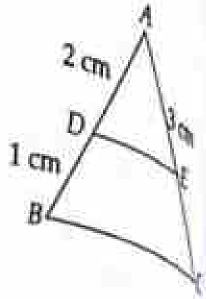
$\therefore \angle ADE = \angle ABC$ *(corresponding angle)*

$\therefore \Delta ADE \sim \Delta ABC$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{2}{1} = \frac{3}{EC}$

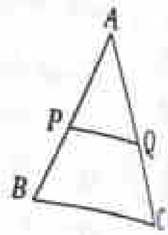
$\Rightarrow EC = \frac{3}{2} = 1.5$ cm



18. **(d)** ΔAPQ and ΔABC are similar.

$\therefore \frac{\text{area of } \Delta APQ}{\text{area of } \Delta ABC}$

$= \frac{AP^2}{AB^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$



19. **(a)** Given, $BD = 2DC$

$\therefore BC + CD = 2DC$

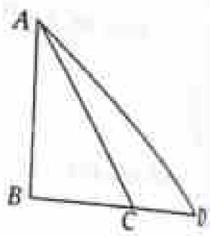
$BC = CD$ ... (i)

In ΔABC

$AC^2 = AB^2 + BC^2$ ... (ii)

In ΔABD

$AD^2 = AB^2 + BD^2$ ... (iii)



Subtracting equation (ii) from equation (iii)

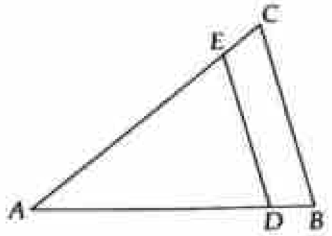
$AD^2 - AC^2 = BD^2 - BC^2$

$= (BD - BC)(BD + BC)$

$= CD(2CD + CD) = 3CD^2$

$\therefore AC^2 = AD^2 - 3CD^2$

20. (c) Solve as question no. 5



21. (b) $\because \quad DE \parallel BC$

$\Rightarrow \quad \angle B = \angle D$ and $\angle E = \angle C$

$\therefore \quad \Delta ADF \sim \Delta ABC$

$\therefore \quad \dfrac{AD}{DB} = \dfrac{AE}{EC}$

$\Rightarrow \quad \dfrac{x}{x-2} = \dfrac{x+2}{x-1}$

$\Rightarrow \quad x(x-1) = (x+2)(x-2)$

$\Rightarrow \quad x^2 - x = x^2 - 4$

$\Rightarrow \quad x = 4$

22. (c) $\because \quad 20^2 = 400$ and $16^2 + 13^2$

$= 256 + 169 = 425$

since sum of squares of two sides is greater than square of third side triangle is acute angled triangle

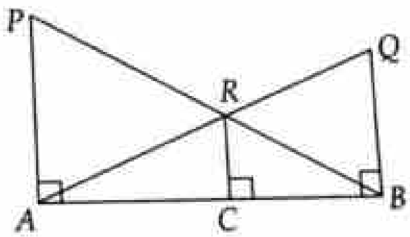
23. (b) $\therefore \quad \dfrac{BC}{EF} = \sqrt{\dfrac{\text{area of }(\Delta ABC)}{\text{area of }(\Delta DEF)}}$

$= \sqrt{\dfrac{16}{49}} = \dfrac{4}{7}$

$\therefore \quad EF = \dfrac{7}{4} BC$

$= \dfrac{7}{4} 2\sqrt{2} = (3.5)\sqrt{2}$

24. (c) In ΔBRC and ΔPAB



$\angle RCB = \angle PAB$

$\angle RBC = \angle PBA$

$\therefore \quad \Delta BRC \sim \Delta PAB$

$\therefore \quad \dfrac{RC}{PA} = \dfrac{BC}{AB}$

$\Rightarrow \quad \dfrac{y}{x} = \dfrac{BC}{AB} \qquad \ldots \text{(i)}$

similarly in ΔARC and ΔABQ

$\Delta ARC \sim \Delta ABQ$

$\therefore \quad \dfrac{RC}{QB} = \dfrac{AC}{AB}$

$$\Rightarrow \frac{y}{z} = \frac{AB - BC}{AB}$$

$$= 1 - \frac{BC}{AB} = 1 - \frac{y}{z}$$

$$\Rightarrow \frac{y}{x} + \frac{y}{z} = 1$$

$$\Rightarrow xy + yz = xz$$

25. **(c)** See article 10.2

26. **(d)** $\dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta DAE} = \left(\dfrac{BC}{DE}\right)^2$

$$= \left(\frac{5}{3}\right)^2 = \frac{25}{9} = 25 : 9$$

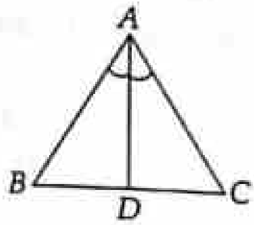
27. **(d)** $DE = \frac{1}{3}BC$

$$= \frac{1}{3} \times 15 \text{ cm} = 5 \text{ cm}$$

28. **(a)** $\because$ AD is bisector of $\angle A$

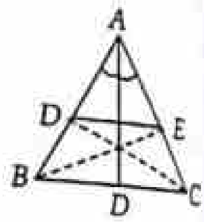
$\therefore$ $\dfrac{AB}{AC} = \dfrac{BD}{DC} = \dfrac{5}{7.5 - 5}$

$$= \frac{5}{2.5} = 2 : 1$$



29. **(b)** $\because$ area of ΔBED = area of ΔCED

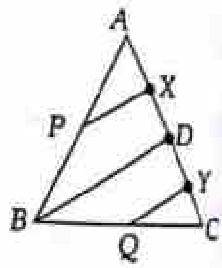
Hence, area of ΔACD = area of ΔABE



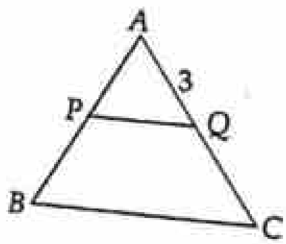
30. **(b)** $\because$ $PX \parallel BD$ and $PX = \frac{1}{2} BD$

$QY \parallel BD$ and $QY = \frac{1}{2} BD$

$\therefore$ $PX : QY = 1 : 1$



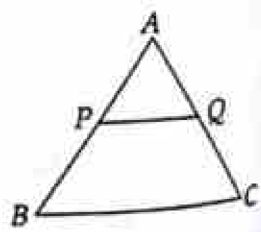
31. **(b)** According to question,



$$AC = \frac{3}{1} \times 3 = 9 \text{ cm}$$

32. **(c)** $\because$ $PQ \parallel BC$

$\therefore$ $\dfrac{AP}{AB} = \dfrac{AQ}{AC}$



$\Rightarrow \frac{AB}{AP} = \frac{AC}{AQ}$

$\Rightarrow \frac{AP+PB}{AP} = \frac{AQ+QC}{AQ}$

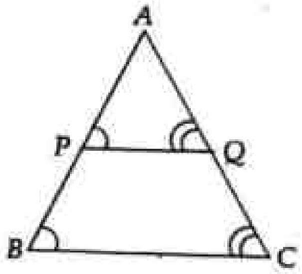
$\Rightarrow \frac{PB}{AP} = \frac{QC}{AQ} = \frac{AP}{AQ}$

$\Rightarrow AP^2 = PB \times AQ = 4 \times 9 = 36$

$\therefore AP = 6$ unit

33. (c) Drawing diagram $PQ \parallel BC$

and $PQ = 5$ cm



$\therefore$ ABC is an equilateral triangle

$\therefore \angle A = \angle B = \angle C$

Again $\because$ $PQ \parallel BC$

$\therefore \angle APQ = \angle B$

and $\angle AQP = \angle C$

Hence $\angle A = \angle APQ = \angle AQP$

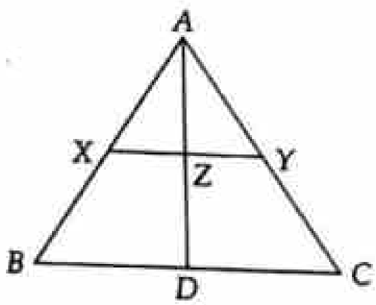
Thus ΔAPQ is an equilatral triangle with each side $= 5$ cm

$\therefore$ area of $\Delta APQ = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 5 \times 5$

$= \frac{25\sqrt{3}}{4}$ sq. $(\text{cm})^2$

34. (d) $\because \Delta ABC \sim \Delta AXY$



$\frac{\text{Area } \Delta \text{ ABC}}{\text{Area } \Delta \text{ AXY}} = \left(\frac{\text{AB}}{\text{AX}}\right)^2$

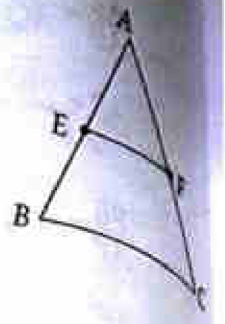
$\Rightarrow \frac{2}{1} = \left(\frac{AB}{AX}\right)^2$

$\Rightarrow \sqrt{2}\, AX = AB$

$\Rightarrow \sqrt{2}(AB - BX) = AB$

$\Rightarrow (\sqrt{2}-1) AB = \sqrt{2}\, BX$

$\therefore BX : AB = \sqrt{2} - 1 : \sqrt{2}$

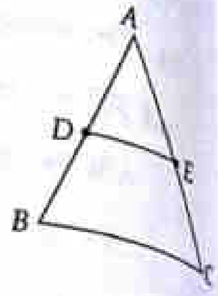


35. (d) $\because \quad \Delta AEF \sim \Delta ABC$

$\therefore \quad \dfrac{\text{area of } \Delta AEF}{\text{area of } \Delta ABC} = \left(\dfrac{EF}{BC}\right)^2 = \left(\dfrac{3}{7}\right)^2 = \dfrac{9}{49}$

area of trapezium $BEFC = 49k - 9k = 40k$

$\therefore \quad \dfrac{\text{area of } \Delta AEF}{\text{area of trapezium } BEFC} = \dfrac{9k}{40k} = \dfrac{9}{40}$



36. (c) $\because$ area of $\Delta ABC = 16k$ and area of trapezium $DECB = 15k$

area of $\Delta ADE = 16k - 15k = k$

$\Delta ADE \sim \Delta ABC$

$\therefore \quad \dfrac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \left(\dfrac{AE}{AC}\right)^2$

or, $\dfrac{k}{16k} = \left(\dfrac{AE}{AC}\right)^2$

or, $\dfrac{1}{4} = \dfrac{AE}{AC}$

37. (c) ratio of area of perimeter $= \sqrt{\text{ratio of area of square}}$

$$= \sqrt{\frac{16}{9}} = \frac{4}{3}$$

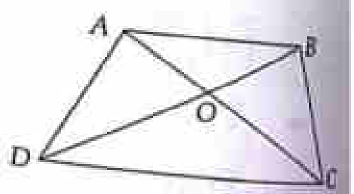
38. (a) When area of two similar triangles are equal they are congruent.
Hence required ratio is $1 : 1$

39 (b) Since ratio of area of two similar triangles is equal to square of ratio of their angle bisector.

$\therefore \quad \dfrac{64}{81} = \left(\dfrac{4}{x}\right)^2$

or, $x^2 = \dfrac{16 \times 81}{64} = \dfrac{81}{4}$

or, $x = \sqrt{\dfrac{81}{4}} = \dfrac{9}{2} = 4.5$ cm



40. (d) $\because \dfrac{AO}{OC} = \dfrac{BO}{OD} = \dfrac{1}{2}$

and $\angle AOB = \angle COD$

$\therefore \quad \Delta AOB \sim \Delta\, COD \Rightarrow \dfrac{AO}{OC} = \dfrac{AB}{DC}$

$\Rightarrow \dfrac{1}{2} = \dfrac{16}{DC}$

$\Rightarrow DC = 32$ cm

41. (a) DE ||

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

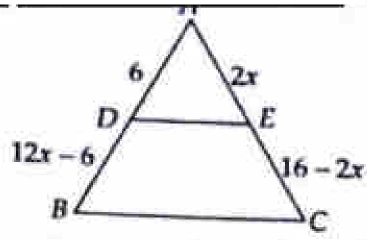
$\Rightarrow \frac{6}{12x-6} = \frac{2x}{16-2x}$

$\Rightarrow \frac{1}{2x-1} = \frac{x}{8-x}$

$\Rightarrow 8 - x = 2x^2 - x$

$\Rightarrow x^2 = 4$

$\therefore \quad x = 2$



42. (b) In ΔBAC and ΔADC

$\angle ADC = \angle BAC$ and $\angle C$ is common

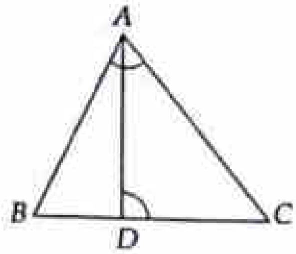
$\therefore \quad \Delta BAC \sim \Delta ADC \Rightarrow \frac{BC}{AC} = \frac{AC}{DC}$

$\Rightarrow AC^2 = BC \cdot DC$

$= BC\,(BC - BD)$

$= 8\,(8 - 6) = 16 \text{ cm}^2$

$\therefore \quad AC = 4$ cm



43. (b) $\because\ 6 + 9 < 16$

$\therefore$ this group doesnot form triangle

$\because\ 11^2 < 7^2 + 8^2$, triplet mentioned in option (b) formed acute angled triangle.

44. (b) $6 + 7 = 13$, doesnot form a triangle

$\because\ 8^2 > 5^2 + 6^2$,

$\therefore$ 5, 6, 8 are sides of an obtuse angled triangle

5. (d) Let third side $= x$ cm

If this be the greatest side then $8 + 12 > x \Rightarrow x < 20$

if this be the smallest side then $x + 8 > 12 \Rightarrow x > 4$

$\therefore \quad 4 < x < 20$

(c) if x be greatest side then $\quad x^2 < 8^2 + 5^2$

$\Rightarrow x^2 < 289$

$\Rightarrow x < 17$

if x be smallest side then $15^2 < 8^2 + x^2$

$$\Rightarrow x^2 > 225 - 64$$
$$\Rightarrow x^2 > 161$$
$$\Rightarrow x > \sqrt{161}$$
$$\therefore \sqrt{161} < x < 17$$

47. (d) triangle will be formed if $15 - 8 < x < 15 + 8$

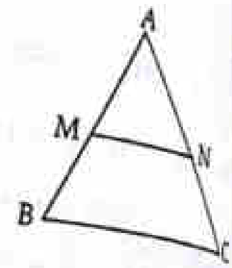
or, $7 < x < 23$

As in above question when $\sqrt{161} < x < 17$ acute angled triangle will be formed

when $x = \sqrt{161}$ then $x^2 + 8^2 = 15^2$ and $x = 17$ then $8^2 + 5^2 = 17^2$, in these cases right angle triangled are formed. In remaining cases obtuse angle triangel are formed, so when $7 < x < \sqrt{161}$ and $17 < x < 23$ obtuse angled triangle are formed

48. (c) here area $\Delta AMN = \frac{1}{2}$ (area ΔABC)

or, $\dfrac{\text{area of } \Delta AMN}{\text{area of } \Delta ABC} = \dfrac{1}{2}$



or, $\left(\dfrac{AM}{AB}\right)^2 = \dfrac{1}{2}$

or, $\sqrt{2}\ AM = AB$

or, $\sqrt{2}\ AM = (AM + MB)$

or, $(\sqrt{2} - 1)\ AM = MB$

or, $\dfrac{AM}{BM} = \dfrac{1}{\sqrt{2} - 1}$

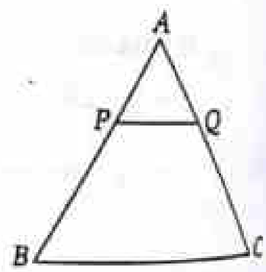
$= \dfrac{1}{\sqrt{2} - 1} \times \dfrac{\sqrt{2} + 1}{\sqrt{2} + 1}$

$= \sqrt{2} + 1$

49. (b) $\dfrac{AP}{PB} = \dfrac{AQ}{QC} = \dfrac{1}{3}$

$\Rightarrow \Delta APQ \sim \Delta ABC$

$\therefore \dfrac{AP}{AB} = \dfrac{1}{1+3} = \dfrac{1}{4}$

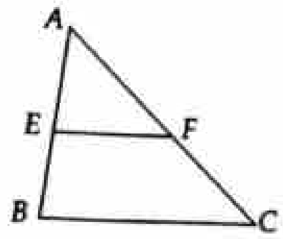


$\therefore \dfrac{\text{area of } \Delta APQ}{\text{area of } \Delta ABC} = \left(\dfrac{1}{4}\right)^2 = \dfrac{1}{16}$

$\dfrac{\text{area of } \Delta APQ}{\text{area of } PBCQ} = \dfrac{1}{16 - 1} = \dfrac{1}{15}$

50. (b) Use similar triangles

51. (a) $\Delta AEF \sim \Delta ABC$



$\therefore \left(\frac{AE}{AB}\right)^2 = \frac{\text{area of } \Delta AEF}{\text{area of } \Delta ABC}$

$\left(\frac{AE}{AB}\right)^2 = \frac{1}{1+3}$

$\Rightarrow \frac{AE}{AB} = \frac{1}{2}$

$\Rightarrow$ E is midpoint of AB.

$\therefore$ $EB : AB = 1 : 2$

Exercise—5B

1. Two sides of atriangle are of length 4 cm and 10 cm. If the length of the third side is 'a' cm, then

   (a) $a < 6$ (b) $6 < a < 14$ (c) $a > 5$ (d) $6 \le a \le 12$

   [SSC Tier-I 2012]

2. Consider ΔABD such that $\angle ADB = 20°$ and C is a point on BD such that $AB = AC$ and $CD = CA$. Then the measure of $\angle ABC$ is.

   (a) 40° (b) 45° (c) 60° (d) 30°

   [SSC Tier-I 2012]

3. ABC is a right-angled triangle. AD is perpendicular to the hypotenuse BC. If $AC = 2AB$, then the value of BD is.

   (a) $\frac{BC}{2}$ (b) $\frac{BC}{3}$ (c) $\frac{BC}{4}$ (d) $\frac{BC}{5}$

   [SSC Tier-I 2012]

4. In ΔABC, AD is drawn perpendicular from A on BC. If $AD^2 = BD \cdot CD$, then $\angle BAC$ is.

   (a) 30° (b) 45° (c) 60° (d) 90°

5. In a triangle ABC, $AB = AC$. BA is produced to D in such a manner that $AC = AD$. The circular measure of $\angle BCD$ is.

   (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{2}$

   [SSC Tier-I 2012]

6. The perimeter of an isosceles right-angled triangle is $2p$ unit. The area of the same triangle is .

   (a) $(3-\sqrt{2})p^2)$ sq. unit (b) $(2-\sqrt{2})p^2$ sq. unit

   (c) $(3-2\sqrt{2})p^2$ sq. unit (d) $(3+2\sqrt{2})p^2$ sq. unit

   [SSC Tier-I 2012]

7. The perimeters of two similar triangles ΔABC and ΔPQR are 36 cm and 24 cm respectively. If $PQ = 10$ cm, then AB is.
(a) 10 cm (b) 15 cm (c) 20 cm (d) 25 cm
[SSC Tier-I 2012]

8. ΔABC and ΔDEF are similar and their areas be respectively 64 cm^2 and 121 cm^2. If $EF = 15.4$ cm, BC is.
(a) 11.2 cm (b) 12.1 cm (c) 11.0 cm (d) 12.3 cm
[SSC Tier-I 2012]

Answers—5B

1. (b) 2. (a) 3. (d) 4. (d) 5. (d) 6. (c) 7. (b) 8. (a)

Explanation

1. **(b)** If a is smallest side then $a + 4 > 10 \Rightarrow a > 6$

If a is biggest side, then $4 + 10 > a \Rightarrow a < 14$

$\therefore$ $6 < a < 14$

2. **(a)** Let $\angle ABC = \theta$ then $\angle ACB = \theta$ $(\because AB = AC)$

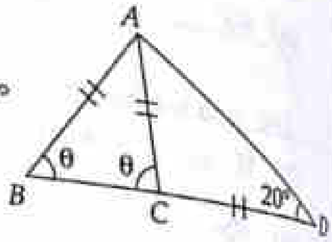
$\therefore$ $AC = CD$

$\therefore$ $\angle CAD = \angle ADC = 20°$

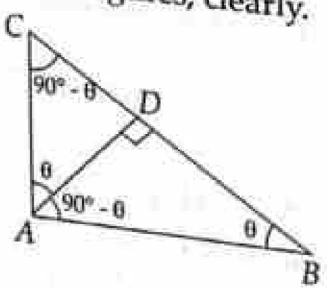
In ΔACD, $\angle ACD = 180° - 20° - 20° = 140°$

On straight line BCD, $\theta + 140° = 180°$

$\Rightarrow$ $\theta = 40°$



3. **(d)** See, the angles in given figures, clearly.



$\Delta BAC \sim \Delta BDA$

$\therefore$ $\frac{BA}{BD} = \frac{BC}{BA}$ ...(i)

But $BC^2 = AC^2 + AB^2$

$= 4AB^2 + AB^2 = 5AB^2$

$\therefore$ From (i) $BC = \frac{BA^2}{BD}$ ...(ii)

or, $BC = \frac{BC^2}{5BD}$ (from (ii)) $AB^2 = \frac{BC^2}{5}$)

or, $BD = \frac{BC}{5}$

4. (d) From $AD^2 = BD \cdot CD$

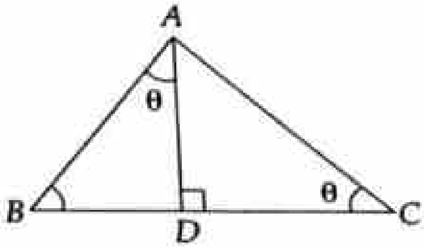
$\frac{AD}{BD} = \frac{CD}{AD}$ and $\angle ADB = \angle ADC = 90°$

$\therefore$ $\Delta ACD \sim \Delta BAD$

Let $\angle ACD = \angle BAD = \theta$

In ΔACD, $\angle DAC = 90° - \theta$

$\therefore$ $\angle BAC = \angle ABD + \angle DAC = \theta + 90° - \theta = 90°$



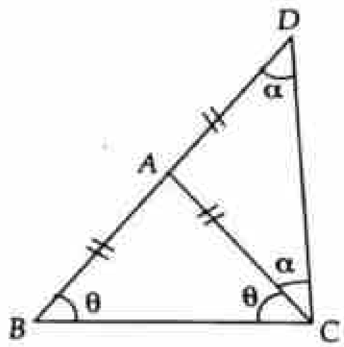
5. (d) If $\angle ABC = \theta$, then $\angle ACB = \theta$

If $\angle ACD = \alpha$ then $\angle ADC = \alpha$

In ΔBCD, $\theta + \theta + \alpha + \alpha = 180°$

or, $2(\theta + \alpha) = 180°$

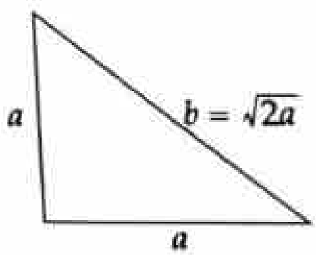
or, $\theta + \alpha = 90° = \angle BCD$



6. (c) Given, $a + a + b = 2p$

$$2a + \sqrt{2}a = 2p$$

or, $a = \frac{2p}{2+\sqrt{2}} = \frac{\sqrt{2}p}{\sqrt{2}+1}$



$\therefore$ Appropriate Area $= \frac{1}{2}a^2 = \frac{1}{2}\left(\frac{\sqrt{2}p}{\sqrt{2}+1}\right)^2$

$$= \frac{p^2}{(\sqrt{2}+1)^2}$$

$$= \frac{p^2(\sqrt{2}-1)^2}{\{(\sqrt{2}+1)(\sqrt{2}-1)\}^2}$$

$$= p^2\left((\sqrt{2}-1)\right)^2$$

$$= p^2(2+1-2\sqrt{2})$$

$$= (3-2\sqrt{2})p^2$$

7. **(b)** $\therefore \quad \frac{\quad}{PQ} = \frac{\quad}{24}$

$\Rightarrow \quad \frac{AB}{10} = \frac{3}{2}$

$\Rightarrow \quad AB = \frac{3}{2} \times 10 = 15 \text{ cm}$

8. **(a)** In similar triangle

(ratio of sides)^2 = ratio of Areas

$\therefore \quad \left(\frac{BC}{EF}\right)^2 = \left(\frac{area\,\Delta ABC}{area\,\Delta DEF}\right)$

$\Rightarrow \quad \frac{BC}{EF} = \sqrt{\frac{64}{121}} = \frac{8}{11}$

$\therefore \quad BC = \frac{8}{11} \times EF$

$= \frac{8}{11} \times 15{\cdot}4 = 8 \times 1.4 = 11.2 \text{ cm}$

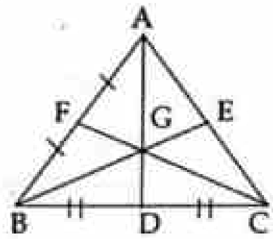
★★★

06

Centre of Triangle

1. Centre of a triangle : What do you mean by centroid, Incentre, Circumcentre and orthocentre of a triangle ?

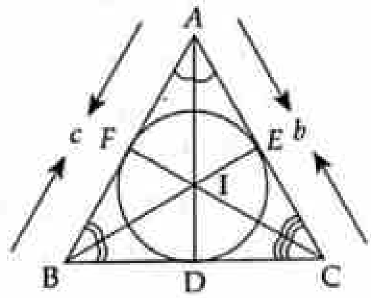
1.1. Centroid : In a triangle line joining the midpoint of a side to the opposite vertex is called a median. The three medians of a triangle meet at a point and the point is called centroid (G) of the triangle. In the adjacent figure points D, E and F are respectively mid point of sides BC, CA and AB. We must learn that



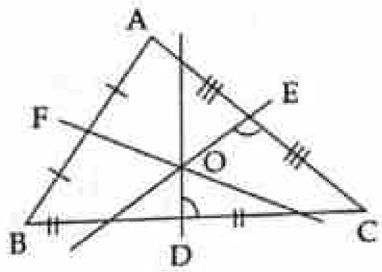
(a) $\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$ i.e., Centroid divides median in the ratio \2 : 1.

(b) area of ΔAGC = area of ΔBGC = area of ΔAGB i.e., lines joining centroid to the vertices of triangle divide the triangle into three equal areas.

1.2. Incentre : Lines bisecting internal angles (in two equal part) of a triangle are called internal bisector of angles. The internal bisectors of a triangle meet at a point and the point is called incentre of the triangle. In the figure, it is important to note that $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$

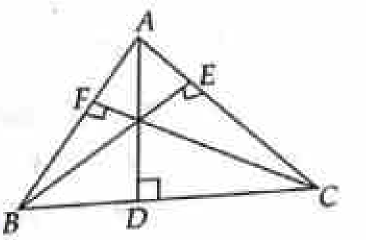


1.3. Circumcentre : Perpendiculars drawn on mid points of sides of a triangle (i.e. perpendicular bisector) meet at a point and the point is called circumcentre of the triangle. Geometrically, circumcentre is equidistant from vertices of a triangle; thus assuming this as centre we can draw a circle passing through all the three vertices of the triangle so,

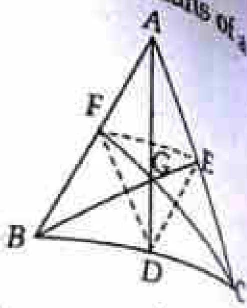


$$AO = BO = CO.$$

1.4. Orthocentre : In a triangle, perpendicular drawn from vertices to the opposite sides (called altitudes) meet at a point and the point is called orthocentre.



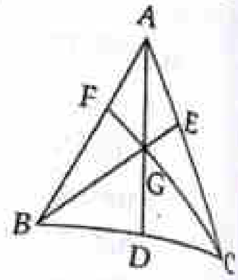
2. Important properties of centroid : If AD, BE and CF are medians of a triangle ABC and G be its centroid then.



2.1. $\frac{AG}{GD} = \frac{BG}{GE} = \frac{GC}{GF} = \frac{2}{1}$

or, $\frac{AG}{AD} = \frac{2}{3}$,

$\frac{GD}{AD} = \frac{1}{3}$ etc.



2.2. A medians divides triangle into two equal areas

i.e. ar (ΔABD) = ar $\Delta (ACD) = \frac{1}{2}$ ar (ΔABC)

ar $\Delta (BEC)$ = ar $\Delta (BEA) = \frac{1}{2}$ ar (ΔABC) etc.

2.3. Lines joining centroid to vertices of a triangle divide the triangle into three equal areas.

i.e. ar (ΔAGB) = ar (ΔBGC) = ar $(\Delta CGA) = \frac{1}{3}$ ar ΔABC

2.4. Since GD is the median of triangle BGC.

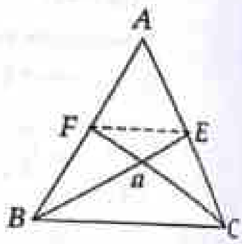
$\therefore$ ar (ΔBGD) = ar $(\Delta CGD) = \frac{1}{2}$ ar $(\Delta BGC) = \frac{1}{2} \cdot \frac{1}{3}$ ar (ABC)

$\therefore$ ar $(\Delta BGD) = \frac{1}{6} \times$ ar (ΔABC) etc.

2.5. G is also the centroid of ΔDEF.

2.6. Since E and F are respectively mid points of AB and AC

therefore, $EF \parallel BC$ and $EF = \frac{1}{2} BC$



2.7. If E and F, respectively mid point of AC and AB then

$\angle AEF = \angle ACB$ $\quad$ $(\because EF \parallel BC)$

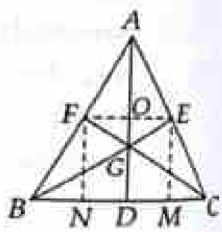
$\angle AFE = \angle ABC$ $\quad$ $(\because EF \parallel BC)$

$\therefore$ $\Delta AFE \sim \Delta ABC$.

2.8. If G be centroid and O is the point of interection of AG and EF then

$\Delta AOE \sim \Delta ADC$ $\quad$ $(\because EF \parallel DC \Rightarrow \angle AEO = \angle ACD$

$\angle AOE = \angle ADC)$



$\therefore$ $\frac{AO}{AD} = \frac{AE}{AC} = \frac{1}{2}$

$\Rightarrow$ $2AO = AD \Rightarrow AO = OD$ $\quad$ $(\because$ E is mid point)

i.e., point O is midpoint of AD.

2.9. In the above figure if $FN \perp r\ BC$ and $EM \perp r\ BC$ then $FN = EM$ [Also see solved example 16]

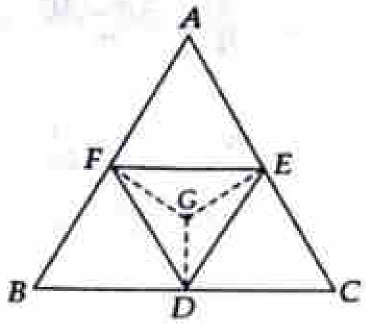
$\therefore$ area of $\Delta BFC = \frac{1}{2} \times BC \times FN$; area of $\Delta BEC = \frac{1}{2} \times BC \times EM$

$\because$ $FN = EM$

$\therefore$ area of ΔBFC = area of ΔBEC

2.10. Since D, E, F are mid point of sides then, $\Delta BDF \cong \Delta EFD \cong \Delta FEA \cong \Delta DCE$ and ar (ΔBDF) = ar (ΔCDE) = ar (ΔAEF)

$$= \text{ar } (\Delta FDE) = \frac{1}{4} (\text{ar } \Delta ABC)$$



$\because$ G, is also centroid of ΔDEF

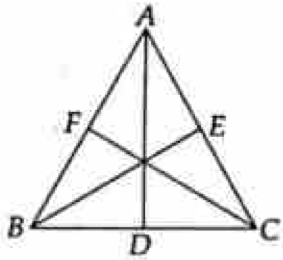
$\therefore$ ar (ΔDGE) = ar (ΔEGF) = ar Δ (DGF)

$$= \frac{1}{4} \times \frac{1}{3} \text{ ar } (\Delta ABC)$$

$$= \frac{1}{12} \text{ ar } (\Delta ABC)$$

3. Relation among sides and medians of a triangle : Suppose ABC is a triangle whose medians are AD, BE and CF.

If $AB = c$, $BC = a$ and $AC = b$, then



3.1. $AD = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

3.2. $BE = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$

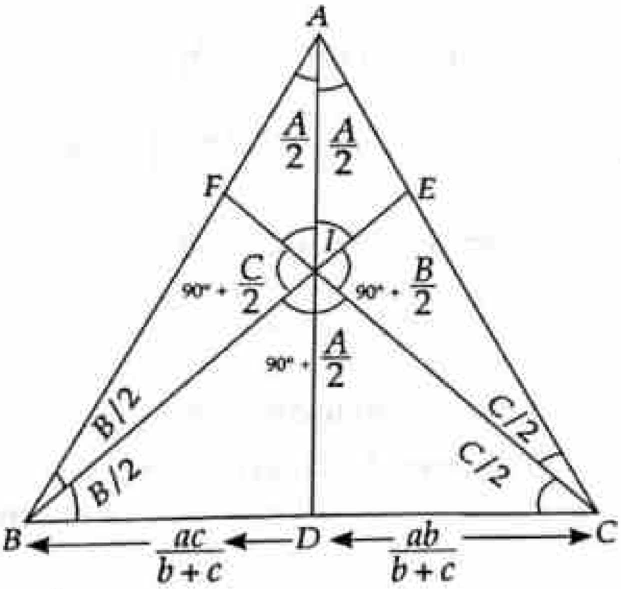
3.3. $CF = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$

3.4. $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$

or, $AB^2 + BC^2 + CA^2 = \frac{4}{3}(AD^2 + BE^2 + CF^2)$

3.5. area of $ABC = \frac{4}{3}$ × (area formed by taking AD, BE, CF as sides of a triangle)

4. Important properties of Incentre : In the given figure AD, BE and CF are respectively bisectors of $\angle A$, $\angle B$ and $\angle C$. These internal bisectors meet at I which is incentre of the triangle.



Clearly,

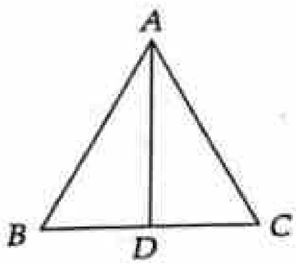
$$\angle BAD = \angle CAD = \frac{\angle A}{2}$$

$$\angle ABE = \angle CBE = \frac{\angle B}{2}$$

and $$\angle BCF = \angle ACF = \frac{\angle C}{2}$$

4.1. If AD is bisector of $\angle A$ then $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$

i.e., angle bisector AD divides side BC in the ratio $AB : AC$.



Similarly, $\frac{CE}{EA} = \frac{BC}{BA}, \frac{AF}{FB} = \frac{CA}{CB}$

4.2. $\frac{AI}{ID} = \frac{AB + AC}{BC} = \frac{c+b}{a}$ (How to recall : AB and AC are connected with AI while ID stand on BC)

Similarly, $\frac{BI}{IE} = \frac{BA + BC}{AC}, \frac{CI}{IF} = \frac{CA + CB}{AB}$

4.3. $BD = \frac{ac}{b+c}, CD = \frac{ab}{b+c}$

$CE = \frac{bc}{c+a}, EA = \frac{ba}{c+a}$

$AF = \frac{cb}{a+b}, BF = \frac{ca}{a+b}$

Explanation : Since $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$

let $BC = ck$ and $DC = bk$

$\therefore$ $BD + DC = ck + bk$

or, $BC = (c + b)k$

or, $a = (b + c)k$

or, $k = \frac{a}{b+c}$

$\therefore$ $BD = ck = \frac{ac}{b+c}, CD = bk = \frac{ba}{b+c}$

(How to recall : Since $BD : CD = c : b$, thus multiplying by $\frac{a}{b+c}$

We get, $BD = \frac{ac}{b+c}, CD = \frac{ab}{b+c}$ etc)

4.4. $\angle BIC = 180° - \frac{B}{2} - \frac{C}{2}$

$= 180° - \left(\frac{B+C}{2}\right) = 180° - \left(\frac{180° - A}{2}\right) = 90° + \frac{\angle A}{2}$

Similarly, $\angle AIC = 90° + \frac{\angle B}{2}$

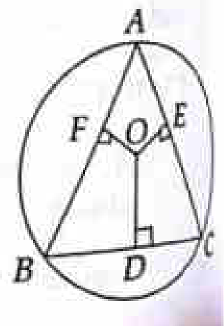
$\angle AIB = 90° + \frac{\angle C}{2}$ [SSC Tier-I 2014]

4.5. Radius of incircle of ΔABC i.e., inradius $\boxed{r = \frac{\Delta}{s}}$

Where, Δ = area of triangle,

and s = semiperimeter of the triangle.

5. Circumcentre : In the given figure O is the circumentre of ΔABC. Hence,

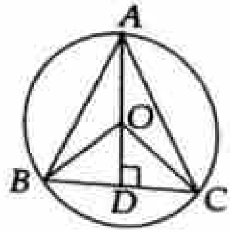


5.1. OD, OE and OF are respectively perpendicular bisector of sides BC, AC and AB i.e., $BD = DC$ and $OD \perp r$ BC etc.

5.2. Circumcentre O is equidistant from vertices A, B, C of the triangle i.e., $OA = OB = OC = R$

5.3. R is called circumradius and $\boxed{R = \frac{abc}{4\Delta}}$

Where, Δ is area of triangle.

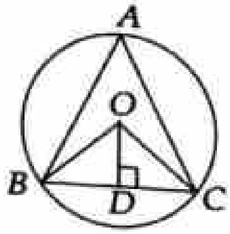


5.4. Angle subtends by arc of a circle at centre is double the angle subtends by it at circumference.

i.e., $\angle BOC = 2\angle A$

$\angle COA = 2\angle B$

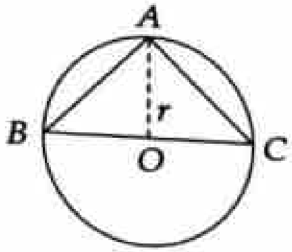
and $\angle AOB = 2\angle C$



5.5. $\Delta OBD \cong \Delta OCD$

$\therefore \quad \angle BOD = \angle COD = \angle A$

and $\angle OBC = \angle OCB = 90° - A$

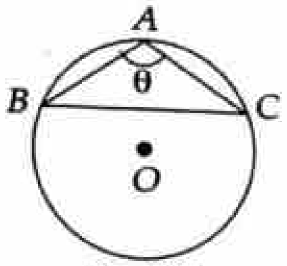


5.6. If ABC is a right angled triangle (with $\angle A = 90°$) then circumcentre O is the mid point of hypotenuse BC.

Since, $OB = OC = OA = r$,

Hence in a right angled triangle, mid point of hypotenuse is equidistant from the vertices of the triangle.

5.7. If ABC is an obtused angle triangle its circumcentre lies out side the triangle ABC.



$\theta > 90°$

6. Important properties of orthocentre

In the given figure $AD \perp r\ BC$, $BE \perp r\ AC$ and $CF \perp r\ AB$. Altitudes AD, BE and CF meet at P which is orthocentre of the ΔABC

6.1. In ΔABD

$$\angle BAD = 180° - 90° - \angle B \qquad (\because \angle ADB = 90°)$$

$$= 90° - \angle B$$

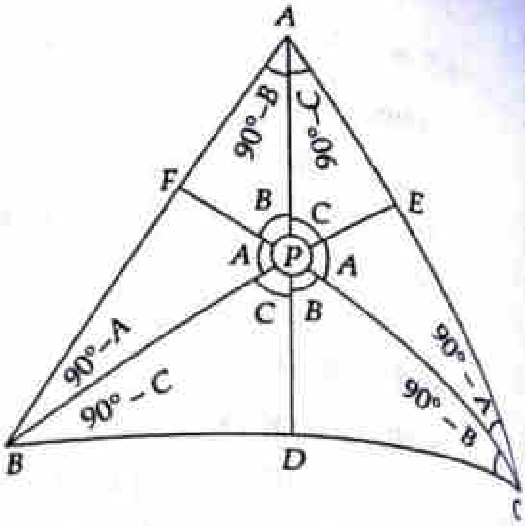
Similarly in ΔADC, $\angle CAD = 90° - \angle C$ etc

See the remaining angles in the figure.

6.2. Angle around orthocentre P :

In ΔBPD,

$$\angle BPD + \angle PBD = 90°$$



or, $\angle BPD + 90° - \angle C = 90°$

or, $\angle BPD = \angle C$

See the remaining angles in the figure,

6.3. $\angle BPC = \angle B + \angle C$

$= 180° - \angle A$

$\angle CPA = \angle C + \angle A = 180° - \angle B$

$\angle APB = \angle A + \angle B = 180° - \angle C$

6.4. $BD = \dfrac{AB^2 + BC^2 - AC^2}{2BC}$

$= \dfrac{c^2 + a^2 - b^2}{2a}$

$CD = \dfrac{AC^2 + BC^2 - AB^2}{2BC}$

$= \dfrac{b^2 + a^2 - c^2}{2a}$

$\therefore\ BD : DC = c^2 + a^2 - b^2 = b^2 + a^2 - c^2$

6.5. Pair of similar triangles are

$\Delta PEC \sim \Delta PFB$

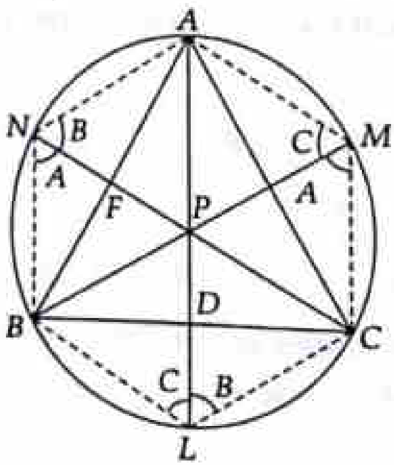
$\Delta PDC \sim \Delta PFA$

$\Delta PEA \sim \Delta PDB$

Write the ratio of sides of triangle yourself. Questions may be asked on these ratio.

6.6. It must be noted that ΔPDB, ΔPDC, ΔPEC, ... etc. are right angled triangles.

6.7. P is orthocentre of ΔABC. Draw a circumcircle to the triangle ABC. Since angles in the same segment (or the same base or in the same arc) of a circle are equal,



$\therefore$ On base BL, $\angle BCL = \angle BAL = 90° - \angle B$

On base CL, $\angle CBL = \angle CAL = 90° - \angle C$

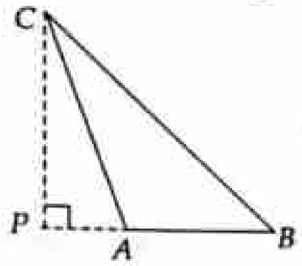
On base BC, $\angle BMC = \angle A$ etc.

See the remaining angles in the figure.

8. The orthocentre of a right angles triangle is that point where triangle forms the right angle.

9. The orthocentre of an obtuse angled triangle lies out side the triangle.

   In figure, orthocentre P lies outside the triangle.



Mixed properties of centres of a triangle.

7.1. In an equilateral triangle all the four centres are coincident i.e., centroid, incentre, circumcentre and orthocentre of an equilateral triangle lie at the same point.

7.2. Centroid (G), orthocentre (P) and circumcentre (O) of a triangle are always collinear (i.e., lie in a straight line) and $PG : GO = 2 : 1$.

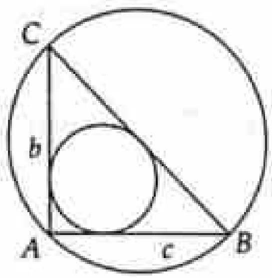
7.3. The orthocentre of a right angled triangle lies at the right angled vertex while its circumcentre is mid point of hypotenuse.

7.4. Circumcentre and orthocentre of an obtuse angled triangle always lie outside the triangle.

7.5. The sum of diameters of circumcircle and incircle of a right angled triangle is equal to the sum of its perpendicular sides.

In the given figure ABC is a right angled triangle with $\angle A = 90°$. If radius of circumcircle and incircle of the triangle be respectively R and r then $2(R + r) = b + c$

(See solved example–21)



7.6. The distance between incentre and circumcentre of a triangle is $\sqrt{R^2 - 2rR}$ where R is circumradius and r is inradius.

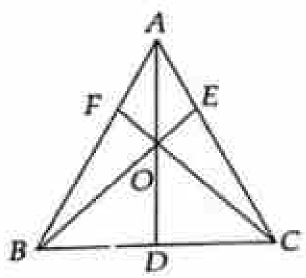
7.7. In an equilateral triangle, length or radius of the circumcircle is equal to twice the radius of its incircle i.e., if ΔABC is equilateral then $R = 2r$.

7.8. Ceva Theorem : If O is any point inside the triangle ABC and AO, BO, CO meet sides BC, CA, AB respectively at point D, E, F then

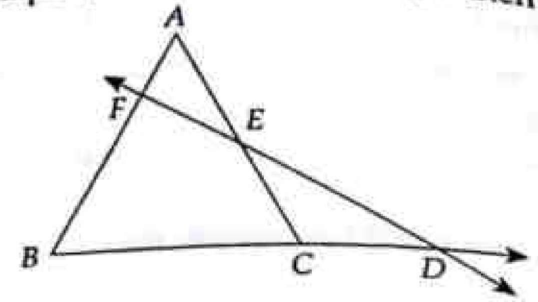
$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$$

Since Ceva Theorem is true for any point inside the triangle, it is therefore also true for centroid, incentre, orthocentre and circumcentre of the triangle.

Converse of Ceva Theorem is also true.



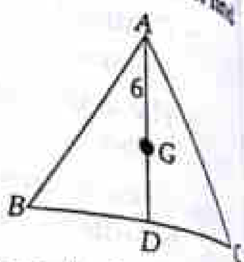
7.9. **Menelaus Theorem :** If a transverse cuts the sides BC, CA and AB (or its produced part) of a triangle at D, E, F then $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = -1$



Converse of the theorem is also true.

Solved Example

1. If distance of centroid of triangle ABC from vertex A is 6 cm them find the length of median through point, A.



Solution : Since, $AG : GD = 2 : 1$

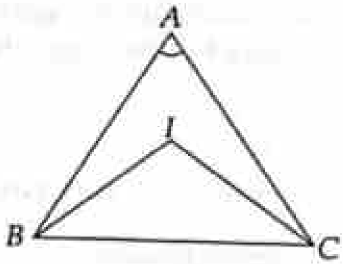
$\therefore \quad \frac{6}{GD} = \frac{2}{1}$

$\Rightarrow \quad GD = 3$

$\therefore \quad AD = AG + GD = 6 + 3 = 9$

2. If I be the incentre of triangle ABC and $\angle A = 70°$ then find the value of $\angle BIC$.

Solution : Recall that $\angle BIC = 90° + \frac{A}{2}$ (See Article 4 (iv))



Hence, $\angle BIC = 90° + \frac{70°}{2} = 125°$

3. In a triangle ABC if $\angle A = \theta$ and perpendiculars drawn from vertices B and C to respective opposite sides meet in P then find the value of $\angle BPC$ in terms of θ.

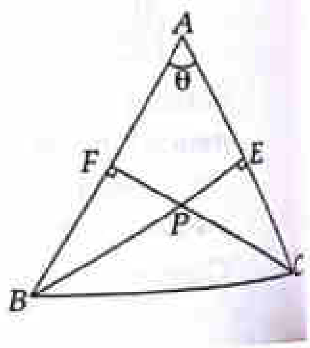
Solution : See the figure, In Quadrilateral $AEPF$

$$\theta + 90° + 90° + \angle EPF$$

$$= 360° \text{ or, } \angle EPF = 180° - \theta$$

$\therefore \quad \angle BPC = \angle EPF = 180° - \theta$

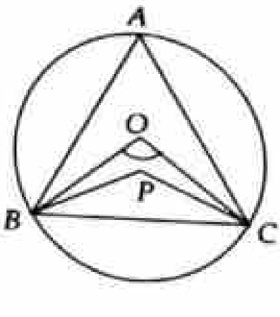
(Vertically opposite angle)



Shortcut : Learn that $\angle BPC = \angle B + \angle C = \pi - A$

O is the circumcentre of a triangle ABC whose $\angle A = 50°$. If bisector of $\angle OBC$ and $\angle OCB$ intersect at P then what is the measure of $\angle BPC$?

Solution : Since angle subtened at the centre of the circle is double the angle subtened at circumference



$\therefore \angle BOC = 50° \times 2 = 100°$

$\because OB = OC$

$\therefore \angle OBC = \angle OCB = \dfrac{180° - 100°}{2} = 40°$

In ΔBPC, $\angle BPC + \angle PBC + \angle PCB = 180°$

or, $\angle BPC + \frac{1}{2} \times 40° + \frac{1}{2} \times 40° = 180°$

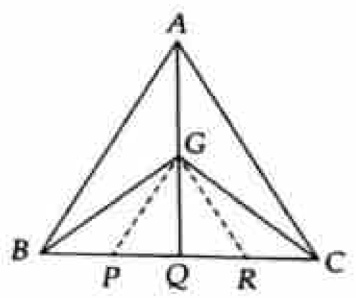
$\therefore \angle BPC = 180° - 20° - 20° = 140°$

Shortcut : $\angle BPC = 90° + \dfrac{\angle BOC}{2} = 90° + A = 90° + \dfrac{100°}{2} = 140°$

Three points P, Q, R lie on the side BC of triangle ABC such that $BP = PQ = QR = RC$. If G be centroid of ΔABCthen what is ratio of areas of ΔPGR and ΔABC.

Solution : Clearly Q is mid point of side BC i.e., AQ is median of ΔABC.

We know that



Area of $\Delta BGC = \frac{1}{3} \times$ area of ΔABC

But height of ΔBGC and ΔPGR are equal.

Let this height be h.

$\therefore$ area $\Delta BGC = \frac{1}{2} \times BC \times h$

and area of $\Delta PGR = \frac{1}{2} \times PR \times h$

Now, $\dfrac{\text{area of } \Delta PGR}{\text{area of } \Delta ABC} = \dfrac{\text{area of } \Delta PGR}{3 \times \text{area of } \Delta BGC}$

$$= \frac{\frac{1}{2} \times PR \times h}{3 \times \frac{1}{2} \times BC \times h} = \frac{PR}{3BC}$$

$$= \frac{PR}{3 \cdot 2PR} = \frac{1}{6} \qquad (\because PR = \tfrac{1}{2} BC)$$

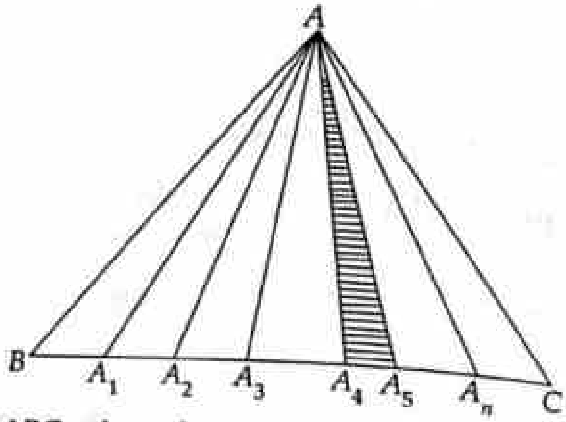
A triangle DEF is formed by joining mid points of sides of triangle ABC. Again mid points of sides of triangle DEF are joined together to form a new triangle PQR. If sides of triangle ABC are respectively 4, 5 and 6 cm then what is the distance between centroid of ΔPQR and ΔDEF.

Solution : Centroid of a given triangle and a triangle formed by mid points of the given triangle are coincident (i.e., lie at the same point);

So required distance = 0

7. n equidistant points $A_1, A_2, A_3 \ldots\ldots A_n$ are taken on base BC of ΔABC ... $A_1B = A_1A_2 = A_nC$ and area of $\Delta A A_4 A_5$ is k cm^2, then what is the area of ΔABC.

Solution : See the figure, a total of $(n+1)$ triangles will be formed when base are same and height are equal.



$\therefore$ Area of $\Delta ABC = (n+1)$ area of ΔAA_4A_5

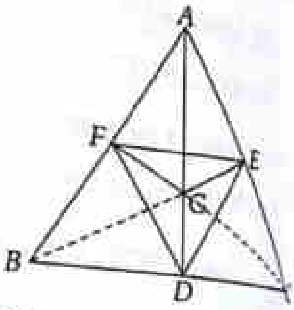
$= (n+1)k$ cm^2

8. Points E and F lie respectively on side AC and AB of a triangle ABC such that $EF \parallel BC$ and $2EF = BC$. If G be the centroid of ΔABC then find the area of triangle formed by joning mid points of sides of triangle EFG with respect to area of ΔABC.

Solution : Since $EF \parallel BC$ and $EF = \frac{1}{2} BC$;

therefore E and F are respectively mid points of sides AC and AB.

We know that line joining the mid points of sides of a triangle divides the triangle in four equal areas.



$\therefore$ Area of $\Delta DEF = \frac{1}{4} \times$ area of ΔABC *(here D is mid point of side BC)*

Now, G is also the centroid of triangle DEF and lines joining centroid and vertices of a triangle divides the triangle into three equal areas.

Hence, area of $\Delta EGF = \frac{1}{3} \times$ area of $\Delta DEF = \frac{1}{3} \times \frac{1}{4} \times$ area of ΔABC

$\therefore$ Area of formed by mid points of sides of $\Delta EGF = \frac{1}{4} \times$ area of ΔEGF

$= \frac{1}{4} \times \frac{1}{3} \times \frac{1}{4} \times$ area of $\Delta ABC = \frac{1}{48} \times$ area of ΔABC

9. The angles of a triangle are in the ratio $3:4:5$. If I be the incentre of ΔABC then find the measure of $\angle ADC$ and $\angle DIC$ where AD, is the bisector of $\angle A$.

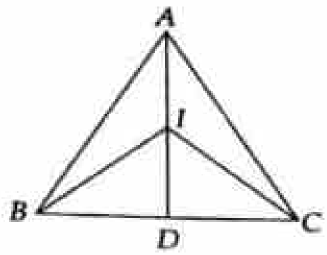
Solution : Let $\angle A = 3k$, $\angle B = 4k$ and $\angle C = 5k$

then, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 3k + 4k + 5k = 180°$

or $\quad k = \frac{180°}{12} = 15°$

$\therefore \quad \angle A = 45°, \angle B = 60°, \angle C = 75°$



In triangle ACD, $\angle ADC = 180° - \frac{\angle A}{2} - \angle C$

$= 180° - \frac{45°}{2} - 75°$

$= 105° - 22\frac{1}{2}° = 82\frac{1}{2}°$

In triangle CID, $\angle CID = 180° - \angle ADC - \frac{\angle C}{2} = 180° - 82\frac{1}{2}° - \frac{75°}{2}$

$= 180° - 82\frac{1}{2}° - 37\frac{1}{2}° = 180° - 120° = 60°$

10. In triangle ABC, $AB = 6$, $AC = 7$ and $BC = 8$. If AD is bisector of $\angle A$ and I is the incentre of trinagle ABC then find the length of BD and CD. Also find the value of $AI : ID$.

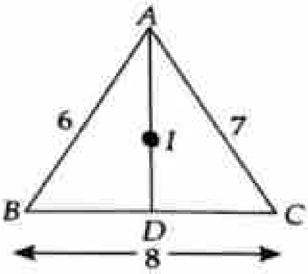
Solution : We know that if AD is bisector of $\angle A$ then $\frac{BD}{DC} = \frac{AB}{AC} = \frac{6}{7}$

Let $\quad BD = 6k, DC = 7k$

then, $BD + DC = BC$

$\Rightarrow \quad 6k + 7k = 8$

$\Rightarrow \quad k = \frac{8}{13}$



$BD = 6k = 6 \times \frac{8}{13} = \frac{48}{13}$

$CD = 8k = \frac{7 \times 8}{13} = \frac{56}{13}$

Shortcut : $BD = \frac{ac}{b+c} = \frac{8 \times 7}{6+7} = \frac{56}{13}$

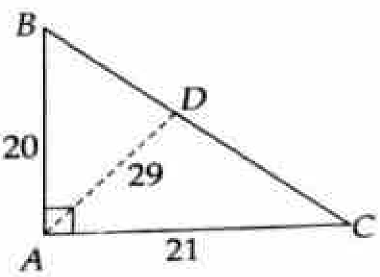
Now, $\frac{AI}{ID} = \frac{AB + AC}{BC} = \frac{6+7}{8} = \frac{13}{8}$

11. In a triangle ABC, if $AB = 20$ cm, $AC = 21$ cm and $BC = 29$ cm, then find the distance between vertex A and mid point of BC.

Solution : Since $20^2 + 21^2 = 400 + 441 = 841 = 29^2$

$\therefore$ ΔABC is a right angled triangle, whose hypotenuse is BC.

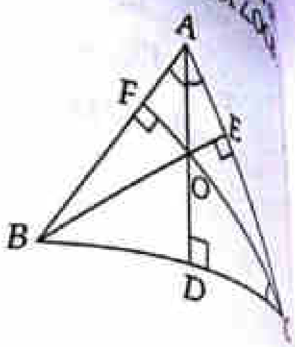
Since mid point of hypotenuse of a right angled triangle is equidistant from each vertex



$\therefore \quad AD = BD = DC = \frac{29}{2}$ cm

or, $AD = 14.5$ cm

12. In triangle ABC, $6\angle A = 4\angle B = 3\angle C$. If AD, BE and CF are altitudes of triangle and O is its point of intersection then find the measure of $\angle COD$, $\angle BOD$ and $\angle BOC$.



Solution : Let $6A = 4B = 3C = K$

then, $\angle A = \frac{K}{6}, \angle B = \frac{K}{4}, \angle C = \frac{K}{3}$

$\therefore \quad \angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \quad \frac{K}{6} + \frac{K}{4} + \frac{K}{3} = 180^\circ$

$\Rightarrow \quad \frac{2K + 3K + 4K}{12} = 180^\circ \Rightarrow K = \frac{180^\circ \times 12}{9} = 240^\circ$

$\therefore \quad A = 40^\circ, B = 60^\circ$ and $C = 80^\circ$

In right angled ΔBCF, $90^\circ + \angle BCF + \angle B = 180^\circ$

or, $90^\circ + \angle BCF + 60^\circ = 180^\circ$

$\therefore \angle BCF = 180^\circ - 90^\circ - 60^\circ = 30^\circ$

In right angled ΔOCD, $\angle ODC + \angle OCD + \angle COD = 180^\circ$

or, $90^\circ + 30^\circ + \angle COD = 180^\circ$

or, $\angle COD = 60^\circ$

In right angled ΔBEC, $90^\circ + \angle C + \angle EBC = 180^\circ$

or, $90^\circ + 80^\circ + \angle EBC = 180^\circ$

or, $\angle EBC = 10^\circ$

In right angled ΔBOD, $90^\circ + \angle OBD + \angle BOD = 180^\circ$

or, $90^\circ + 10^\circ + \angle BOD = 180^\circ$

or, $\angle BOD = 80^\circ$

$\therefore \quad \angle BOC = \angle COD + \angle BOD = 60^\circ + 80^\circ = 140^\circ$

Shortcut : See the figure of orthcentre in theory part. All the angles can be found directly.

13. If O be the orthocentre of ABC, $OF \perp r\ AB$ and $OE \perp r\ AC$. If $OE = 2$ cm and $BE = 5$ cm then find the value of $OF \times OC$.

Solution : In ΔOBF and ΔOCE,

$\angle OFB = \angle OEC = 90^\circ$ and $\angle BOF = \angle EOC$

$\therefore \quad \Delta OBF \sim \Delta OCE$ *(Vertically opposite angle)*

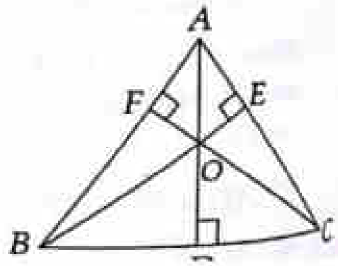
Hence, $\frac{OB}{OC} = \frac{OF}{OE}$ or, $\frac{OB}{OF} = \frac{OC}{OE}$

or, $OB \times OE = OF \times OC$

or, $OF \times OC = OB \times OE$

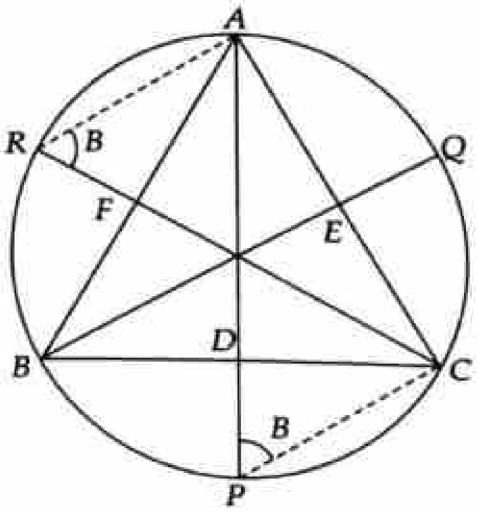
$= (BE - OE)(OE)$

$= (5 - 2) \times 2 = 6 \text{ cm}^2$



14. A circle is drawn circumscribing the ΔABC. If produced part of altitudes AD, BE and CF are bisect on P, Q, R respectively then prove that $\frac{CD}{AF} = \frac{CP}{AR} = \frac{DP}{FR}$

Solution : In ΔCPD, $\angle CDP = 90°$



and $\angle DPC = \angle B$ (angle on the same segment of base AC)

In ΔAFR, $\angle AFR = 90°$, In ΔPDC, $\angle PDC = 90°$

and $\angle ARF = \angle B$ (angle on same segment of base AC)

$\therefore$ $\Delta CDP \sim \Delta AFR \Rightarrow \frac{CD}{AF} = \frac{CP}{AR} = \frac{DP}{FR}$

15. If the distance between centroid and orthocentre of a triangle is 12 cm then find the distance between its orthocentre and circumcentre.

Solution : We know that orthocentre (P), centroid (G) and circumcentre (O) are collinear and $\frac{PG}{GO} = \frac{2}{1}$

According to question $GP = 12.1G$ $(\because GO = \frac{1}{2} \cdot PG)$

$= \frac{3}{2} PG = \frac{3}{2} \times 12 = 18$ cm

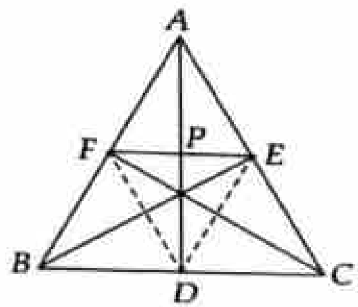
16. If AD, BE and CF are medians of triangle ABC then prove that median AD divides line segment EF.

Solution : Join $E - D$ and $E - F$

$\therefore$ $AFDE$ will be a parallelogram

$(\because ED \parallel AB \Rightarrow ED \parallel AF$ and $FD \parallel AC \Rightarrow FD \parallel AE)$

Hence AD and EF are diagonals of a parallelogram. Its point of intersection P divides diagonal AD of parallelogram which is median AD of ΔABC.



17. If H be orthocentre of ΔABC and mid point of AH, BH, CH are respectively P, Q, R then prove that H is also the orthocentre of ΔPQR and $\Delta PQR \sim \Delta ABC$ (Learn the property)

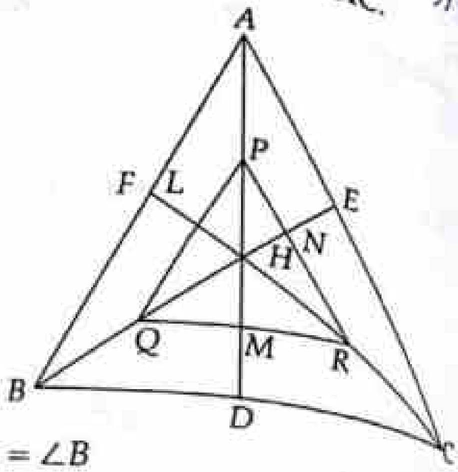
Solution : In ΔHBC, Q is mid point of HB and R is mid point of HC.

Hence $QR \parallel BC$ and $QR = \frac{1}{2} BC$

But $AD \perp r\ BC \Rightarrow PD \perp r\ QR$

Similarly we can prove that $QE \perp r\ PR$ and $RF \perp r\ PQ$

Hence, PD, QE and RF lies on altitudes of ΔPQR. So H, is orthocentre of ΔPQR.



Seccond part : $QR \parallel BC \Rightarrow \angle EQR = \angle EBC$

and $QP \parallel AB \Rightarrow \angle PQE = \angle ABE$

Adding we get $\angle PQR = \angle ABC$ i.e., $\angle Q = \angle B$

Similarly we can prove that $\angle A = \angle P$ and $\angle R = \angle C$

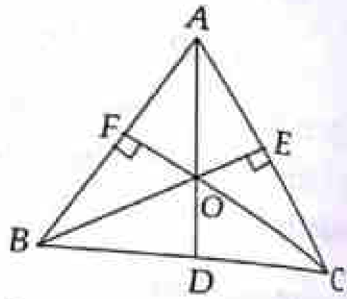
$\therefore \quad \Delta ABC \sim \Delta PQR$

[do your self : $\Delta HQM \sim \Delta HBD$, $\Delta HQR \sim \Delta HBC$ etc]

18. If O be the orthocentre of ΔABC then orthocentre of ΔOBC is A. Justify the statement.

(Learn the property).

Solution : In figure, AD, BE and CF are respectively altitudes on sides BC, CA and AB. O is orthocentre.



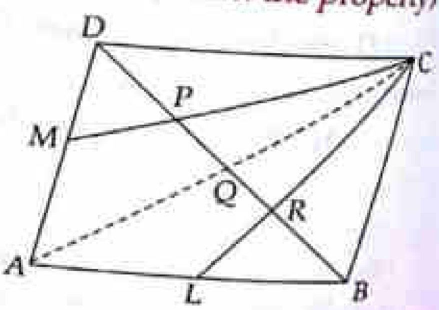
In ΔOBC, CE is perpendicular to produced part of BO.

BF is perpendicular to produced part of CO.

Clearly point of intersection of produced part of BF and CE is A. Thus A is orthocentre of ΔOBC.

19. $ABCD$ is a parallelogram. L and M are respectively mid points of sides AB and AD. Prove that LC and MC divides diagonals BD in three equal parts. (Learn the property)

Solution : See the figure; Let CM, CA and CL intersects diagonals DB respectively at the points P, Q and R.



Q is mid point of AC.

CM and DQ are medians of ΔACD which intersects at P.

Hence P is centroid of ΔACD.

$\therefore \quad \frac{DP}{PQ} = \frac{2}{1}$

Similarly in ΔABC, $\frac{BR}{RQ} = \frac{2}{1}$ ... (ii)

[diagonals of parallelogram bisect each other]

But $DQ = QB$

$\Rightarrow DP + PQ = QR + RB$ ... (iii)

$\Rightarrow 2PQ + PQ = QR + 2QR$ (from (i) &(ii))

$\Rightarrow 3PQ = 3QR \Rightarrow PQ = QR$

Now, from (3), $DP + PQ = QR + RB$

$(\because PQ = QR)$

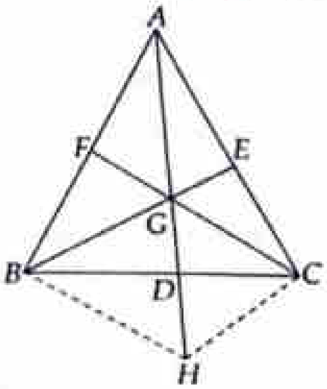
$\Rightarrow DP = RB$ ... (iv)

$\Rightarrow PR = PQ + QR = 2PQ = DP$ ... (v)

From (iv) and (v), $DP = RB = PR$

20. Prove that sum of any two medians of a triangle is greater than the third median. *(Learn the property)*

Solution : In the given figure,



AD, BE and CF are medians of triangle ABC.

G is centroid of ΔABC.

GD is produced to H such that $AG = GH$

Now, see the triangle ΔABH,

Here F is mid point of side AB and G, is the mid point of a side AH.

$\therefore \quad FG \parallel BH \Rightarrow GC \parallel BH$ ... (i)

Now, see the ΔACH.

here E is mid point of side AC and G is mid point of side AH.

$\therefore \quad EG \parallel CH \Rightarrow GB \parallel HC$ ... (ii)

from (i) and (ii), '$BHCG$ is a parallelogram.

$\therefore \quad BG = CH$ and $GC + BG > GH$ [$CH = BG$]

or, $BG + GC > AG$ $(\because AG = GH$ and $GC = BH)$

or, $\frac{3}{2}BG + \frac{3}{2}GC + \frac{3}{2}AG$

or, $BE + CF > AD$ $(\because \frac{3}{2}BG = BE$ etc.)

21. In a right angled ΔABC, $\angle A = 90°$. If $AC = b$, $BC = a$, $AB = c$ and r and R are respectively radii of incircle and circumcircle of the triangle then prove that $2(r + R) = b + c$ *[Learn the property]*

Solution : Clearly hypotenuse $BC = 2R$

$\therefore \quad a = 2R$

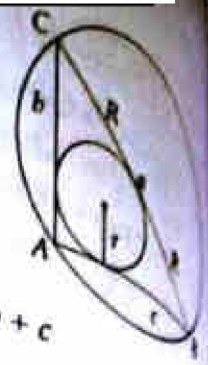
and from $r = \frac{\Delta}{s}$, $r = \dfrac{\frac{1}{2}bc}{\left(\frac{a+b+c}{2}\right)} = \dfrac{bc}{a+b+c}$

$\therefore 2R + 2r = a + \overline{a+b+c}$

$$= \frac{a^2 + ab + ac + 2bc}{a+b+c}$$

$$= \frac{b^2 + c^2 + ab + ac + 2bc}{a+b+c} \quad (\because a^2 = b^2 + c^2)$$

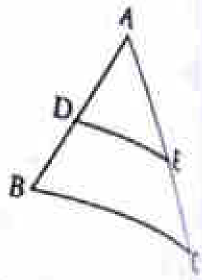
$$= \frac{(b+c)^2 + a(b+c)}{a+b+c} = \frac{(b+c)(b+c+a)}{a+b+c} = b + c$$



Thus sum of diameters of two circles = Sum of mutually perpendicular sides

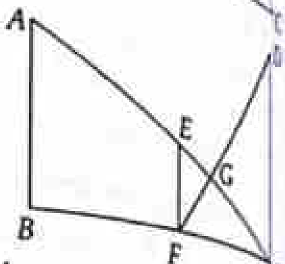
Exercise–6A

1. In the adjacent figure DE is parallel to BC and ratio of areas of ΔADE and trapezium $BDEC$ is 4 : 5. What is the value of $DE : BC$?



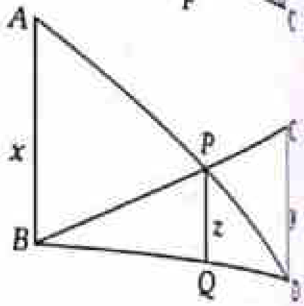
(a) 1 : 2 (b) 2 : 3
(c) 4 : 5 (d) None of these

2. In the given figure AB, EF and CD are parallel lines. It is given that $EG = 5$cm, $GC = 10$cm, $AB = 15$ cm and $DC = 18$ cm, What is the value of AC ?



(a) 20 cm (b) 24 cm
(c) 25 cm (d) 28 cm

3. In the adjacent figure, $\angle ABD = \angle PQD = \angle CDQ = \frac{\pi}{2}$. If $AB = x$, $PQ = z$ and $CD = y$ then which one of the following is true.



(a) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ (b) $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$
(c) $\frac{1}{z} + \frac{1}{y} = \frac{1}{x}$ (d) $\frac{1}{x} + \frac{1}{y} = \frac{2}{z}$

4. ΔPQR is right angled at Q; $PR = 5$ cm and $QR = 4$ cm. Another ΔABC is given whose side are respectively 3 cm, 4 cm and 5 cm then which one of the following is true ?
(a) area of ΔPQR is double the area of ΔABC
(b) area of ΔABC is double the area of ΔPQR
(c) $\angle B = \frac{\angle Q}{2}$
(d) Both triangles are congruent

5. If ratio of length of medians of two equilateral triangles are 3 : 2 then what is the ratio of their sides ?
(a) 1 : 1 (b) 2 : 3 (c) 3 : 2 (d) $\sqrt{3} : \sqrt{2}$

In which of the following triangle centroid and orthocentre are coincident?
(a) Scalene triangle (b) Isosceles triangle
(c) Equilateral triangle (d) Right angled triangle

Consider the triangle ABC. Let D, E are respectively mid points of sides BC, CA while AD and BE intersects at G. Suppose O is a point on AD such that $AO : OD = 2 : 7$.

Assertion (A) : $AO = \frac{(2GD)}{3}$

Reason (R) : $OD = \frac{(2AG)}{3}$

(a) Both Assertion A and Reason R are correct and Reason R is a correct explanation of Assertion A.
(b) Both Assertion A and Reason R are correct but Reason R is not the correct explanation of Assertion A.
(c) Assertion A is correct, Reason R is wrong.
(d) Assertion A is wrong, Reason R is correct.

8. ABC is a given triangle. AD, BE and CF are altitudes of ΔABC.

Assertion (A) : $(AB^2 + BC^2 + CA^2) > (AD^2 + BE^2 + CF^2)$

Reason (R) : $(AE^2 - AF^2) + (BF^2 - BD^2) + (CD^2 - CE^2) = 0$

(a) Both Assertion A and Reason R are correct and Reason R is a correct explanation of Assertion A.
(b) Both Assertion A and Reason R are correct but Reason R is not the correct explanation of Assertion A.
(c) Assertion A is correct, Reason R is wrong.
(d) Assertion A is wrong, Reason R is correct.

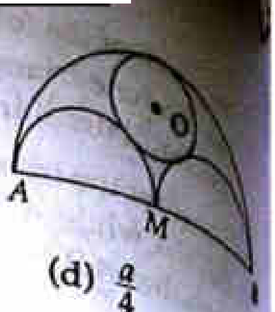
9. ABC is a given triangle. An external point X of ΔABC is such that $CD = CX$, where D is the point of intersection of BC and AX. If $\angle BAX = \angle XAC$, then which one of the following is true ?
(a) ΔABD and ΔACX are similar (b) $\angle ABD < \angle ACD$
(c) $AC = CX$ (d) $\angle ADB > \angle DXC$

10. How many point (s) in the plane of ΔABC is equidistant from its vertices ?
(a) 0 (b) 1 (c) 2 (d) 3

11. In a triangle ABC, interal bisector of $\angle ABC$ and external bisector of $\angle ACB$ meet in D. Which one of the following is true ?
(a) $\angle BDC = \angle BAC$ (b) $\angle BDC = \frac{1}{2} \angle ABC$
(c) $\angle BDC = \angle DBC$ (d) None of these

12. The median BD of ΔABC meets side AC at D. If $BD = \frac{1}{2} AC$, then which one of the following is true.
(a) $\angle ACB = 1$ right angle (b) $\angle BAC = 1$ right angle
(c) $\angle ABC = 1$ right angle (d) None of the above

13. In the given figure, M is the mid point of line segment AB whose length is $2a$. Semicircles having diameters AM, MB and AB are drawn at the same side of the line. The radius of a circle touching all the three semicircle is



(a) $\frac{2a}{3}$ (b) $\frac{a}{2}$ (c) $\frac{a}{3}$ (d) $\frac{a}{4}$

14. Point of concurrency of altitudes of a triangle is called
(a) Circumcentre (b) Orthocentre
(c) Incentre (d) Centroid

15. Number of circles passing through all the three vertices of a triangle is
(a) one (b) two (c) three (d) infinity

16. Consider the following statements :

Statement–I : Suppose PQR is a triangle with $PQ = 3$ cm, $QR = 4$ cm and $RP = 5$ cm. If D is a point either outside or inside of the plane of triangle then $DP + DQ + DR > 6$ cm.

Statement–II : ΔPQR is a right angled triangle.

Regarding two statements described above which one of the following is true.

(a) Both statement I and II are true and statement II is a correct explanation of statement I.
(b) Both statements I and II are true but statement II is not the correct explanations of statement I.
(c) Statement I is true and statement II is false.
(d) Statement I is false and statement II is correct.

17. ΔABC is a given triangle and AD is perpendicular to BC. It is given that length of three sides AB, BC, CA are rational numbers. Which one of the following is true ?
(a) AD and BD both must be rational.
(b) AD must be rational but BD is not necessarily rational.
(c) BD must be rational but AD is not necessarily rational.
(d) neither AD nor BD is necessarily rational.

18. Centroid of ΔABC is 8 cm away from vertex A. What is the length of median passing through vertex A ?
(a) 20 cm (b) 16 cm (c) 12 cm (d) 10 cm

19. If distance of a vertex of an equilateral triangle from its centroid is 6 cm then area of the triangle is
(a) 24 cm^2 (b) $27\sqrt{3}$ cm^2 (c) 12 cm^2 (d) $12\sqrt{3}$ cm^2

20. In ΔPQR, $PQ = 4$ cm, $QR = 3$ cm and $RP = 3.5$ cm, ΔDEF is similar to ΔPQR. If $EF = 9$ cm then perimeter of ΔDEF is—
(a) 10.5 cm (b) 21 cm (c) 31.5 cm
(d) Cannot be determined as data is insufficient

21. AD is an angle bisector of ΔABC and $BD : DC = 2 : 3$. If $AB = 7$ cm then $AC : BC$ is
(a) $2 : 3$ (b) $3 : 2$
(c) $21 : 10$ (d) data insufficient

22. Assertion (A) : AD is angle bisector of $\angle A$ of the triangle ABC. If $AB = 6$ cm, $BC = 7$ cm, $AC = 8$ cm then $BD = 3$ cm and $CD = 4$ cm.
Reason (R) : The angle bisector AD of the triangle divides base BC in the ratio $AB : AC$.
(a) Both Assertion A and Reason R are correct and Reason R is a correct explanation of Assertion A.
(b) Both Assertion A and Reason R are correct but Reason R is not the correct explanation of Assertion A.
(c) Assertion A is correct, Reason R is wrong.
(d) Assertion A is wrong, Reason R is correct.

23. O is the incentre of ABC and $\angle A = 30^\circ$. Accordingly what is $\angle BOC$?
(a) 100° (b) 105° (c) 110° (d) 90°

24. O is centroid of ΔABC and AD, BE, CF are its three medians. If area of ΔAOE is 15 cm^2, then area of quadrilateral $BDOF$ is—
(a) 20 cm^2 (b) 30 cm^2 (c) 40 cm^2 (d) 25 cm^2

25. O and C are respectively orthocentre and circumcentre of ΔPQR. Point P and O are joined and produced part meets side QR in S. If $\angle PQS = 60^\circ$ and $\angle QCR = 130^\circ$, then $\angle RPS = ?$
(a) 30° (b) 35° (c) 100° (d) 60°

26. From the circumcentre O of the triangle ABC perpendicular OD is drawn to BC. If $\angle BAC = 60^\circ$ then what is the measure of $\angle BOD$?
(a) 30° (b) 90° (c) 60° (d) 45°

27. O is the circumcentre of a triangle ABC. If $\angle BAC = 85^\circ$ and $\angle BCA = 75^\circ$ then what is the value of $\angle OAC$?
(a) 40° (b) 60° (c) 70° (d) 90°

28. In a triangle ABC medians CD and BE intersect at point O. What is the ratio of area of ΔODE and ΔABC ?
(a) $1 : 6$ (b) $6 : 1$ (c) $1 : 12$ (d) $12 : 1$

29. Suppose O be incentre of ΔABC and D is a point on side BC of ΔABC such that $OD \perp BC$. If $\angle BOD = 15^\circ$ then $\angle ABC = ?$
(a) 75° (b) 45° (c) 150° (d) 90°

30. The radius of incircle of an equilateral triangle is 3 cm. What is the length of each median of the triangle ABC ?
(a) 12 cm (b) $\frac{9}{2}$ cm (c) 4 cm (d) 9 cm

31. I is the incentre of the triangle ABC. If $\angle ABC = 60^\circ$ and $\angle ACB = 50^\circ$ then $\angle BIC$ is
(a) 55° (b) 125° (c) 70° (d) 65°

the value of $\angle BAC$? ... $\angle BOC = 110°$, then what ...

(a) $20°$ (b) $40°$ (c) $55°$ (d) $110°$

33. Which groups of centres of a triangle given below always lie in a straight line (i.e., centres are collinear)
(a) Incentre, circumcentre, centroid
(b) Incentre, orthocentre, centroid
(c) Circumcentre, orthocentre, centroid
(d) None of these

34. If distance between orthocentre and circumcentre of a triangle is 6 cm then what is the distance between its centroid and circumcentre?
(a) 4 cm (b) 2 cm (c) $\frac{8}{3}$ cm (d) $\frac{4}{3}$ cm

35. Which pair of centres given below lie outside the triangle?
(a) Circumcentre and centroid (b) Incentre and centroid
(c) Circumcentre and orthocentre
(d) None of these

36. What is the distance between circumcentre and orthocentre of a right angled triangle?
(a) Equal to hypotenuse (b) Half to hypotenuse
(c) One third to hypotenuse (d) Two third to hypotenuse

37. If hypotenuse of a right angle is 15 cm then what is the distance between its orthocentre and centroid?
(a) 5 cm (b) 10 cm (c) $\frac{10}{3}$ cm (d) $\frac{20}{3}$ cm

38. A non right angle bisector of a right angled isosceles triangle divide the triangle in those two parts, whose area are in the ratio.
(a) $1:1$ (b) $1:\sqrt{2}$ (c) $1:2$ (d) $1:\sqrt{2}-1$

39. In a ΔABC, $BC = 9$ cm, $AC = 40$ cm and $AB = 41$ cm. If bisector of angle A meets side BC at D then ratio of area of ΔABD ad ΔABC is
(a) $40:41$ (b) $9:40$
(c) $9:41$ (d) $41:81$

Directions (40–42) : In a triangle ABC, $AB = 5$ cm, $BC = 6$ cm and $CA = 7$ cm. If bisectors of $\angle A$, $\angle B$, $\angle C$ respectively meet sides BC, CA and AB at D, E, F and I be the incentre of the triangle then

40. What is $BD : DC$?
(a) $5:7$ (b) $7:5$ (c) $5:6$ (d) $6:5$

41. What is the length of AE?
(a) $\frac{42}{11}$ cm (b) 6 cm (c) $\frac{13}{2}$ cm (d) $\frac{35}{11}$ cm

42. What is $CI : IF$?
(a) $2:1$ (b) $11:7$ (c) $3:1$ (d) $13:5$

43. The semiperimeter of a triangle is S and its centroid is G. What is the distance between G and centroid of the triangle formed by mid points of the sides of the given triangle ?

(a) $\frac{5}{3}$ (b) $\frac{5}{6}$ (c) $\frac{5}{18}$ (d) 0

44. If length of three medians of a triangle are respectively 9 cm, 12 cm and 15 cm then what is the area of the triangle in cm^2 ?

(a) 48 (b) 72 (c) 96 (d) 36

45. If incentre of a isosceles right angled triangle is I then ratio of area of triangle formed by joining I to the respective vertices of triangle is

(a) $1:1:\sqrt{2}$ (b) $\frac{2-\sqrt{2}}{2}:\frac{2-\sqrt{2}}{2}:\sqrt{2}-1$

(c) $\sqrt{2}-1:\sqrt{2}-1:\sqrt{2}+1$ (d) None of these

46. In a right angled isosceles triangle $\angle C = 90°$ and I is its incentre their ratio of area of ΔAIB and ΔABC is

(a) $1:\sqrt{2}+1$ (b) $1:\sqrt{2}-1$ (c) $1:\sqrt{2}$ (d) $1:2$

47. A triangle is formed by joining mid points of sides of a triangle and a triangle is formed again by joining mid points of its sides. The ratio of area of this triangle to the area of original triangle is

(a) $1:4$ (b) $1:8$ (c) $1:16$ (d) $1:64$

Directions (48–51) : In a triangle ABC if side $AB = c = 4$ cm, side $AC = b = 6$ cm and $BC = a = 7$, then answer the following questions

48. The length of median AD is

(a) $\frac{53}{2}$ cm (b) $\frac{1}{2}\sqrt{55}$ cm (c) $\sqrt{\frac{53}{2}}$ cm (d) $\sqrt{\frac{63}{2}}$ cm

49. If AD is bisector of angle A then length of BD is

(a) $\frac{16}{5}$ (b) $\frac{21}{5}$ (c) $\frac{12}{5}$ (d) $\frac{14}{5}$

50. If AD be the altitude then what is BD in cm ?

(a) $\frac{29}{12}$ (b) $\frac{39}{12}$ (c) $\frac{29}{14}$ (d) $\frac{69}{14}$

51. If AD be the altitude then $BD:DC$?

(a) $29:69$ (b) $69:29$ (c) $29:39$ (d) $39:29$

52. If area of a triangle is 81 cm^2 and its semiperimeter is 27 cm then area of incircle of the triangle is

(a) 6π cm^2 (b) 3π cm^2 (c) 18π cm^2 (d) 9π cm^2

53. If sides of a triangle are respectively 5 cm, 6 cm and 7 cm then radius of the circumcircle of the triangle is

(a) 9 cm (b) $\frac{35}{\sqrt{6}}$ cm (c) $\frac{35}{4\sqrt{6}}$ cm (d) $\frac{17}{2}$ cm

54. The sides of a triangle are ... radius of the circumcircle is

(a) 5 cm (b) 7.5 cm (c) 7 cm (d) 8.5 cm

55. The sides of a triangle are 8 cm, 15 cm and 17 cm. The sum of radii of circumcircle and incircle of the triangle is

(a) 23 cm (b) 11.5 cm (c) 25 cm (d) 12.5 cm

56. The sides of a triangle are 9 cm, 40 cm and 41 cm. The distance between its orthocentre and circumcentre is

(a) 29 cm (b) 20.5 cm (c) $\sqrt{29}$ cm (d) 15 cm

57. If ratio of sides of a triangle are 4 : 5 : 6 then what is the ratio of circumradius and inradius ?

(a) 2 : 1 (b) 16 : 7 (c) 12 : 7 (d) 16 : 5

58. The greatest side of a triangle is two more than double of its smallest side while middle one is one unit less than greatest side. If the smallest side is equal to the least odd prime number then ratio of circumradius and inradius of the triangle is

(a) 12 : 7 (b) 7 : 2 (c) 7 : 3 (d) 4 : 1

59. In a triangle ABC if $A = 90°$, $b = 3$ and $c = 4$ then $R : r$ is

(a) 5 : 3 (b) 7 : 3 (c) 3 : 2 (d) 5 : 2

60. If sides of a triangle are 3 cm, 4 cm and 5 cm then what is the distance between its incentre and circumcentre ?

(a) $\frac{5}{4}$ cm (b) $\sqrt{\frac{5}{2}}$ cm

(c) $\frac{\sqrt{5}}{2}$ cm (d) None of then

61. If triangle formed by medians of a right angled triangle is also a right angled triangle then what is the ratio of sides of the original right angled triangle ?

(a) $1 : \sqrt{2} : \sqrt{3}$ (b) $2 : \sqrt{3} : \sqrt{7}$

(c) $\sqrt{2} : \sqrt{3} : \sqrt{5}$ (d) 3 : 4 : 5

Answers–6A

1. (b)	2. (c)	3. (a)	4. (d)	5. (c)	6. (c)	7. (c)	8. (b)
9. (a)	10. (b)	11. (d)	12. (c)	13. (c)	14. (b)	15. (a)	16. (a)
17. (c)	18. (c)	19. (b)	20. (c)	21. (c)	22. (a)	23. (b)	24. (b)
25. (b)	26. (c)	27. (a)	28. (c)	29. (c)	30. (d)	31. (b)	32. (b)
33. (c)	34. (b)	35. (c)	36. (b)	37. (a)	38. (b)	39. (d)	40. (a)
41. (d)	42. (d)	43. (d)	44. (b)	45. (b)	46. (a)	47. (c)	48. (b)
49. (d)	50. (c)	51. (a)	52. (d)	53. (c)	54. (a)	55. (b)	56. (b)
57. (b)	58. (b)	59. (d)	60. (c)	61. (a)			

Exaplanation

1. (b) $\because$ $DE \parallel BC$ and : area (ΔADE) : area (trapezium $BDEC$) = 4 : 5

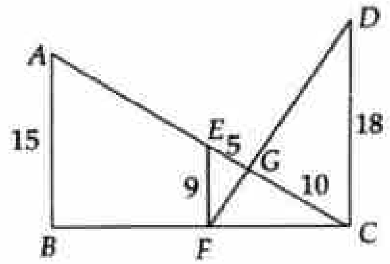
$\therefore$ $\Delta ABC \sim \Delta ADE$

$\Rightarrow \dfrac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \left(\dfrac{DE}{BC}\right)^2$

$\Rightarrow \dfrac{4}{4+5} = \left(\dfrac{DE}{BC}\right)^2$

$\Rightarrow DE : BC = 2 : 3$

2. (c) $\because$ $AB \parallel EF \parallel CD$



$\Rightarrow \dfrac{EG}{GC} = \dfrac{EF}{CD}$

$\Delta EGF \sim CGD$

$\Rightarrow \dfrac{EG}{CG} = \dfrac{EF}{CD} \Rightarrow \dfrac{5}{10} = \dfrac{EF}{18}$

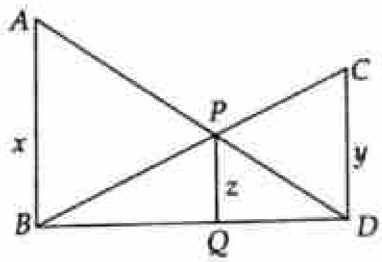
$\Rightarrow EF = 9$ cm

$\because$ $\Delta ABC \sim EFC$

$\therefore \dfrac{EC}{AC} = \dfrac{EF}{AB} \Rightarrow \dfrac{15}{AC} = \dfrac{9}{15}$

$\Rightarrow AC = \dfrac{15 \times 15}{9} = 25$ cm

3. (a) $\because$ $\angle ABD = \angle PQD = 90°$



$\therefore$ $\Delta ABD \sim PQD$

$\Rightarrow \dfrac{x}{z} = \dfrac{BD}{QD}$ ... (i)

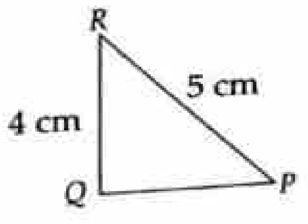
$\because$ $\angle CDB = \angle PQB = 90°$

$\therefore$ $\Delta BCD \sim \Delta BPQ$

$\Rightarrow \dfrac{z}{y} = \dfrac{BQ}{BD} \Rightarrow \dfrac{z}{y} = \dfrac{BD - QD}{BD} \Rightarrow \dfrac{z}{y} = 1 - \dfrac{QD}{BD}$

$\Rightarrow \dfrac{z}{y} = 1 - \dfrac{z}{x} \Rightarrow \dfrac{z}{x} + \dfrac{z}{y} = 1 \Rightarrow \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$

4. (d) In triangle PQR,



$QP^2 = (5)^2 - (4)^2 \Rightarrow QP = 3$

Since sides of ΔABC are also 3 cm, 4 cm, 5 cm therefore the two triangles are congruent.

5. (c) Ratio of medians of two equilateral triangle = ratio of their sides = 3 : 2

6. (c) All the centres in, an equilateral triangle are coincident, so centroid and orthocentre are also coincident

7. (c) Given, $AO : OD = 2 : 7$

$\therefore \quad OA = \frac{2}{9}AD, OD = \frac{7}{9}AD$

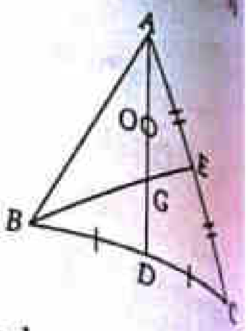
We know that centroid divides median in the ratio 2 : 1,

Therefore, $AG = \frac{2}{3}AD, GD = \frac{1}{3}AD$

(A) $OA = \frac{2}{9}AD$

$OA = \left(2 \cdot \frac{1}{3}AD\right)\frac{1}{3}$

$= (2GD)\frac{1}{3} = \frac{2GD}{3}$ [from (i)]



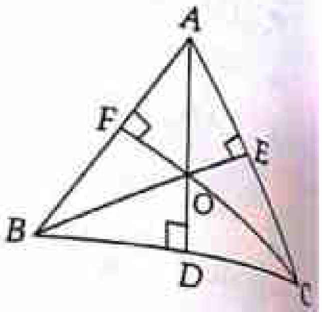
(R) $OD = \frac{7}{9}AD = \left(7 \cdot \frac{2}{3}AD\right) \cdot \frac{1}{3 \times 2} = \frac{7AG}{6}$

Hence (A) is true and (R) is false.

8. (b) (A) We know that in a right angled triangle hypotenuse is the greatest side.

In ΔABD,

$AB^2 > AD^2$ ... (i)



In ΔBEC,

$BC^2 > BE^2$ ... (ii)

In ΔACF,

$AC^2 > CF^2$ ... (iii)

adding (i), (ii) and (iii)

$(AB^2 + BC^2 + AC^2) > (AD^2 + BE^2 + CF^2)$

Now (R), $(AE^2 - AF^2) + (BF^2 - BD^2) + (CD^2 - CE^2)$

$= (OA^2 - OE^2) - (OA^2 - OF^2) + (OB^2 - OF^2)$

$- (OB^2 - OD^2) + (OC^2 - OD^2) - (OC^2 - OE^2)$

$= 0$

Both (A) and (R) ave correct but (R), (A) is not the correct explanation of (A).

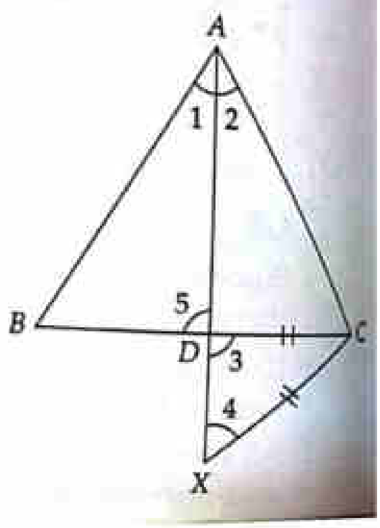
9. (a) In ΔDCX,

$CD = CX$ (given)

$\angle 3 = \angle 4$ (opposite angles of equal sides)

but, $\angle 3 = \angle 5$

Hence, $\angle 4 = \angle 5$



In ΔABD and ΔACX,

$\angle 1 = \angle 2$ (given)

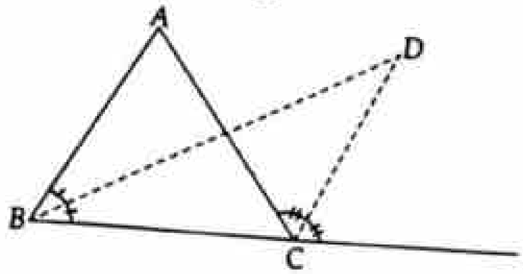
$\angle 4 = \angle 5$

$\therefore \quad \Delta ABD \sim \Delta ACX$ ($A - A$ condition)

10. (b) In the plane of the triangle circumcentre is the only point which is equidistant from all the three vertices of the triangle.

11. (d) In ΔBCD,

$\angle DBC = \frac{B}{2}, \angle BCD = \frac{A+B}{2}$



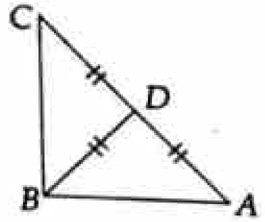
$\therefore \quad \angle BDC = \pi - \frac{B}{2} - \frac{A+B}{2}$

$= \pi - \frac{A}{2} - B$

12. (c) Given,

$CD = BD = DA$

It is possible only when ΔABC is a right angled triangle.



13. (c) $AB = 2a \Rightarrow AM = a$

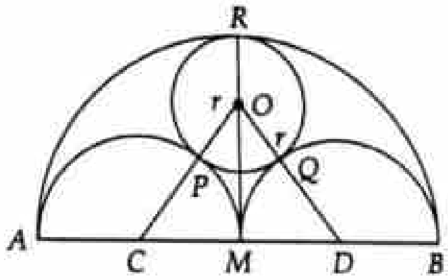
and $AC = CM = BD = MD = \frac{a}{2}$

Now, $OC = OP + PC = OP + CM$

$= r + \frac{a}{2}$

and $OD = OQ + QD = OQ + MD$

$= \left(r + \frac{a}{2}\right)$



$\therefore \quad \Delta OCD$ is an isosceles triangle and M is mid point of CD

$(\because \quad OC = OD)$

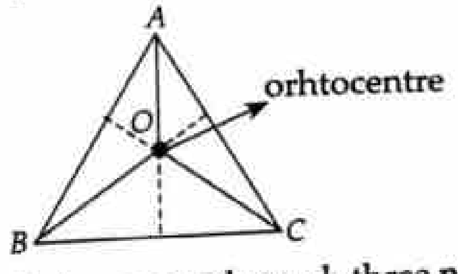
$\Rightarrow \angle OMC = 90^\circ$

In ΔOMC, $OC^2 = OM^2 + CM^2$

$\Rightarrow \left(r + \frac{a}{2}\right)^2 = (a - r)^2 + \left(\frac{a}{2}\right)^2$

$\Rightarrow r = \frac{a}{3}$

14. (b) Point of concurrency of altitudes of a triangle is called orthocentre



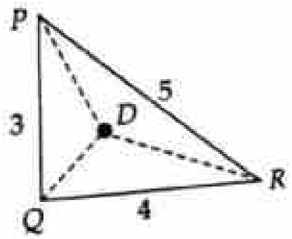
15. (a) One and only one circle passes through three non collinear points.

16. (a) $3^2 + 4^2 = 5^2 \Rightarrow$ triangle is right angled.

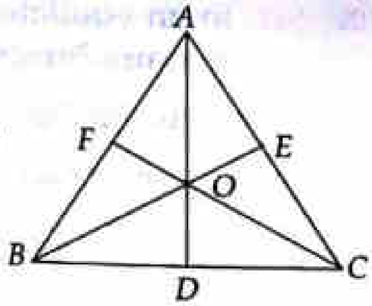
In ΔDQR, $DQ + DR > 4$

In ΔDPR, $DP + DR > 5$

In ΔDQP, $DQ + DP > 3$



24. (b) See the figure,

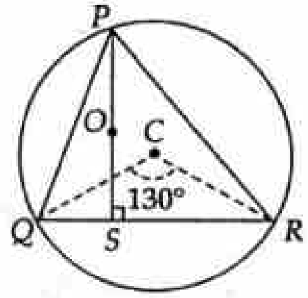


Δ will be divided into 6 equal parts.

Area of quadrilateral $BDOF = 2 \times$ area of ΔOAE

$$= 2 \times 15 = 30 \text{ cm}^2$$

25. (b) In ΔPQR,



$$\angle QPR = \frac{1}{2}\angle QCR = \frac{1}{2} \times 130° = 65°$$

$\angle PQR = \angle PQS = 60°$ (given)

$\therefore$ $\angle PRQ = 180° - 65° - 60° = 55°$

$\because$ O is the orthocentre

$\therefore$ $\angle PSR = 90°$

Thus in ΔPSR

$\angle RPS = 180° - 90° - \angle PRS$

$= 180° - 90° - 55°$ $\quad (\because \angle PRS = \angle PRQ = 90°)$

$= 35°$

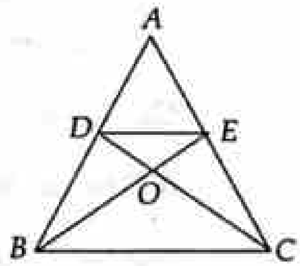
$\therefore$ $\angle RPS = 35°$

26. (c) $\angle BOD = \frac{1}{2} \times \angle BOC = \frac{1}{2} \times 120° = 60°$

27. (a) $\angle B = 180° - 75° - 85° = 20°$

$\therefore$ $\angle OAC = 2\angle B = 40°$

28. (c) In ΔODE and ΔBOC,



$\angle BOC = \angle DOE$

$\angle DEO = \angle OBC$

$\angle ODE = \angle OCB$

Both triangles are similar.

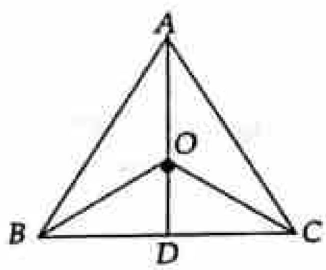
$$\frac{\Delta ODE}{\Delta BOC} = \frac{DE^2}{BC^2}$$

$DE \parallel BC$ and $DE = \frac{1}{2}BC$

Area of $\Delta ABC = 3 \times$ Area of ΔOBC

$$\therefore \quad \frac{\Delta ODE}{\Delta ABC} = \frac{\Delta ODE}{3 \times \Delta BOC} = \frac{1}{3} \cdot \frac{DE^2}{BC^2} = \frac{1}{3} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

29. (c) BO is bisector of $\angle B$



$\angle ODB = 90°$;

$\angle BOD = 15°$

$\angle OBD = 180° - 90° - 15° = 75°$

$\angle ABC = 2 \times 75° = 150°$

30. (d) In an equilateral triangle ratio of inradius to circumradius is 1 : 2.

$\therefore$ Circumradius = 6 cm

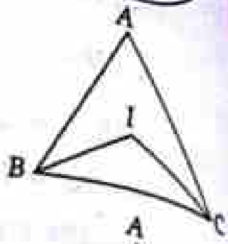
$\therefore$ Length of each median = 3 + 6 = 9 cm

31. (b) Shortcut : $\angle BIC = 90^\circ + \frac{A}{2}$

$$= 90^\circ + \frac{180^\circ - B - C}{2}$$

$$= 90^\circ + \frac{180^\circ - 60^\circ - 50^\circ}{2}$$

$$= 90^\circ + 35^\circ = 125^\circ$$

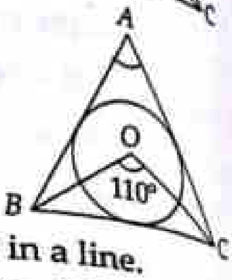


32. (b) Shortcut :

$\angle BOC = 90^\circ + \frac{A}{2}$

$\Rightarrow 110^\circ = 90^\circ + \frac{A}{2}$

$\Rightarrow A = 40^\circ.$



33. (c) Note that except incentre all the three centres are in a line.

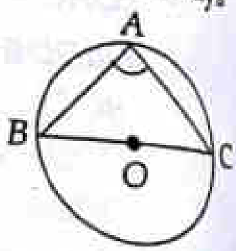
34. (b) The orthocentre (P), centroid (G) and circumcentre (O) always lie on a straight line and $PG : GO = 2 : 1$.

As in question $PO = 6$ cm

$\therefore$ $OG = \frac{1}{3} \times 6 = 2$ cm

35. (c) In an obtused angled triangle circumcentre and orthocentre always lie outside the circle.

36. (b) In the given figure ΔBAC is a right angled triangle. which subtends right angle at A. Clearly A is the orthocentre and O is the circumcentre.



$\therefore$ $OA = OB = OC = \text{radius} = \frac{BC}{2} = \frac{\text{hypotenuse}}{2}$

37. (a) If P be orthocentre, G be centroid, O be circumcentre then $\frac{PG}{OG} = \frac{2}{1}$

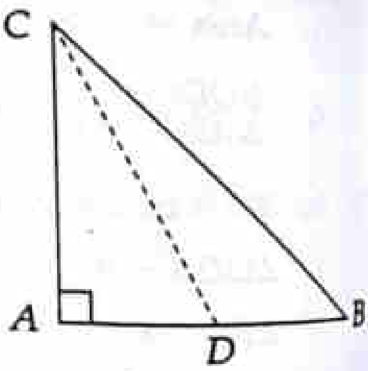
But in right angled triangle $OP = \frac{\text{hypotenuse}}{2} = \frac{15}{2}$ cm

$\therefore$ $PG = \frac{2}{3}\, OP = \frac{2}{3} \times \frac{15}{2} = 5$ cm

38. (b) In figure, $\angle A = 90^\circ$, $\angle B = \angle C = 45^\circ$

CD, is bisector of non right angle C.

We have $\frac{AD}{DB} = \frac{AC}{BC} = \frac{k}{\sqrt{k^2 + k^2}} = \frac{1}{\sqrt{2}}$



Hence, $\frac{\text{Area } \Delta ACD}{\text{Area } \Delta ABD} = \frac{\frac{1}{2} \times AD \times height}{\frac{1}{2} \times DB \times height}$

$= \frac{AD}{DB}$, (height of both triangles are equal) $= 1 : \sqrt{2}$

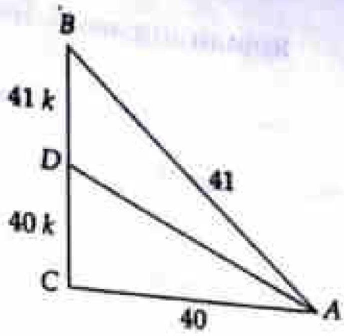
39. (d) $\because 9^2 + 40^2 = 41^2$

$\therefore \Delta ABC$ is a right angled triangle with $\angle C = 90°$

See the figure,

$\because$ AD is bisector of $\angle A$.

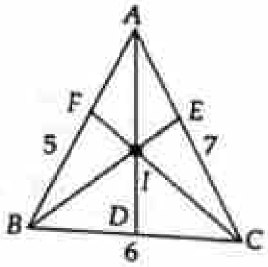
$\therefore \frac{CD}{BD} = \frac{AC}{AB} = \frac{40}{41} \Rightarrow CD = 40k,\ BD = 41k$



$$\frac{\text{Area } \Delta ABD}{\text{Area } \Delta ABC} = \frac{\frac{1}{2}\times BD \times AC}{\frac{1}{2}\times BC \times AC}$$

$$= \frac{BD}{BC} = \frac{41k}{40k + 41k} = \frac{41}{81}$$

40. (a) $BD : DC = \frac{AB}{AC} = \frac{5}{7}$



41. (d) $\because \quad \frac{AE}{CE} = \frac{AB}{BC} = \frac{5}{6}$

$\therefore \quad AE = \frac{5}{5+6}\times 7 = \frac{35}{11}$ cm

42. (d) Recall that $\frac{CI}{IF} = \frac{CA + CB}{AB} = \frac{7+6}{5} = \frac{13}{5}$

43. (d) The two triangles mentioned in the questions have centroid at the same point.

44. (b) $\because \quad 9^2 + 12^2 = 81 + 144 = 225 = 15^2$

$\therefore \quad$ 9, 12 and 15 are sides of a right angled triangle.

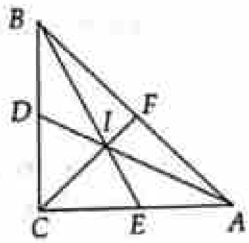
Area of triangle $= \frac{4}{3}$ (Area of triangle formed by taking medians as side of the triangle)

$$= \frac{4}{3} \times \left(\frac{1}{2}\times 9 \times 12\right) = 4 \times 9 \times 2 = 72 \text{ cm}^2$$

45. (b) Let in ΔABC, $\angle C = 90°$, $AC = BC = x$

then $AB = \sqrt{x^2 + x^2} = \sqrt{2}x$

$\frac{CI}{IF} = \frac{CA + CB}{AB} = \frac{x + x}{\sqrt{2}x} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2}}{1}$ (here, $CF \perp r\ AB$)



$$\therefore \quad \frac{\text{area } \Delta AIB}{\text{area } \Delta ABC} = \frac{\frac{1}{2}\times IF \times AB}{\frac{1}{2}\times CF \times AB}$$

$$= \frac{IF}{CF} = \frac{IF}{CI + IF} = \frac{1}{\sqrt{2} + 1}$$

$$= \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \sqrt{2}-1$$

Remaining area = area of (Δ CIA) + area of (Δ CID) $= 1-(\sqrt{2}-1) = 2-\sqrt{2}$

By symmetry area of Δ CIA = area of Δ $CIB = \frac{2-\sqrt{2}}{2}$

46. (a) See the solution of question no. 45.

47. (c) Required ratio $= \frac{\frac{1}{4}\times\frac{1}{4}}{1} = \frac{1}{16}$

48. (b) $AD = \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$

$$= \frac{1}{2}\sqrt{2(6)^2+2(4)^2-7^2} = \frac{1}{2}\sqrt{72+32-49} = \frac{1}{2}\sqrt{55}$$

49. (d) $BD = \frac{ac}{b+c} = \frac{7\times 4}{6+4} = \frac{28}{10} = \frac{14}{5}$

50. (c) $BD = \frac{AB^2+BC^2-AC^2}{2BC} = \frac{c^2+a^2-b^2}{2a} = \frac{4^2+7^2-6^2}{2\times 7} = \frac{29}{14}$

51. (a) From above question, $BD = \frac{29}{14}$

and $CD = \frac{b^2+a^2-c^2}{2a} = \frac{36+49-14}{2\times 7} = \frac{69}{14}$

$\therefore$ $BD : DC = 29 : 69$

52. (d) $r = \frac{\Delta}{s} = \frac{81}{27} = 3$ cm

$\therefore$ Area $= \pi r^2 = \pi(3)^2 = 9\pi$ cm^2

53. (c) Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{9\times 4\times 3\times 2} = 6\sqrt{6} \text{ cm}^2 \qquad (\because s = \frac{5+6+7}{2} = 9)$$

$\therefore$ $R = \frac{abc}{4\Delta} = \frac{5\times 6\times 7}{4\times 6\sqrt{6}} = \frac{35}{4\sqrt{6}}$ cm

54. (a) $\because$ $6^2+8^2 = 10^2$

$\therefore$ Given triangle is right angled.

$\therefore$ Hypotenuse = diameter of circumcircle

or, $10 = 2r$

$\Rightarrow$ $r = 5$

55. (b) Given triangle is a right angled triangle. If radius of its incircle is r and that of circumcircle is R then

$2(r+R) = a+b$, (where a and b are perpendicular sides)

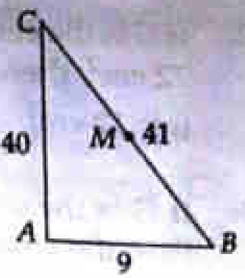
or, $2(r+R) = 8+15 = 23$

or, $r+R = \frac{23}{2} = 11.5$ cm

6. (b) $\because$ $9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$

$\therefore$ Given triangle is right angled.

If $AB = 9$, $AC = 40$ and $BC = 41$ then A is orthocentre and mid point of hypotenuse BC is circumcentre of the triangle.



$\therefore$ $AM = BM = CM =$ radius of circumcircle.

or, $AM = \frac{41}{2} = 20.5$ cm

57. (b) Let $a = 4k$, $b = 5k$, $c = 6k$, $s = \frac{4k + 5k + 6k}{2} = \frac{15k}{2}$

$$\therefore \frac{R}{r} = \frac{\left(\frac{abc}{4\Delta}\right)}{\left(\frac{\Delta}{s}\right)} = \frac{abcs}{4\Delta^2} = \frac{abcs}{4s(s-a)(s-b)(s-c)}$$

$$= \frac{abc}{4(s-a)(s-b)(s-c)} = \frac{(4k)(5k)(6k)}{4\left(\frac{7k}{2}\right)\left(\frac{5k}{2}\right)\left(\frac{3k}{2}\right)}$$

$$= \frac{4\times5\times6\times8}{4\times7\times5\times3} = \frac{6\times8}{7\times3} = \frac{16}{7}$$

58. (b) Least odd prime number $= 3 =$ smallest side

$\therefore$ Greatest side $= 2 \times 3 + 2 = 8$ and middle side $= 8 - 1 = 7$

Now, solve as in above questions.

59. (d) Given triangle is a right angled triangle.

$$\therefore a = \sqrt{3^2 + 4^2} = 5 = 2R \Rightarrow R = \frac{5}{2}$$

$$r = \frac{\Delta}{s} = \frac{\frac{1}{2}\cdot bc}{\frac{a+b+c}{2}} = \frac{bc}{a+b+c} = \frac{3\times4}{3+4+5} = 1$$

$\therefore$ $R : r = 5 : 2$

60. (c) Distance between incentre and circumcentre $= \sqrt{R^2 - 2Rr}$

From above question $R = \frac{5}{2}$ and $r = 1$

$$\text{Required distance} = \sqrt{\left(\frac{5}{2}\right)^2 - 2\times\frac{5}{2}\times1} = \sqrt{\frac{25}{4} - 5} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2} \text{ cm}$$

61. (a) Required ratio is $1 : \sqrt{2} : \sqrt{3}$. Learn it and try to prove it.

Exercise–6B

1. In ΔABC, AD is the median and $AD = \frac{1}{2} BC$. If $\angle BAD = 30°$, then measure of $\angle ACB$ is

(a) 30° (b) 60° (c) 90° (d) 45°

[SSC Tier-I 2012]

2. If G is the centroid and AD, ... medians of ΔABC with ... 72 cm², then the area of ΔBDG is
(a) 12 cm² (b) 16 cm² (c) 24 cm² (d) 8 cm²
[SSC Tier-I 20...

3. If G is the centroid and AD be a median with length 12 cm of ΔAB... then the value of AG is
(a) 4 cm (b) 8 cm (c) 10 cm (d) 6 cm
[SSC Tier-I 20...

4. O is the orthocentre of the triangle ABC. If $\angle BOC = 120°$, then $\angle BAC$ ...
(a) 150° (b) 60° (c) 135° (d) 90°
[SSC Tier-I 20...

5. Circumcentre of ΔABC is O. If $\angle BAC = 85°$, $\angle BCA = 80°$, then $\angle OAC$ ...
(a) 80° (b) 30° (c) 60° (d) 75°
[SSC Tier-I 20...

6. The length of the circum-radius of a triangle having sides of length 12 cm, 16 cm and 20 cm is
(a) 15 cm (b) 10 cm (c) 18 cm (d) 16 cm
[SSC Tier-I 20...

7. If D is the mid-point of the side BC of ΔABC and the area of ΔABD is 16 cm², then the area of ΔABC is
(a) 16 cm² (b) 24 cm² (c) 32 cm² (d) 48 cm²
[SSC Tier-I 2012]

8. ABC is a triangle. The medians CD and BE intersect each other at O. Then $\Delta ODE : \Delta ABC$ is
(a) 1 : 3 (b) 1 : 4 (c) 1 : 6 (d) 1 : 12
[SSC Tier-I 2012]

9. AB is a diameter of the circumcircle of ΔAPB ; N is the foot of the perpendicular drawn from the point P on AB. If $AP = 8$ cm and $BP = 6$ cm, then the length of BN is
(a) 3·6 cm (b) 3 cm (c) 3·4 cm (d) 3·5 cm
[SSC Tier-I 2012]

10. The bisector of $\angle A$ of ΔABC cuts BC at D and the circumcircle of the triangle at E. Then
(a) $AB : AC = BD : DC$
(b) $AD : AC = AE : AB$
(c) $AB : AD = AC : AE$
(d) $AB : AD = AE : AC$
[SSC Tier-I 2012]

11. O is the centre of the circle passing through the points A, B and C such that $\angle BAO = 30°$, $\angle BCO = 40°$ and $\angle AOC = x°$. What is the value of x?
(a) 70° (b) 140° (c) 210° (d) 280°
[SSC Tier-I 2012]

12. In an obtuse angled triangle ABC, $\angle A$ is the obtuse angle and O is the orthocentre. If $\angle BOC = 54°$, then $\angle BAC$ is
(a) 108° (b) 126° (c) 136° (d) 116°
[SSC Tier-I 2012]

13. Let BE and CF be the two medians of a ΔABC and G be their intersection. Also let EF cut AG at O. Then $AO : OG$ is

(a) $1:1$ (b) $1:2$ (c) $2:1$ (d) $3:1$

[SSC Tier-I 2012]

14. If S is the circumcentre of ΔABC and $\angle A = 50°$, then the value of $\angle BCS$ is

(a) $20°$ (b) $40°$ (c) $60°$ (d) $80°$

[SSC Tier-I 2012]

15. If I is the in-centre of ΔABC and $\angle A = 60°$, then the value of $\angle BIC$ is

(a) $100°$ (b) $120°$ (c) $150°$ (d) $110°$

[SSC Tier-I 2012]

16. O is the circum centre of the triangle ABC with circumradius 13 cm. Let $BC = 24$ cm and OD is perpendicular to BC. Then the length of OD is

(a) 3 cm (b) 4 cm (c) 5 cm (d) 7 cm

[SSC Tier-I 2012]

17. If G is the centroid of ΔABC and $AG = BC$ then $\angle BGC$ is

(a) $45°$ (b) $90°$ (c) $60°$ (d) $75°$

[SSC Tier-I 2012]

18. The three medians AD, BE and CF of ΔABC intersect at point G. If the area of ΔABC is 60 sq. cm then the area of the quadrilateral $BDGF$ is

(a) 15 sq. cm (b) 20 sq. cm (c) 30 sq. cm (d) 10 sq. cm

[SSC Tier-I 2012]

19. In a ΔABC, $\angle B = 90°$, $\angle C = 45°$ and D is mid point of AC. If $AC = 4\sqrt{2}$ unit then BD is

(a) $\frac{5}{2}$ unit (b) 2 unit (c) $2\sqrt{2}$ unit (d) $4\sqrt{2}$ unit

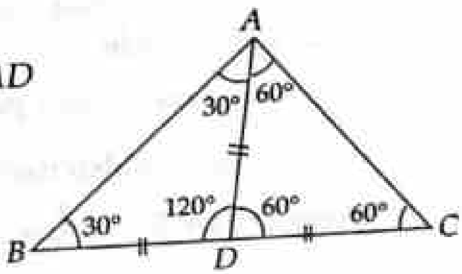
[SSC Tier-I 2012]

Answers–6B

1. (b)	2. (a)	3. (b)	4. (b)	5. (d)	6. (b)	7. (c)	8. (d)
9. (a)	10. (a)	11. (b)	12. (b)	13. (d)	14. (b)	15. (b)	16. (c)
17. (b)	18. (b)	19. (c)					

Exaplanation

1. (b) $AD = \frac{1}{2}BC \Rightarrow AD = CD = BD$



In ΔABD, $AD = BD \Rightarrow \angle ABD = \angle BAD$

or, $\angle ABD = 30°$

$\therefore \angle ADB = 180° - 30° - 30° = 120°$

And $\angle ADC = 180° - 120° = 60°$

But, $AD = CD \Rightarrow \angle ACD = \angle DAC$

$\Rightarrow \angle ACD + \angle DAC = 180° - 60° = 120°$

$\angle ACD = \frac{120°}{2} = 60° = \angle ACB$

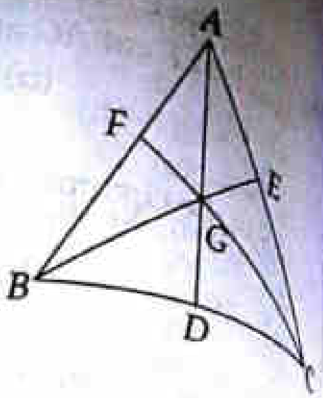
2. (a) If G be the centroid of the triangle then

area of $\Delta BGC = \frac{1}{3}$ (area of ΔABC)

$= \frac{1}{3} \times 72 = 24 \text{ cm}^2$

$\therefore$ area of $\Delta BGD = \frac{1}{2} \times$ area of ΔBGC

$= \frac{1}{2} \times 24 \text{ cm}^2 = 12 \text{ cm}^2$



3. (b) $AG = \frac{2}{3} \times AD = \frac{2}{3} \times 12 = 8$ cm

4. (b) In figure AD, BE and CF are altitudes

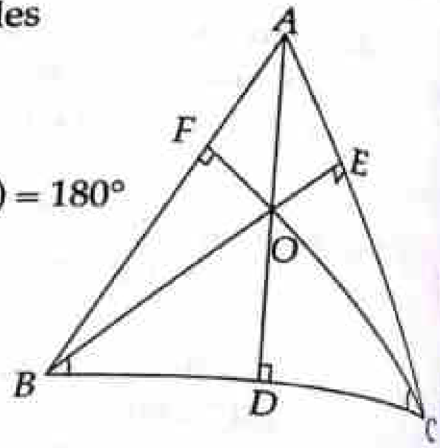
In ΔBFC, $\angle BCF = 90° - B$

In ΔBEC, $\angle CBE = 90° - C$

$\therefore$ In ΔBOC, $120° + (90° - B) + (90° - C) = 180°$

or, $B + C = 120°$

$\therefore$ $\angle A = 180° - 120° = 60°$



5. (d) $\angle ABC = 180° - 80° - 85° = 15°$

$\therefore$ $\angle AOC = 2 \times \angle ABC = 30°$

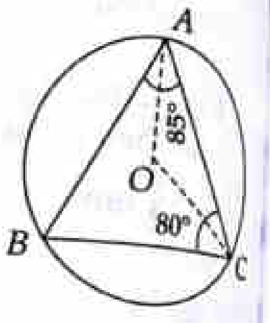
In ΔOAC,

Let $\angle OAC = \theta$

$\therefore$ $\angle OCA = \theta$ $\quad$ ($\because$ $OA = OC$ = radius)

$\therefore$ $\theta + \theta + 30° = 180°$

$\Rightarrow$ $\theta = 75°$



6. (b) $\because$ $12^2 + 16^2 = 20^2$

$\therefore$ This is a right angled triangle. The diameter of the circumcircle of the triangle is hypotenus of the triangle (Recall that angle of semicircle is right angle)

$\therefore$ Circumradius $= \frac{\text{hypotenuse}}{2} = \frac{20}{2} = 10$ cm

7. (c) Area of $\Delta ABC = 2 \times$ Area of $\Delta ABD = 2 \times 16 = 32 \text{ cm}^2$

(Recall that medians divides triangle into two equal parts)

8. (d) Area of triangle

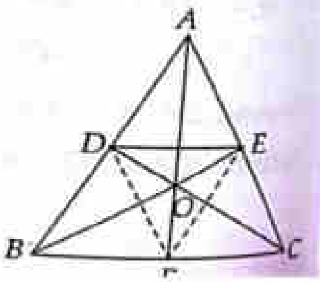
DEF formed by mid points D, E, $F = \frac{1}{4} \times$ (Area of ΔABC)

Centroid O is also the centroid of ΔDEF.

$\therefore$ Area of $\Delta DOE = \frac{1}{3} \times$ (Area of ΔDEF)

$= \frac{1}{3} \times \left(\frac{1}{4} \text{Area of } \Delta ABC\right)$

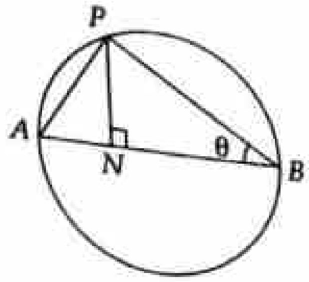
$= \frac{1}{12} \times$ Area of ΔABC



9. (a) Since AB is a diameter of circumcircle of ΔAPB, therefore $\angle APB = 90°$.
Thus triangle is right angled.

$\therefore AB = \sqrt{6^2 + 8^2} = 10$

Area of the triangle $\frac{1}{2} \times AP \times BP = \frac{1}{2} \times PN \times AB$



or, $\frac{1}{2} \times 8 \times 6 = \frac{1}{2} \times PN \times 10$

or, $PN = \frac{48}{10} = \frac{24}{5}$

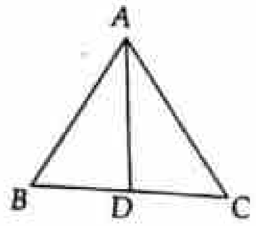
$\therefore BN = \sqrt{PB^2 - (PN)^2}$

$= \sqrt{6^2 - \left(\frac{24}{5}\right)^2} = \frac{\sqrt{30^2 - 24^2}}{5} = \frac{18}{5} = 3{\cdot}6$

10. (a) If AD is bisector of $\angle A$

then $\frac{AB}{AC} = \frac{BD}{DC}$

(It is very important property, learn it)



11. (b) $\because OA = OB =$ radius of circle

$\therefore \angle OBA = \angle OAB = 30°$

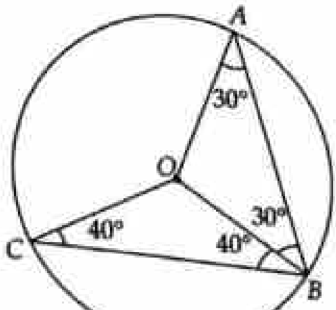
and $\angle AOB = 180° - 30° - 30° = 120°$

Similarly in ΔOBC,

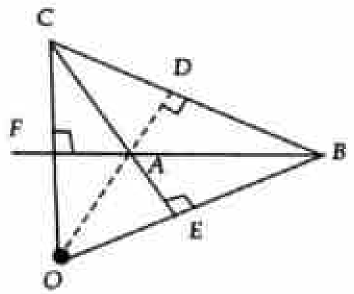
$\angle BOC = 180° - 40° - 40° = 100°$

From figure, $\angle AOC = 360° - \angle AOB - \angle BOC$

$= 360° - 120° - 100° = 140°$



12. (b) In the given figure ABC is an obtused angle triangle. $AD \perp BC$, $CF \perp BA$ (on produced part) and $BE \perp CA$ (on produced part). Altitudes AD, CF and BE, intersect at point O.



Concentrate on Quadrilateral $AFOE$,

Here, $\angle AFO = 90°$, $\angle AEO = 90°$

and $\angle EOF = \angle BOC = 54°$

$\therefore \angle FAE = 360° - 90° - 90° - 54° = 126°$

From vertically opposite angle, $\angle BAC = \angle FAE = 126°$

13. (d) $\Delta AOE \sim \Delta ADC$ $\quad (\because FE \parallel BC \Rightarrow \angle AEO = \angle ACD)$

$\therefore \frac{AO}{AD} = \frac{AE}{AC} = \frac{1}{2}$ $\quad (\because E$ is mid point of $AC)$

But $\frac{AG}{AD} = \frac{2}{3}$

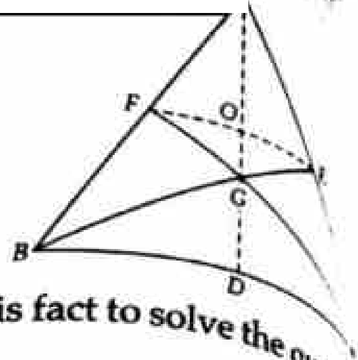
$\because \quad AD \wedge AG = 2 \wedge 2 = 4$

$\Rightarrow \quad 4AO = 3AG$

$\Rightarrow \quad 4AO = 3(AO + OG)$

$\Rightarrow \quad AO = 3OG$

$\therefore \quad \dfrac{AO}{OG} = \dfrac{3}{1}$



[Shortcut : O is mid point of AD. Take help of this fact to solve the ques...

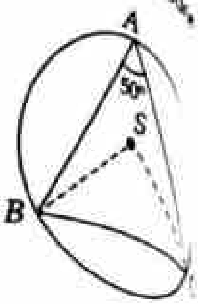
14. (b) $\angle BSC = 2 \times 50^\circ = 100^\circ$

$\because \quad \angle SBC = \angle SCB$

$\therefore \quad$ In ΔBSE

$100^\circ + 2\angle BCS = 180^\circ$

$\Rightarrow \quad \angle BCS = \dfrac{80^\circ}{2} = 40^\circ$

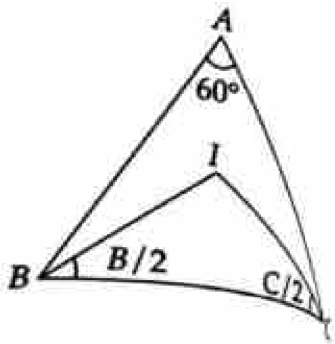


15. (b) $\angle BIC = 180^\circ - \dfrac{B}{2} - \dfrac{C}{2}$

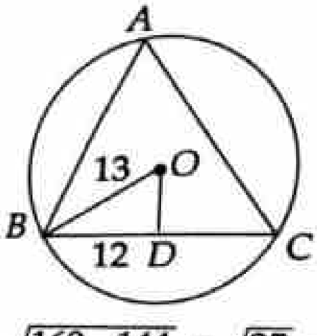
$= 180^\circ - \left(\dfrac{B+C}{2}\right) = 180^\circ - \left(\dfrac{180^\circ - A}{2}\right)$

$= 90^\circ + \dfrac{A}{2}$ (shortcut, learn it direct)

$= 90^\circ + \dfrac{60^\circ}{2} = 120^\circ$



16. (c) See the figure



$$OD = \sqrt{OB^2 - BD^2} = \sqrt{169 - 144} = \sqrt{25} = 5$$

17. (b) $\because GD = \dfrac{1}{2} AG$

$\therefore GD = \dfrac{1}{2} BC$ $(\because AG = BC)$

or, $GD = CD$ and $GD = BD$

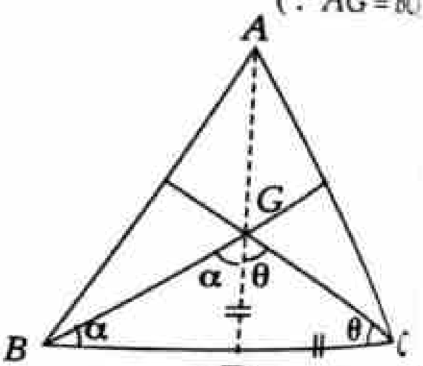
$\therefore \quad \angle DGC = \angle DCG = \theta$ (See the figure)

$\angle DBG = \angle BGD = \alpha$ (See the figure)

In ΔBGC,

$\alpha + \alpha + \theta + \theta = 180$

or, $\alpha + \theta = 90^\circ = \angle BGC$

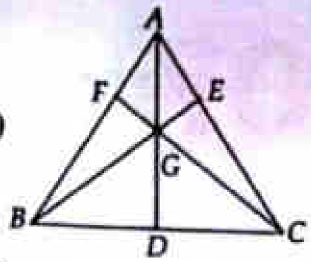


18. (b) area of $BDGF$

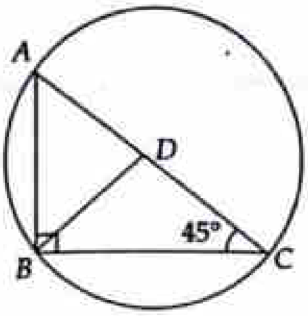
$= \text{Area of } (\Delta BDG) + \text{Area of } (\Delta BGF)$

$= \frac{1}{2} \text{Area of } (\Delta BGC) + \text{Area of } (\Delta ABG)$

$= \frac{1}{2}\left(\frac{1}{3}\times 60 + \frac{1}{3}\times 60\right) = 20 \text{ cm}^2.$



19. (c) ΔABC lies on the semicircle whose centre is D.



$\therefore \; BD = CD = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

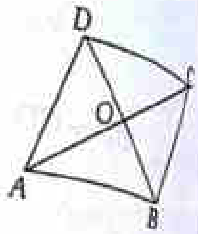
★★★

07

Quadrilateral

1. Types of Quadrilateral and their properties

1.1 Parallelogram : If opposite sides of a quadrilateral are parallel, it is called a parallelogram. Its opposite sides are also equal in length and its diagonals bisect each other. In the adjacent figure ABCD is a parallelogram where $AB \parallel DC$ and $AD \parallel BC$, Hence



1.1.1. $AB = CD$ and $AD = BC$

1.1.2. $AO = OC$ and $BO = OD$

1.1.3. $\angle A + \angle D = 180°$, $\angle B + \angle C = 180°$ etc.

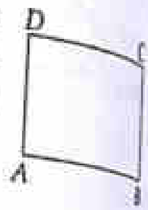
1.1.4. $\Delta AOB \cong \Delta COD$ and $\Delta AOD \cong \Delta COB$

1.1.5. $\Delta ABC \cong \Delta CDA$

1.1.6. area of ΔAOB, ΔBOC, ΔCOD and ΔAOD are equal.

1.2. Rectangle : A parallelogram is called a rectangle if its all angles are 90°. Hence every rectangle is a parallelogram but every parallelogram is not a rectangle.

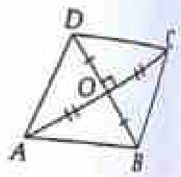
If diagonals of a parallelogram are equal (i.e. $AC = BD$) then it is a rectangle.



1.3. Rhombus : If all the sides of a parallelogram are equal it is a rhombus. Diagonals of a rhombus bisect each other at right angle. i.e.

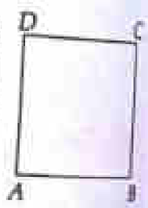
(a) $AO = OC$ and $OB = OD$

(b) $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

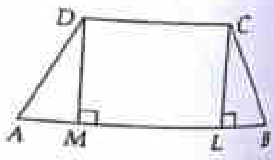


1.4. Square : If all the sides of a parallelogram are equal and all its angle are 90° then it is a square, or if all sides of a rectangle are equal then it is a square, or it all the angles of a rhombus are 90° then it is a square.

So every square is a rhombus but every rhombus is not a square.



1.5. Trapezium : If two sides of a quadrilateral are parallel and other two sides are non parallel then if is called a trapezium. In a trapezium two altitudes DM and CL (see figure) are equal.

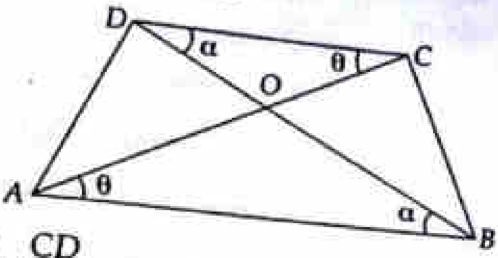


1.6. If midpoints of sides of a quadrilateral are joined, it is a parallelogram.

2. Important properties of a trapezium :

2.1. Diagonals of a trapezium intersect each other in the same ratio. Thus in trapezium $ABCD$, where $AB \parallel CD$, $\frac{DO}{OB} = \frac{CO}{OA}$

Explanation : See the figure



$\because$ $AB \parallel DC$

$\therefore$ $\angle CAB = \angle ACD = \theta$

and $\angle DBA = \angle BDC = \alpha$

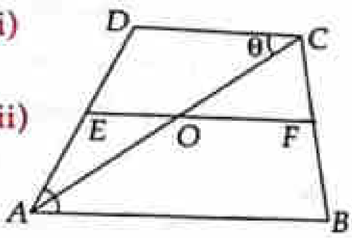
$\therefore$ $\Delta OCD \sim \Delta OAB \Rightarrow \frac{OC}{OA} = \frac{OD}{OB} = \frac{CD}{AB}$

2.2. If diagonal of a quadrilateral intersect each other in the same ratio, then at least on opposite pair of the quadrilateral are parallel and hence it is a trapezium

Explanation : This statement is converse of above statement. If the ratio is 1 : 1, it is a parallelogram

2.3. If a line is drawn parallel to parallel sides of a trapezium, it intersects the non parallel sides in same ratio. Thus in a trapezium $ABCD$ with $AB \parallel CD$, if EF is drawn parallel to AB and CD then $\frac{DE}{EA} = \frac{CF}{FB}$

Explanation : Join A-C



$\Delta AOE \sim \Delta ACD \Rightarrow \frac{AO}{OC} = \frac{AE}{ED} = \frac{OE}{CD}$ ... (i)

$\Delta ACF \sim \Delta OCF \Rightarrow \frac{OC}{CA} = \frac{CF}{CB} = \frac{OF}{AB}$ ... (ii)

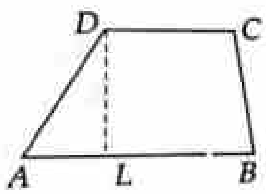
dividing (i) by (ii)

$\frac{AO}{OC} = \frac{AE}{ED}$ or, $\frac{OC}{OA} = \frac{CF}{FB}$

or, $\frac{AE}{ED} = \frac{FB}{CF}$ (It can also be proved by Thale's Theorem)

2.4. Area of trapezium $ABCD = \frac{1}{2}(AB + DC) \times DL$

Where DL is the distance between parallel lines AB and CD



3. Important properties of a rhombus :

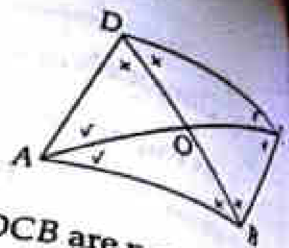
Suppose $ABCD$ is a rhombus whose diagonals AC and BD intersect at O, then

3.1. $AO = OC$ and $BO = OD$

3.2. $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

3.3. diagonal AC is bisector of $\angle A$ and $\angle C$

3.4. diagonal BD is bisector of $\angle B$ and $\angle D$



3.5. $\Delta OAB \cong \Delta OCB \cong \Delta OCD \cong \Delta OAD$

(It must be noted that in a parallelogram $ABCD$ when $AB \neq BC$, diagonal AC doesnot bisect $\angle A$ and $\angle C$ and hence ΔOAB and ΔOCB are not congruent)

3.6. ar (ΔOAB) = ar (ΔOCB) = ar (ΔOCD) = ar (ΔOAD)

(These areas are also equal when $ABCD$ is a parallelogram)

3.7. $\because$ Area of rhombus $ABCD = \frac{1}{2} \times BD \times AC = \frac{1}{2} \cdot d_1 d_2$

(where $d_1 = BD$ and $d_2 = AC$ are diagonals)

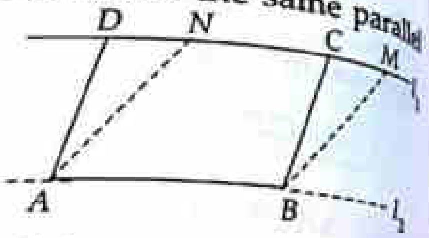
$\therefore$ Area of $\Delta OAB = \frac{1}{4} \cdot \frac{1}{2} \cdot d_1 d_2 = \frac{1}{8} d_1 d_2$

3.8. Sum of squares of sides of a rhombus = Sum of squares of its diagonals.

i.e. $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$ (see solved example 11)

4. Some important propertics about Areas of a parallelogram

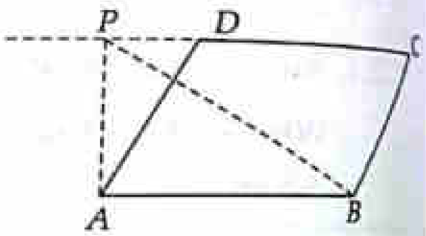
4.1. Parallelograms on the same base and between the same parallel lines are equal in area.



In the adjacent figure l_1 and l_2 are two parallel lines. AB is a base taken on line l_2. Points M, C, N, D lie on line l_1 such that $ABCD$ and $ABMN$ are parallelogram.

Thus we have, area ($\square ABCD$) = area ($\square ABMN$) as their base and height are same.

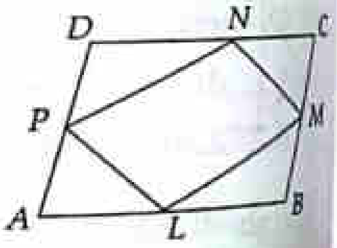
4.2. The area of a triangle is half the area of parallelogram having the same base and between the same parallel lines.



In the figure ΔAPB and parallelogram $ABCD$ have same base and same parallel lines AB and CD (or CP). Thus

Area of $\Delta ABP = \frac{1}{2} \times$ area of parallelogram $ABCD$.

4.3. If a parallelogram is formed by joining midpoints of sides of a given parallelogram then its area is half the area of the original parallelogram.

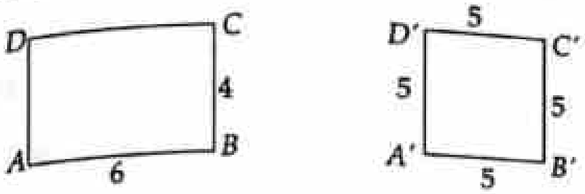


In the adjacent figure L, M, N, P are

respectively mid points of sides AB, BC, CD and DA then

Area of parallelogram $LMNP = \frac{1}{2}$ (Area of parallelogram $ABCD$)

4.4. If a square and a rectangle have equal perimeter then area of the square is greater than area of the rectangle. For example, in the figure given, below.



$ABCD$ is a rectangle whose unequal sides are 6 cm and 4 cm and $A'B'C'D'$ is a square whose each side is 5 cm.

Perimeter of rectangle $ABCD = 2(6 + 4) = 20$ cm

Perimeter of square $A'B'C'D' = 4 \times 5 = 20$ cm

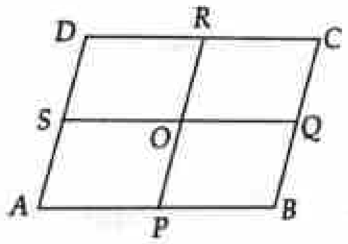
Area of rectangle $ABCD = 6 \times 4 = 24$ cm^2

Area of square $ABCD = 5 \times 5 = 25$ cm^2

Clearly area of square > area of rectangle. It can be verified for other rectangles of the same perimeter.

4.5. If a square and a rectangle have equal area then perimeter of square is less than perimeter of rectangles

4.6. Lines joining midpoints of opposite sides of a parallelogram divide the parallelogram in four equal Areas.



In the figure P, Q, R, S are respectively midpoint of sides AB, BC, CD and DA of a parallelogram $ABCD$. Thus we have

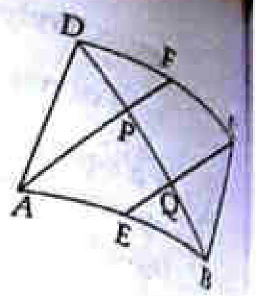
ar ($\square APOS$) = ar ($\square PBQO$) = ar ($\square OQCR$) = ar ($\square ORDS$)

Types of quadrilateral formed by joining midpoints of a given quadrilateral.

5.1. When midpoints of sides of a quadrilateral taken in order are joined, a parallelogram is formed. For different types of quadrilateral, the shape of resultant quadrilateral will be as follows.

Original quadrilateral	Quadrilateral formed by joining midpoints of sides
Parallelogram	Parallelogram
Rectangle	Rhombus
Rhombus	Rectangle
Square	Square
Trapezium	Parallelogram

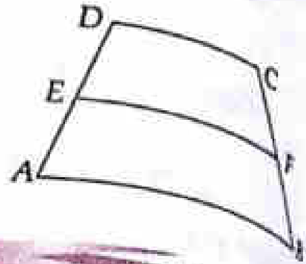
5.2. Suppose $ABCD$ is a parallelogram. E and F are respectively midpoints of sides AB and CD. If BD intersect diagonal AF and EC respectively at P and Q then



(i) $DP = PQ = QB$ (ii) $AF \parallel EC$

(iii) $\Delta ADF \cong \Delta CBE$

6. If $ABCD$ is a trapezium with $AB \parallel DC$ and E and F are respectively midpoint of AD and BC then,



$$EF \parallel DC \parallel AB \text{ and } EF = \frac{1}{2}(AB + DC)$$

Solved Examples

1. If angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13 then find all the angles of the quadrilateral.

Solution : Let angles be $3x, 5x, 9x$ and $13x$

$\because$ Sum of angles of a quadrilateral is $360°$

$\therefore$ $3x + 5x + 9x + 13x = 360°$

or, $30x = 360°$ or, $x = \frac{360°}{30} = 12°$

Hence angles are $3x = 3 \times 12° = 36°$

$5x = 5 \times 12° = 60°$

$9x = 9 \times 12° = 108°$

and $13x = 13 \times 12° = 156°$

2. One of the angle of a parallelogram is $\frac{4}{5}$ of its adjacent angle. Find the measure of both angles.

Solution : Let $\angle A$ and $\angle B$ be adjacent angles of a parallelogram $ABCD$.

According to question $\angle A = \frac{4}{5}\angle B$ ...(i)

$\because$ Sum of adjacent angles of a parallelogram is $180°$

$\therefore$ $\angle A + \angle B = 180°$

or, $\frac{4}{5}\angle B + \angle B = 180°$ (from (i))

or, $\frac{9}{5}\angle B = 180°$

or, $\angle B = \frac{5 \times 180°}{9} = 100°$ $\therefore$ $\angle A = \frac{4}{5} \times 100° = 80°$

Hence two angles are respectively $100°$ and $80°$.

The diagonals AC and BD of a parallelogram intersects at O. If $\angle OAD = 40^\circ$, $\angle OAB = 20^\circ$ and $\angle COD = 75^\circ$, then evaluate the following,

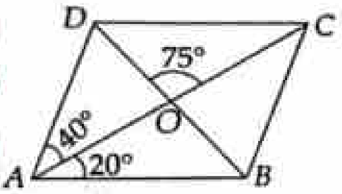
(i) $\angle ABD$ (ii) $\angle BDC$ (iii) $\angle ACB$ (iv) $\angle DBC$ (v) $\angle ADC$

Solution : As per question $ABCD$ is a parallelogram with $\angle COD = 75^\circ$, $\angle OAD = 40^\circ$ and $\angle OAB = 20^\circ$

(i) $\therefore$ $\angle ABD = 180^\circ - (\angle OAB + \angle AOB) = 180^\circ - (20^\circ + 75^\circ)$

($\because \angle OAB = 20^\circ$ and $\angle COD = \angle AOB = 75^\circ$ vertically opposite angle)

$= 180^\circ - 95^\circ = 85^\circ$



(ii) $\angle BDC = \angle ABD = 85^\circ$ *(alternate angle)*

(iii) $\angle ACB = \angle CAD = 40^\circ$ *(alternate angle)*

(iv) $\because \angle DAB + \angle ABC = 180^\circ$ *(adjacent angle)*

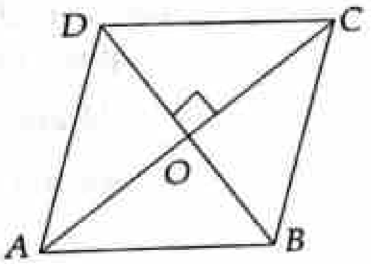
or, $40^\circ + 20^\circ + 85^\circ + \angle DBC = 180^\circ$

or, $145^\circ + \angle DBC = 180^\circ$

$\therefore$ $\angle DBC = 180^\circ - 145^\circ = 35^\circ$

(v) $\angle ADC = 180^\circ - \angle DAB = 180^\circ - 60^\circ = 120^\circ$ *(adjacent angle)*

4. If length of each side of a rhombus is 5 cm and length of one of its diagonal is 8 cm, find the length of other diagonal.



Solution : Let $ABCD$ is a rhombus with

$AB = BC = CD = DA = 5$ cm

and $AC = 8$ cm

$\therefore$ $OC = \frac{1}{2} AC = 4$ cm

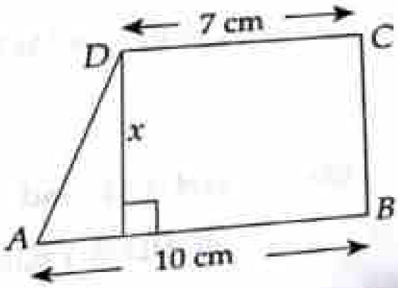
Now, $OD = \sqrt{CD^2 - OC^2} = \sqrt{5^2 - 4^2}$ ($\because$ ΔOCD is right angle)

$= \sqrt{25 - 16}$

$= \sqrt{9} = 3$

$\therefore$ $BD = 2 \cdot OD = 2 \times 3 = 6$ cm.

5. $ABCD$ is a trapezium with $AB \parallel CD$. If $AB = 10$ cm, $CD = 7$ cm and area of trapezium $= 102$ cm^2, then find the height of trapezium.



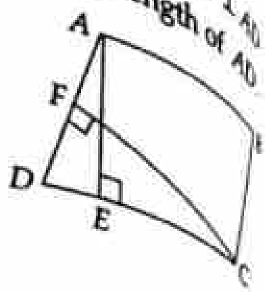
Solution : ar (trapezium $ABCD$)

$= \frac{1}{2}(10 + 7) \times x$, where x is its height

$\therefore$ $\frac{1}{2} \times 17 \times x = 102$, (given)

$\therefore$ $x = \frac{102 \times 2}{17} = 12$ cm $=$ height

6. In the given figure $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm then find the length of AD.



Solution : Since, $ABCD$ is a parallelogram

$\therefore \quad AB = CD \qquad$ or, $CD = 16$ cm

Now, ar $(\square ABCD) = (CD) \times (AE)$

$= 16 \times 8 \text{ cm}^2$

$= 128 \text{ cm}^2$ ...(i)

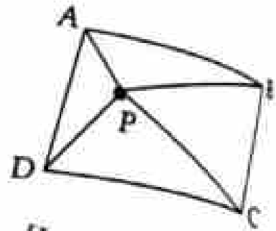
Again, ar $(\square ABCD) = (AD) \times (CF)$

$= (AD) \times 10$ cm ...(ii)

$\therefore$ from (i) and (ii) $AD \times 10 = 128$

$\therefore \quad AD = \frac{128}{10} = 12.8$ cm

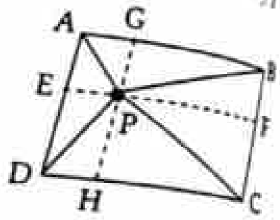
7. In the adjacent figure P is a point inside the parallelogram $ABCD$. Prove that



(i) ar (APB) + ar $(PCD) = \frac{1}{2}$ ar $(ABCD)$

(ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD) [Learn the property]

Solution : Given that $ABCD$ is a parallelogram and P is a point inside it.



Draw : $EF \parallel AB$ and $GH \parallel AD$

(i) $\because \; EF \parallel AB$ and $AE \parallel BF \qquad (\because \; AD \parallel BC)$

$\therefore \quad AEFB$ is a parallelogram

Now, ar $(\Delta APB) = \frac{1}{2}(AB) \times (\text{height}) = \frac{1}{2}$ ar $(\square AEFB)$

$(\because$ ar $\square AEFB = AB) \times$ height) ...(i)

(Here base and height of $\square AEFB$ and ΔAPB are same)

Again ar $(\Delta DPC) = \frac{1}{2}(DC) \times (\text{height})$

$= \frac{1}{2}$ ar $(\square DEFC)$ ...(ii)

adding (i) and (ii)

ar (ΔAPB) + ar $(\Delta DPC) = \frac{1}{2}$ {ar $(\square AEFB)$ + ar $(\square DEFC)$}

$= \frac{1}{2}$ ar $(\square ABCD)$; Proved

(ii) $\because \quad GH \parallel AD$ and $AG \parallel DH \qquad (\because \; AB \parallel DC)$

$\therefore \quad AGHD$ is a parallelogram

Now ... (height)

$= \frac{1}{2}$ (□AGHD)

and ar $(\Delta BPC) = \frac{1}{2}(BC) \times$ (height) ... (iii)

$= \frac{1}{2}$ (□BGHC)

adding (iii) and (iv) ... (iv)

ar (ΔAPD) + ar $(\Delta BPC) = \frac{1}{2}$ {ar (□AGHD) + ar (□BGHC)}

$= \frac{1}{2}$ ar (□ABCD)

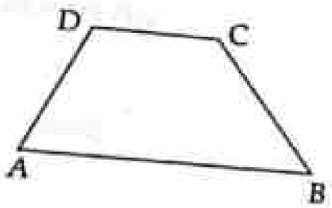
$\therefore$ from (i) ar (ΔAPD) + ar (ΔBPC) = ar (ΔAPB) + ar (ΔPCD); Proved

8. In the given figure $ABCD$ is a trapezium with $AB \parallel DC$ and $AD = BC$ prove that

(i) $\angle A = \angle B$ (ii) $\angle C = \angle D$

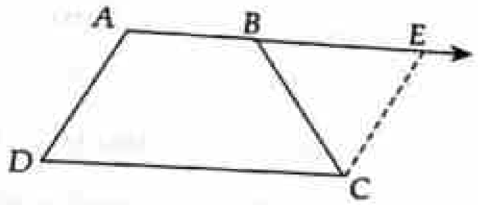
(iii) $\Delta ABC \cong \Delta BAD$

(iv) diagonal AC = diagonal BD



Solution : Given that $ABCD$ is a trapezium with $AB \parallel DC$ and $AD = BC$

Produce : AB to E and draw $CE \parallel AD$



(i) $\because AB \parallel DC$ (given)

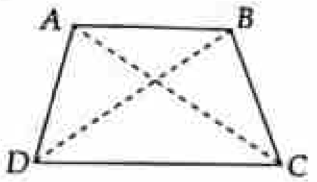
and $CE \parallel AD$ (by construction)

Hence, $AECD$ is a parallelogram

$\therefore AD = CE$

or, $BC = CE$ ($\because BC = AD$ given)

$\therefore \angle CBE = \angle CEB$ ... (i)



Now, $\angle A + \angle CEB = 180°$

or, $\angle A = 180° - \angle CEB$

or, $\angle A = 180° - \angle CBE$ ($\because$ from (i) $\angle CBE = \angle CEB$)

or, $\angle A = \angle ABC$ (linear pair of angles)

or, $\angle A = \angle B$ ... (ii)

(ii) $\because AB \parallel DC$ and AD is a transverse line

$\therefore \angle A + \angle D = 180°$ ... (iii)

$\because AB \parallel DC$ and BC is a transverse line

$\therefore \angle B + \angle C = 180°$ ... (iv)

Hence, $\angle A + \angle D = \angle B + \angle C$

or, $\angle C = \angle D$ ($\because$ $\angle A = \angle B$)

(iii) In ΔABC and ΔBAD

$AB = AB$ (common)

$AD = BC$ (given)

and $\angle A = \angle B$ (already proved)

$\therefore$ From S-A-S, $\Delta ABC \cong \Delta BAD$

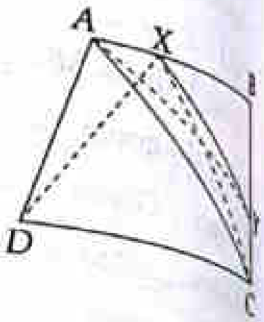
(iv) $\because$ $\Delta ABC \cong \Delta BAD$ $\therefore$ $AC = BD$

($\because$ corresponding part of congruent triangle are equal); Proved

9. $ABCD$ is a trapezium with $AB \parallel DC$. A line parallel to AC cuts the line AB at X and BC at Y. Prove that ar (ADX) = ar (ACY)

Solution : Given situation is shown in the adjacent figure

To prove : ar (ΔADX) = ar (ΔACY)



Join C and X.

$\because$ $AC \parallel XY$ ((given)

$\therefore$ (ΔACX) = ar (ΔACY) ... (i)

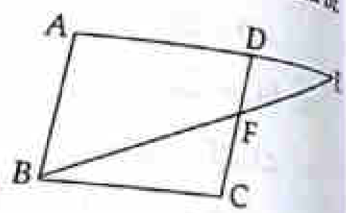
Again, $AB \parallel CD$ (given)

$\therefore$ ar (ΔACX) = ar (ΔADX) ... (ii)

from equation (i) and (ii), ar (ΔADX) = ar (ΔACY); Proved.

10. E is a point on the produced part AD of parallelogram $ABCD$ and BE intersects side CD at F.

Prove that $\Delta ABE \sim \Delta CFB$



Solution : In ΔABE and ΔCFB

$\angle AEB = \angle CBF$ (alternate angle)

and $\angle A = \angle C$ (opposite angle of parallelogram)

$\therefore$ from A-A criterion of similar triangle $\Delta ABE \sim \Delta CFB$.

11. From a point P inside the triangle ABC, perpendiculars PQ, PR and PS are respectively drawn to sides BC, CA and AB.

Prove that $AS^2 + BQ^2 + CR^2 = BS^2 + CQ^2 + AR^2$

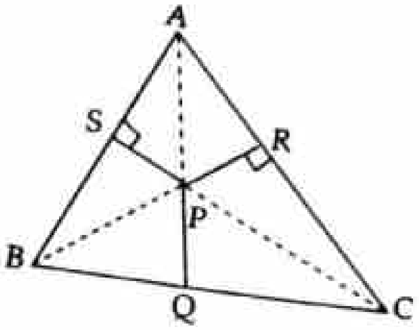
Solution : See the figure, $PQ \perp BC$, $PR \perp CA$ and $PS \perp AB$, Join PA, PB and PC

In right angled triangle PQB and PQC

$PB^2 = PQ^2 + QB^2$

and $PC^2 = PQ^2 - QC^2$

$\therefore \quad PB^2 - PC^2 = QB^2 - QC^2 \qquad \ldots \text{(i)}$



Similarly in right angled ΔPRC and ΔPRA

$PC^2 - PA^2 = CR^2 - AR^2 \qquad \ldots \text{(ii)}$

And in right angled ΔPSA and ΔPSB

$PA^2 - PB^2 = AS^2 - SB^2 \qquad \ldots \text{(iii)}$

Adding equation (i), (ii) and (iii)

$$QB^2 - QC^2 + CR^2 - AR^2 + AS^2 - SB^2 = 0$$

or, $AS^2 + BQ^2 + CR^2 = BS^2 + CQ^2 + AR^2$

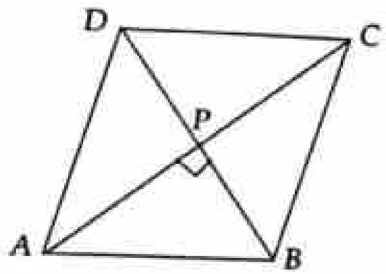
2. In a rhombus prove that sum of squares of sides is equal to sum of square of diagonals or, In a rhombus $ABCD$ prove that

$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

Solution : We know that diagonals of a rhombus bisect each other at right angle.

Let diagonals AC and BD of a rhombus $ABCD$ intersect at P, then

$$\angle APB = \angle BPC = \angle CPD = \angle DPA = 90°$$



and $\quad AP = PC = \dfrac{AC}{2}$

$BP = PD = \dfrac{BD}{2}$

In right angled triangle APB

$$AB^2 = AP^2 + BP^2 = \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2$$

or, $AB^2 = \dfrac{BD^2}{4} + \dfrac{AC^2}{4} \qquad \ldots \text{(i)}$

In right angled triangle BPC,

$BC^2 = BP^2 + PC^2 = \frac{1}{4} BD^2 + \frac{1}{4} AC^2 \qquad \ldots \text{(ii)}$

In right angled triangle CPD

$CD^2 = PD^2 + PC^2 = \frac{1}{4} BD^2 + \frac{1}{4} AC^2 \qquad \ldots \text{(iii)}$

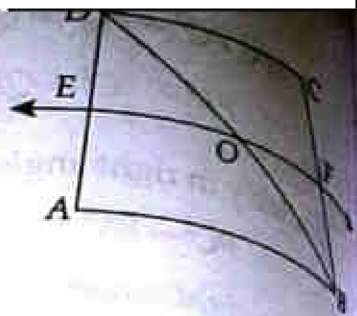
In right angled triangle ΔAPD

$DA^2 = DP^2 + AP^2 = \dfrac{BD^2}{4} + \dfrac{AC^2}{4} \qquad \ldots \text{(iv)}$

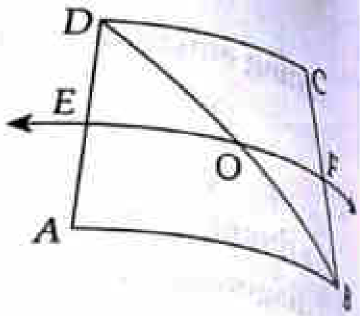
Adding (i), (ii), (iii) and (iv)

$$AB^2 + BC^2 + CD^2 + DA^2 = 4\left(\frac{1}{4} BD^2 + \frac{1}{4} AC^2\right) = BD^2 + AC^2$$

13. $ABCD$ is a trapezium with $AB \parallel DC$. E is midpoint of side AD. From point E a line is drawn parallel to AB that intersects BC at F. Show that F is midpoint of BC.



Solution : Let EF intersects diagonal BD at O



$\therefore$ In ΔADB

$OE \parallel AB$

Thus O is the midpoint of BD

Now $AB \parallel DC$ (given)

and $AB \parallel EF$ (as given in question)

$\therefore$ $DC \parallel EF$

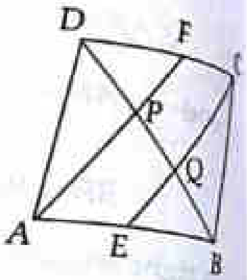
or, $DC \parallel OF$ ...(i)

In ΔDBC,

DC is parallel to OF and O is midpoint of BD.

Hence F is midpoint of BC

14. In a parallelogram $ABCD$, points E and F are respectively midpoint of sides AB and CD (see the adjacent figure). Prove that line segment AF and EC trisect diagonal BD. *[Learn the property]*



Solution : In ΔADF and ΔCBE

$AD = BC$ (parallel sides of parallelogram

$\frac{1}{2}DC = \frac{1}{2}AB$ (parallel sides of parallelogram

or, $DF = BE$

and $\angle ADF = \angle CBE$ (opposite angles of parallelogram

Hence, $\Delta ADF \cong \Delta CBE$ (from S-A-S

or, $\angle ADP = \angle CBQ$ (alternate angle) ...(i

and $\angle DAP = \angle BCQ$ (from CPCT) ...(ii

Now in ΔDAP and ΔBCQ

$AD = BC$, (opposite sides of a parallelogram

$\angle ADP = \angle CBQ$ (from (i

$\angle DAP = \angle BCQ$ (from (ii

$\therefore$ $\Delta DAP \cong \Delta BCQ$

$\therefore \quad DP = BQ$ (by CPCT) ... (iii)

Again, $DC = AB$ (opposite sides of a parallelogram)

or, $\frac{1}{2} DC = \frac{1}{2} AB$ or, $FC = AE$

and $FC \parallel AE$ (given)

$\therefore \quad AFCE$ is a parallelogram

Hence, $AF \parallel CE$

or, $PF \parallel CQ$ ... (iv)

Now in ΔDCQ

$CQ \parallel PF$ (from (iii))

and F is midpoint of DC

$\therefore \quad P$ is also the midpoint of DQ

$\therefore \quad DP = PQ$ ... (v)

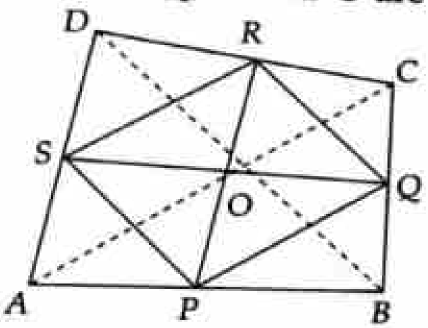
Hence from (iii) and (v)

$$DP = PQ = QB = \left(\frac{1}{3} BD\right)$$

15. Prove that lines joining the midpoints of opposite sides of a quadrilateral bisect each other.

Solution : Suppose $ABCD$ is a quadrilateral. Points P, Q, R and S are respectively midpoints of sides AB, BC, CD and AD. Let PR and QS intersect at O.

[Learn the property]



To prove : $PO = OR$ and $OS = OQ$

Join AC and BD

In ΔABC

$PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ ... (i)

Similarly in ΔADC

$SR \parallel AC$ and $SR = \frac{1}{2} AC$ ... (ii)

Hence $PQ \parallel SR$ and $SR = PQ$ *(from (i) and (ii))*

$\because$ Opposite sides of the quadrilateral are equal and parallel,

$\therefore$ $ABCD$ is a parallelogram

We know that diagonals of a parallelogram bisect each other

Hence, $PO = OR$ and $OS = OQ$; Proved.

16. In the given figure $ABCD$ is a trapezium with $AB \parallel DC$. E and F are respectively midpoint of AD and BC. Prove that, $EF = \frac{1}{2}(AB + CD)$

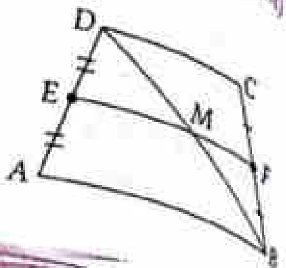
Solution : Join B and D

In ΔABD

$EM = \frac{1}{2} AB$ (since E and M are midpoints)

In ΔBCD

$MF = \frac{1}{2} DC$



adding $EM + MF = \frac{1}{2} AB + \frac{1}{2} DC$

or, $EF = \frac{1}{2}(AB + DC)$

Exercise 7A

1. $ABCD$ is a trapezium length of whose parallel sides AB and CD are respectively 10 cm and 12 cm. If midpoint of AD and BC are respectively E and F then length of EF is.

(a) 11 cm (b) more then 11 cm

(c) less than 11 cm (d) nothing can be said

2. $ABCD$ is a rectangle length of whose two consecutive sides are respectively 9 cm and 40 cm. E and F are respectively midpoints of sides AB and CD. A is joined of F and E is joined to C. They respectively intersect BD at P and Q. Length of PQ is—

(a) $\frac{31}{3}$ cm (b) $\frac{41}{3}$ cm (c) $\frac{49}{3}$ cm (d) $\frac{47}{3}$ cm

3. Length of parallel sides AB and CD of a trapezium are respectively 10 cm and 14 cm. If its diagonals AC and BD intersect at O then $AO : OC$ is

(a) $5 : 7$ (b) $12 : 7$ (c) $7 : 5$ (d) $7 : 12$

4. A line EF is drawn parallel to the parallel sides AB and CD of a trapezium $ABCD$ where E lies on AD and F lies on BC. If $AE : ED = 2 : 1$ then what is $BF : BC$?

(a) $2 : 3$ (b) $3 : 2$ (c) $2 : 1$ (d) $1 : 2$

5. If each side of a rhombus is 10 cm then what is the square root of sum of square of its diagonals ?

(a) $10\sqrt{10}$ cm (b) 20 cm (c) $10\sqrt{20}$ cm (d) $20\sqrt{10}$ cm

6. $ABCD$ is a parallelogram with base $AB = 12$ cm and height 5 cm. If E and F are respectively midpoint of AB and CD and diagonal BD intersects AF and CE respectively at P and Q then area of quadrilateral $PQCF$ is

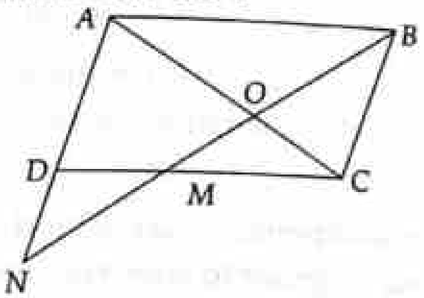
(a) 12 cm^2 (b) 18 cm^2 (c) 20 cm^2 (d) 15 cm^2

7. Select the wrong statement among following.
(a) Join of midpoints of sides of a rectangle taken in order form a rhombus
(b) Join of midpoints of sides of a rhombus taken in order form a rectangle
(c) Join of midpoints of sides of a square taken in order form a rhombus
(d) Join of midpoints of sides of trapezium form a rhombus

8. If $ABCD$ is a trapezium with $AB \parallel DC$ then which of the following is ratio of area of ΔABC and area of ΔBCD.
(a) $AB : CD$ (b) $CD : AB$ (c) $AD : BC$ (d) $BC : AD$

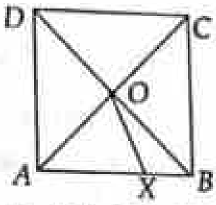
9. $ABCD$ is a trapezium with $AB \parallel DC$ whose diagonals meet at O. If $AB = 2CD$ then ratio of area of ΔAOB and ΔCOD is.
(a) $1 : 4$ (b) $4 : 1$ (c) $1 : \sqrt{2}$ (d) $\sqrt{2} : 1$

10. In the figure given below M is the midpoint of side CD of the parallelogram $ABCD$. What is $ON : OB$?



(a) $3 : 2$ (b) $2 : 1$ (c) $3 : 1$ (d) $5 : 2$

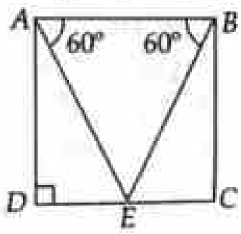
11. In the adjacent figure $ABCD$ is a square with $AO = AX$. $\angle XOB$ is equal to



(a) $22.5°$ (b) $25°$
(c) $30°$ (d) $45°$

12. The quadrilateral formed by joining midpoints of sides AB, BC, CD, DA of quadrilateral $ABCD$ is
(a) a trapezium but not a parallelogram
(b) a quadrilateral but not a trapezium
(c) a parallelogram
(d) a rhombus

13. In the adjacent figure $ABCD$ is a quadrilateral. AB, DC are parallel and AD, BC are parallel. ADC is a right angle. If perimeter of ΔABE is 6 unit, then what is the area of the quadrilateral ?



(a) $2\sqrt{3}$ sq. unit (b) 4 sq. unit
(c) 3 sq. unit (d) $4\sqrt{3}$ sq. unit

14. Suppose $LMNP$ ... to NR ... area of parallelogram is 6 times area of ΔRNP and $RP = 6$ cm, then ... is equal to

(a) 15 cm (b) 12 cm (c) 9 cm (d) 8 cm

15. If a transversal line cuts two parallel lines then bisector of internal angle formed a

(a) rectangle (b) square

(c) rhombus (d) parallelogram

16. In a parallelogram $ABCD$, M is the midpoint of BD and BM is bisector of $\angle B$. The measure of $\angle AMB$ is.

(a) 45° (b) 60° (c) 90° (d) 120°

17. The angle subtended by side of a parallelogram with pair of other parallel lines is 150°. If distance between parallel sides PQ and SR is 20 cm then what is the length of side RQ ?

(a) 40 cm (b) 50 cm (c) 60 cm (d) 70 cm

18. Side AB of a parallelogram $ABCD$ is produced to E such that $BE = AB$. If DE intersects side BC at Q then in what ratio point Q divides side BC.

(a) 1 : 2 (b) 1 : 1 (c) 2 : 3 (d) 2 : 1

19. $ABCD$ is a square. M is midpoint of side AB and N is midpoint of side BC. DM and AN are joined together to construct new sides which intersect at O. Which of the following is true ?

(a) $OA : OM = 1 : 2$ (b) $AN = MD$

(c) $\angle ADM = \angle ANB$ (d) $\angle AMD = \angle BAN$

20. In a parallelogram $ABCD$, $AB = 24$ cm and $AD = 16$ cm. Distance between sides AB and DC is 10 cm. What is the distance between sides AB and BC ?

(a) 16 cm (b) 18 cm (c) 15 cm (d) 26 cm

21. $ABCD$ is a rhombus. A straight line passing through point C meets the produced part of AD at P and produced part of AB at Q. If $DP = \frac{1}{2} AD$ then what is the ratio of length of BQ and AB ?

(a) 2 : 1 (b) 1 : 2 (c) 1 : 1 (d) 3 : 1

22. In a quadrilateral with distinct sides, if diagonals AC and BD intersects at right angle, then which of the following is true—

(a) $AB^2 + BC^2 = CD^2 + DA^2$

(b) $AB^2 + CD^2 = BC^2 + DA^2$

(c) $AB^2 + AD^2 = BC^2 + CD^2$

(d) $AB^2 + BC^2 = 2(CD^2 + DA)^2$

23. Length of diagonal BD of a parallelogram $ABCD$ is 18 cm. If P and Q are respectively centroid of their ΔABC and ΔADC, then what is the length of line segment PQ ?
(a) 4 cm (b) 6 cm (c) 9 cm (d) 12 cm

24. $ABCD$ is a cyclic trapezium in which sides AD and BC are parallel. If $\angle ABC = 72^\circ$, then what is the measure of $\angle BCD$?
(a) 162° (b) 18° (c) 108° (d) 72°

25. Ratio of $\angle A$ and $\angle B$ of a non square rhombus is $4:5$, the measure of $\angle C$ is—
(a) 50° (b) 45° (c) 80° (d) 95°

26. The external angle of a cyclic quadrilateral is 50°. What is the measure of its internal opposite angle ?
(a) 130° (b) 40° (c) 50° (d) 90°

27. $ABCD$ is a cyclic trapezium with $AD \parallel BC$. If $\angle ABC = 70^\circ$, then measure of $\angle BCD$ is—
(a) 60° (b) 70° (c) 40° (d) 80°

28. Each side of a rhombus is 10 cm, the sum of square of its diagonal is
(a) 20 cm^2 (b) 200 cm^2 (c) 400 cm^2 (d) 100 cm^2

29. In a trapezium $ABCD$, AB is parallel to CD. If E is midpoint of side AD and a line drawn from point E, parallel to the parallel sides cuts BC at F then
(a) $BF = CF$ if $AD = BC$ (b) $BF = CF$ is always true
(c) $BF : CF$ is less than 1 if $AD < BC$
(d) $BF : CF$ is greater than 1 if $AD > BC$

30. Points E and F are respectively midpoints of sides AB and CD of a rectangle $ABCD$. If line segment AF and EC respectively intersect diagonal BD at P and Q, then what is the length of PQ if sides of rectangle are respectively 10 cm and 24 cm ?
(a) 10 cm (b) 17 cm (c) $\frac{32}{3}$ cm (d) $\frac{26}{3}$ cm

31. The area of a parallelogram $ABCD$ is equal to that right angled isosceles triangle whose hypotenuse is l cm. If O is a point inside the parallelogram $ABCD$ then sum of areas of ΔAOB and ΔCOD is.
(a) $\frac{l^2}{2}$ cm^2 (b) $\frac{l^2}{4}$ cm^2 (c) $\frac{l^2}{8}$ cm^2 (d) $\frac{l^2}{16}$ cm^2

Answers 7A

1. (a)	2. (b)	3. (a)	4. (a)	5. (b)	6. (d)	7. (d)	8. (a)
9. (b)	10. (b)	11. (a)	12. (c)	13. (a)	14. (b)	15. (a)	16. (c)
17. (a)	18. (b)	19. (b)	20. (c)	21. (a)	22. (b)	23. (b)	24. (d)
25. (c)	26. (c)	27. (b)	28. (c)	29. (b)	30. (d)	31. (c)	

1. (a) Recall that $EF = \frac{1}{2}(AB + CD)$ [see solved exampl...

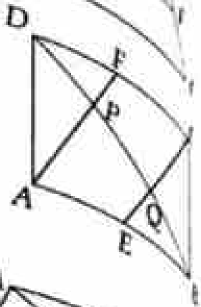
$= \frac{1}{2}(10 + 12) = 11$ cm

2. (b) See solved example 14

Here $DP = PQ = QB = \frac{1}{3}BD$

$= \frac{1}{3}\sqrt{40^2 + 9^2}$

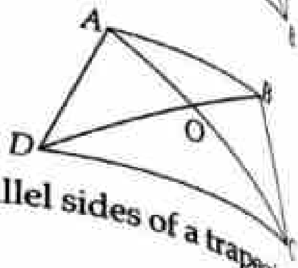
$= \frac{1}{3} \times 41$ cm



3. (a) $\Delta AOB \sim \Delta COD$

$\Rightarrow \frac{AO}{CO} = \frac{AB}{CD}$

$= \frac{10}{14} = 5:7$



4. (a) We know that a line drawn parallel to parallel sides of a trapezium cuts non parallel sides in the same ratio.

$\because \frac{BF}{FC} = \frac{AE}{ED} = \frac{2}{1}$

or, $BF = 2FC = 2(BC - BF)$

or, $3BF = 2BC \quad \therefore \frac{BF}{BC} = \frac{2}{3}$

5. (b) As in solved example 11, for a rhombus, sum of square of diagonals = sum of square of its sides

$= 10^2 + 10^2 + 10^2 + 10^2 = 400$

$\therefore$ Required square root $= \sqrt{400} = 20$

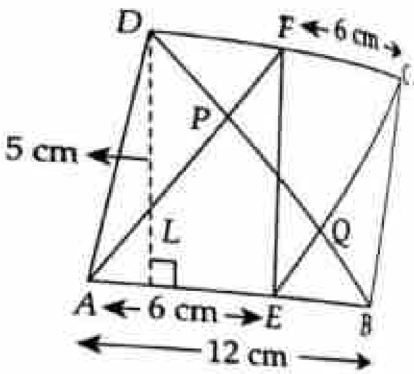
6. (d) area of quadrilateral $PQCF$

$= \frac{1}{2}$ (area of quadrilateral $AECF$)

$= \frac{1}{2}(2 \times \text{area of } \Delta AEF)$

$= \frac{1}{4} \times$ area of quadrilateral $ABCD$

$= \frac{1}{4} \times 12 \times 5 = 15 \text{ cm}^2$



2nd Method :

Area of quadrilateral $PQCF\ \frac{1}{2}$ (area AECF)

$= \frac{1}{2} \times FC \times DL$

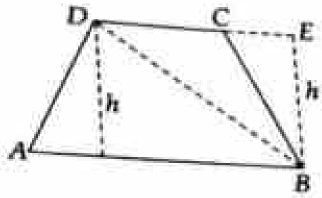
$= \frac{1}{2} \times \frac{1}{2} \times DC \times DL = \frac{1}{4} \times 12 \times 5 = 15 \text{ cm}^2$

7. (d) Statement (c) is correct. When we join midpoints of a square it is a square which is also a rhombus.

When midpoints of a trapezium is joined, a parallelogram is formed. So, statement (d) is wrong.

8. (a) $\dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta BCD} = \dfrac{\frac{1}{2}\cdot AB\cdot h}{\frac{1}{2}\cdot CD\cdot h}$

$= AB : CD$



9. (b) $\Delta AOB \sim \Delta COD$

$$\frac{ar(\Delta AOB)}{ar(COD)} = \frac{AB^2}{CD^2} = \frac{(2CD)^2}{CD^2} = \frac{4CD^2}{CD^2} = 4:1$$

10. (b) $\angle DMN = \angle CMB$

$\angle DNM = \angle CBM$ *(alternate angle)*

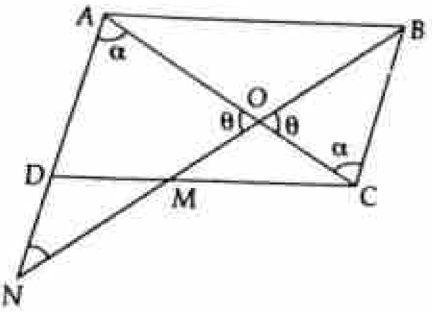
$DM = CM$

$\Delta NDM \cong \Delta BCM$

$\therefore DN = CB$

$AD = BC = DN$

$\therefore AN = AD + DN = AD + AD = 2AD$



In ΔOBC and ΔONA

$\angle BOC = \angle AON$ *(vertically opposite angle)*

$\angle OCB = \angle OAN$ *(alternate angle)*

$\therefore \Delta OBC \sim \Delta ONA \quad \therefore \dfrac{ON}{OB} = \dfrac{NA}{BC} = \dfrac{2}{1}$

11. (a) Let $\angle XOB = \theta$ then $\angle AOX = 90° - \theta$

$\because AO = AX$

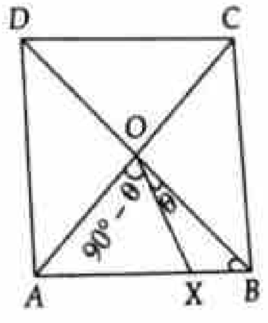
$\therefore \angle AXO = \angle AOX = 90° - \theta$

In ΔAOX, $\angle OAX + \angle AOX + \angle AXO = 180°$

or, $45° + (90° - \theta) + (90° - \theta) = 180°$

or, $2\theta = 45°$

or, $\theta = \dfrac{45°}{2}$



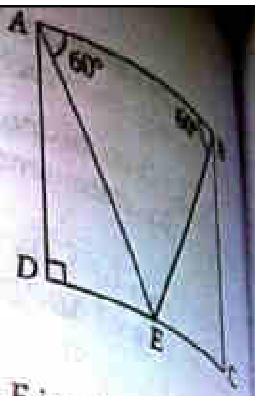
12. (c) When midpoints of a quadrilateral are joined, a parallelograms is formed.

13. (a) $\because AB \parallel DC$ and $AD \parallel BC$

In ΔABE, $\angle EAB = \angle ABE = 60°$

$\Rightarrow \angle AEB = 60°$

$\Rightarrow \Delta ABE$ is an equilateral triangle



Now, perimeter of $\Delta ABE = 6$

$\Rightarrow AB + BE + EA = 6 \Rightarrow AB = 2$ unit

and In ΔADE, $AE^2 = AD^2 + ED^2$

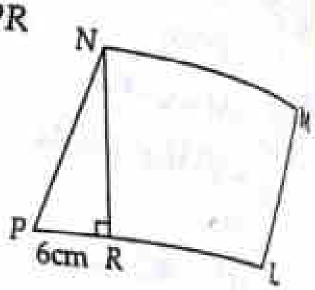
$\Rightarrow 4 = AD^2 + 1 \Rightarrow AD = \sqrt{3}$ unit $\quad$ ($\because$ E is mid point of CD)

Hence, area of quadrilateral $ABCD = AB \times AD = 2 \times \sqrt{3}$

$= 2\sqrt{3}$ square unit

14. (b) According to question,

area of parallelogram $= 6 \times$ area of ΔNPR

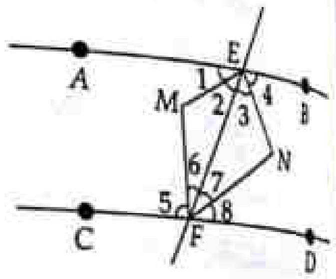


$\Rightarrow NR \times PL = 6 \times \frac{1}{2} \times NR \times PR$

$\Rightarrow PL = 3PR = 3 \times 6 = 18$ cm

$RL = PL - PR = 18 - 6 = 12$ cm

15. (a) Given $\angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$



$\because \angle 1 + \angle 2 = \angle 7 + \angle 8$ *(alternate angle)*

$\therefore 2\angle 2 = 2\angle 7 \Rightarrow \angle 2 = \angle 7 \quad \ldots$ (i)

Similarly $\angle 3 = \angle 6 \quad \ldots$ (ii)

from (i) and (ii)

$\angle 2 + \angle 3 = \angle 6 + \angle 7 \quad \ldots$(iii)

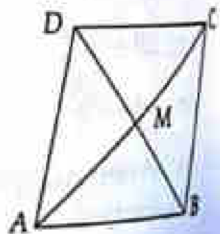
But, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

$\Rightarrow 2(\angle 2 + \angle 3) = 180° \quad$ (from (iii))

$\Rightarrow \angle MEN = 90°$ and $\angle 2 = \angle 7, \angle 3 = \angle 6 \Rightarrow EM \parallel NF, EN \parallel MF$

$\therefore \square MFNE$ is a rectangle.

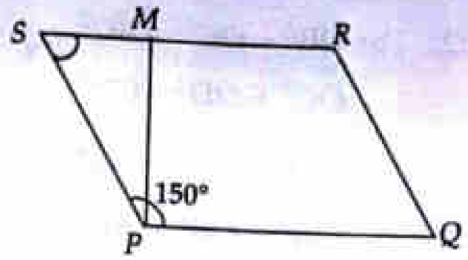
16. (c) Here midpoint of diagonal BD is M and it also bisects $\angle B$, so parallelogram is a rhombus.



$\therefore \angle AMB = 90°$

17. (a) Given $\angle SPQ = 150°$ and $PM = 20$ cm

In parallelogram $PQRS$,



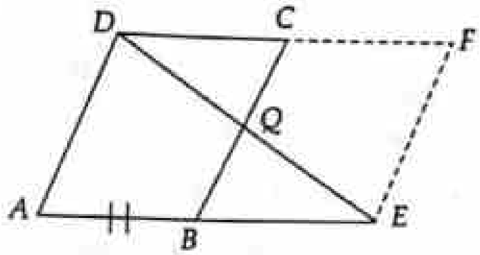
$\angle RSP + \angle SPQ = 180°$

$\Rightarrow \angle RSP = 180° - 150° = 30°$

$\Rightarrow \theta = 30°$

In ΔPSM, $\sin 30° = \frac{PM}{SP}$

$\Rightarrow \frac{1}{2} = \frac{20}{SP} \Rightarrow SP = 40$ cm



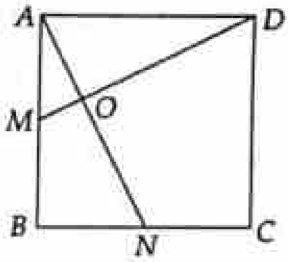
18. (b) See the figure,

In ΔCDQ and BEQ

$DQ = QE$ and $CD = BE$

$\therefore \; BQ = QC$

Hence, $BQ : QC = 1 : 1$



19. (b) Let each side of square is a

$\because$ M, N are midpoints

$\therefore \; AM = BN = \frac{a}{2}$

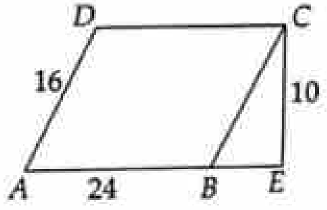
In right angled ΔDAM,

$$MD^2 = AD^2 + AM^2 = a^2 + \left(\frac{a}{2}\right)^2 = \frac{5a^2}{4} \quad \text{... (i)}$$

Similarly in ΔABN

$$AN^2 = AB^2 + BN^2 = a^2 + \left(\frac{a}{2}\right)^2 = \frac{5a^2}{4} \quad \text{... (ii)}$$

from (i) and (ii) $AN = MD$

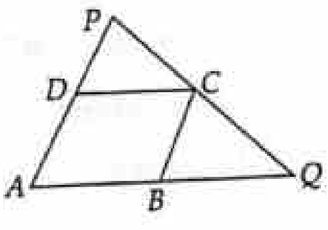


1. (c) Area of parallelogram = base × height

$= 24 \times 10 = 240 \text{ cm}^2$

If required distance is x cm then $240 = 16 \times x$

$\therefore \; x = \frac{240}{16} = 15$ cm



(a) From Thale's Theorem,

$\because \; AB \parallel CD$

$\therefore$ In ΔAPQ

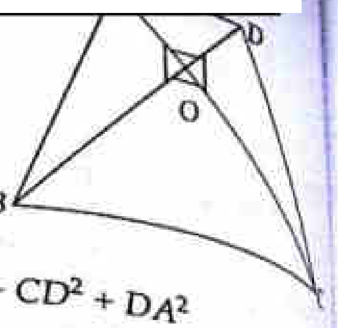
$$\frac{PC}{QC} = \frac{PD}{DA} = \frac{1}{2} \quad \text{... (i)}$$

But, $BC \parallel AD$

$\therefore$ Using Thale's theorem in ΔAQP, $\frac{BQ}{AB} = \frac{QC}{CP} = \frac{2}{1}$

22. (b)

$$OC^2 + OD^2 = CD^2$$
$$OD^2 + OA^2 = AD^2$$
$$OA^2 + OB^2 = AB^2$$



Adding,

$\therefore \quad 2(OB^2 + OA^2 + OD^2 + OC^2) = AB^2 + BC^2 + CD^2 + DA^2$

$\Rightarrow \quad 2(AB^2 + CD^2) = AB^2 + BC^2 + CD^2 + DA^2$

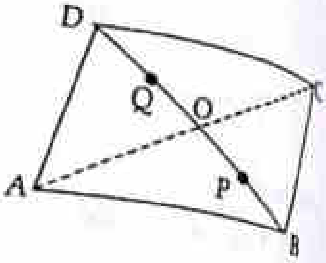
$\Rightarrow \quad AB^2 + CD^2 = BC^2 + DA^2$

23. (b) Since $DQ : QO = 2 : 1$,

$BP : PO = 2 : 1$ and $BO = DO$

$\therefore \quad DQ = 2k, QO = k, BP = 2k, PO = k$

Hence $PQ = PO + OQ = 2k = \frac{2k}{6k} \times 18 = 6$ cm



24. (d) Since sum of opposite angles of a cyclic quadrilateral = 180°

$\because \quad 72° + \alpha = 180°$

$\therefore \quad \alpha = 180° - 72° = 108°$

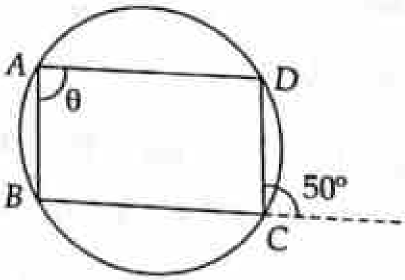
$\therefore \quad \theta = 180° - 108° = 72°$

25. (c) $\because \quad 4x + 5x = 180°$

$\Rightarrow \quad 9x = 180° \Rightarrow x = 20°$

$\therefore \quad \angle C = 4x = 80°$

26. (c) External angle is equal to internal opposite angle.



27. (b) $\angle ADC = 108° - 70° = 110°$ and $\angle ADC + \angle BCD = 180°$

$\Rightarrow \quad \angle BCD = 108° - 110° = 70°$

28. (c) see solved example 12

29. (b) see solved example 13

30. (d) see solved example 14 $PQ = \frac{1}{3} \times 2\sqrt{10^2 + 24^2} = \frac{26}{3}$

31. (c) If perpendicular sides of right angled isosceles triangle are x then

$$x^2 + x^2 = l^2 \quad \Rightarrow \quad x^2 = \frac{l^2}{2}$$

$\therefore$ Area of triangle $= \frac{1}{2} \cdot x \cdot x = \frac{1}{2} \cdot \frac{l^2}{2} = \frac{l^2}{4}$ = area of parallelogram $ABCD$

As in solved example 7.

area of ΔAOB + area of ΔCOD

$$= \frac{1}{2} \text{ (area of quadrilateral } ABCD = \frac{1}{2} \cdot \frac{l^2}{4} = \frac{l^2}{8}$$

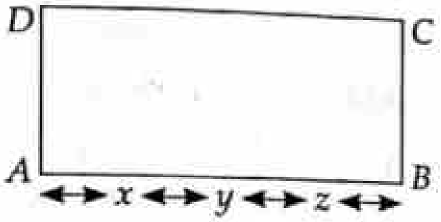
Exercise—7B

1. If the length of the side PQ of the rhombus $PQRS$ is 6 cm and $\angle PQR = 120°$, then the length of QS, in cm, is

(a) 3 (b) 5 (c) 4 (d) 6

[SSC Tier-I 2012]

2. Side AB of rectangle $ABCD$ is divided into four equal parts by points x, y, z. Then ratio of the $\frac{\text{area } (\Delta XYC)}{\text{Area (Rectangle } ABCD)}$ is



(a) $\frac{1}{7}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ (d) $\frac{1}{8}$

[SSC Tier-I 2012]

3. $ABCD$ is a trapezium, such that $AB = CD$ and $AD \parallel BC$. $AD = 5$ cm, $BC = 9$ cm. If area of $ABCD$ is 35 sq. cm, their CD is

(a) $\sqrt{29}$ cm (b) 5 cm (c) 6 cm (d) $\sqrt{21}$ cm

[SSC Tier-I 2012]

4. The area, perimeter and diagonal of a square are a, b, c respectively. Then the value of $\frac{bc}{a}$ is.

(a) 4 (b) 2 (c) $4\sqrt{2}$ (d) $2\sqrt{2}$

5. The length of the side of a square is 14 cm. Find out the ratio of the radii of the inscribed and circumscribed circle of the square.

(a) $\sqrt{2} : 1$ (b) $1 : \sqrt{2}$ (c) $\sqrt{2} : 3$ (d) $2 : 1$

[SSC Tier-I 2012]

6. If P, R, T are the area of a parallelogram, a rhombus and a triangle standing on the same base and between the same parallels, which of the following is true ?

(a) $R < P < T$ (b) $P > R > T$ (c) $R = P = T$ (d) $R = P = 2T$

[SSC Tier-I 2012]

Answers—7B

1. (d) 2. (d) 3. (a) 4. (c) 5. (b) 6. (d)

Explanation

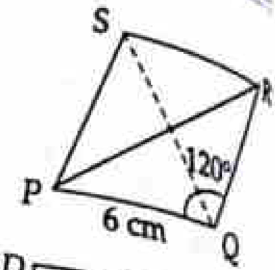
1. **(d)** $\angle SPQ = 180° - 120° = 60°$

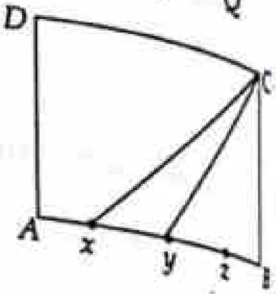
$\angle SQP = \frac{1}{2} \times 120° = 60°$

and $\angle PSQ = 180° - 60° - 60° = 60°$

$\therefore$ ΔPSQ is an equilateral triangle

Hence, $SQ = PQ = 6$ cm



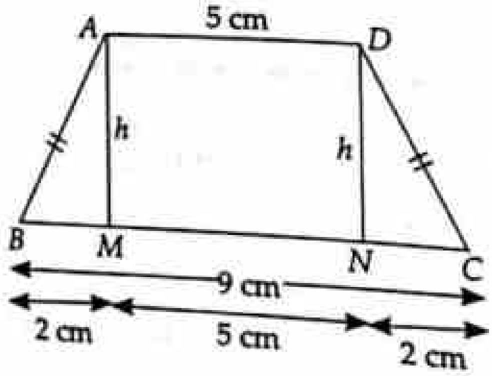


2. **(d)** $\dfrac{\text{area }(\Delta xyc)}{\text{area }(\square ABCD)} = \dfrac{\frac{1}{2} \cdot xy \cdot height}{AB \cdot BC}$

$$= \frac{\frac{1}{2} \cdot xy \cdot BC}{4\,xy\,BC} = \frac{1}{8}$$

($\because$ height of triangle = height of rectangle and $AB = 4xy$)

3. **(a)** See the figure, Let $AM = DN = G$ then

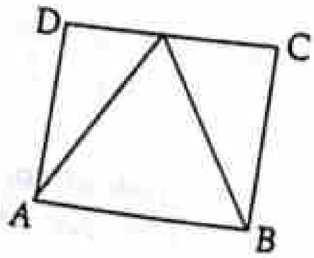


area $(\Delta ABM + \square AMND + \Delta CND) = 35$ cm^2

or, $\frac{1}{2} \cdot 2 \cdot h + 5 \cdot h + \frac{1}{2} \cdot 2 \cdot h = 35$

or, $7h = 35$ or, $h = 5$ $\therefore CD = \sqrt{h^2 + CN^2} = \sqrt{5^2 + 2^2} = \sqrt{29}$

6. **(d)** Both parallelogram and rhombus are same as base are same.

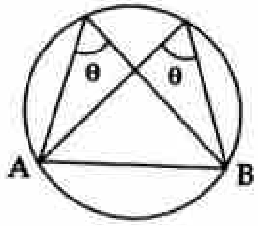


★★★

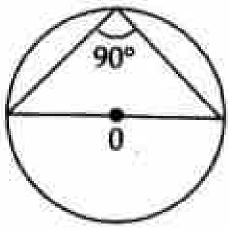
08 Circle and its Tangent lines

1. Main Geometric properties Related to circle

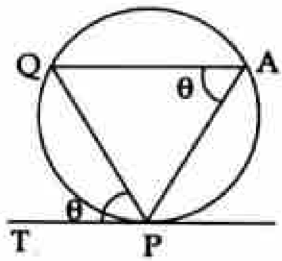
1.1. **Angles in the same segment of a circle are equal**



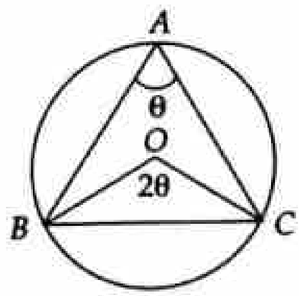
1.2. **The angle in a semicircle is right angled.**



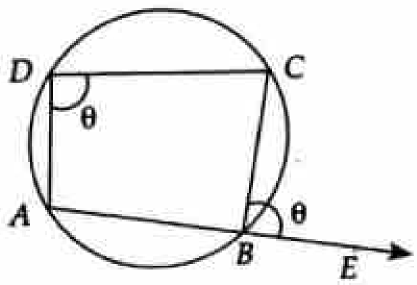
1.3. **If line PT touches a circle at the point P and a chord PQ is drawn from point of contact P, then angle made by PQ in the alternate segment ($\angle PAQ$ in figure) of the circle is equal to angle ($\angle QPT$ in figure) made by the tangent PT to the circle.**



1.4. **The angle at the centre (O in figure) in a circle is double the angle at the circumference standing on the same arc or same base (BC in figure) i.e. in the same segment.**

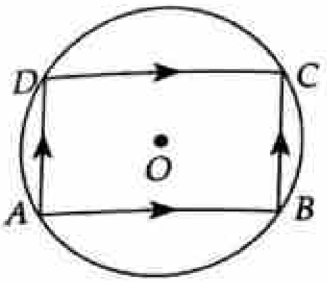


1.5. **A quadrilateral inside the circle formed by taking four points on the circumference of the circle is called a cyclic quadrilateral sum of its opposite angle is 180° (i.e. $\angle A + \angle C = 180°$ and**

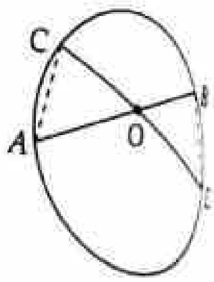


$\angle B + \angle D = 180°$). Its converse is also true. If AB is produced
then $\angle CBE = \angle D = \theta$

1.6. If a parallelogram is inscribed inside a circle, it is either a rectangle or a square.



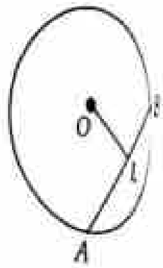
1.7. If two chords AB and CD of a circle intersect at O then ΔAOC and ΔDOB are similar i.e. $\Delta AOC \sim \Delta BOD$ (In the given figure $\angle A = \angle D$, $\angle C = \angle B$ and $\angle BOD = \angle AOC$)



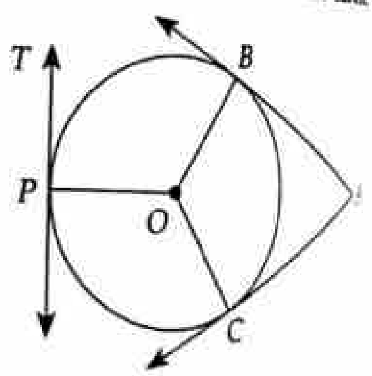
Hence, $\frac{AO}{DO} = \frac{AC}{DB} = \frac{OC}{OB}$

or, $(AO)(OB) = (OC)(OD)$

1.8. Perpendicular drawn from the centre of a circle to any chord bisects the chord. Its converse is also true. In the given figure $OL \perp r\ AB \Leftrightarrow AL = BL$



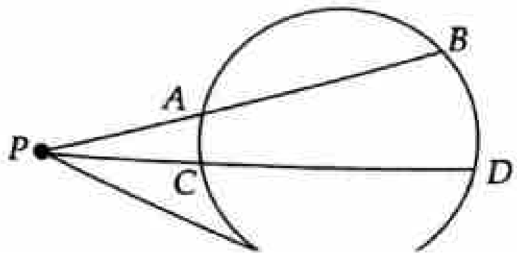
1.9. From a point on the circle, only one tangent can be drawn to the circle (PT in figure). However two tangents (AB and AC) can be drawn to a circle from an external point. Length of these two tangents are equal i.e. $AB = AC$



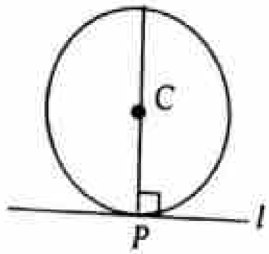
The line joining centre and point of contact of a circle is perpendicular to the tangent drawn at point of contact.

In the adjacent figure $OB \perp r\ AB$, $OC \perp r\ AC$ and $OP \perp r\ PT$

1.10. In the given figure $(PA)(PB) = PT^2 = PC.PD$



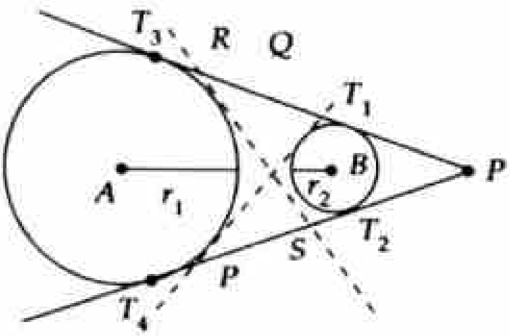
2. Tangent and Normal to a circle : A line that touches a circle at one and only point is called a tangent line or simply tangent to the circle. In the given figure l is a tangent line to the circle that touches the circle at point P. This point is called point of contact of tangent



A line through point P and perpendicular to tangent l is called normal to the circle. Normal to a circle always passes through its center.

3. Number of common tangents to the two circles : There are maximum number of four common tangents and minimum number of zero tangent to the two given circles. They are as follows.

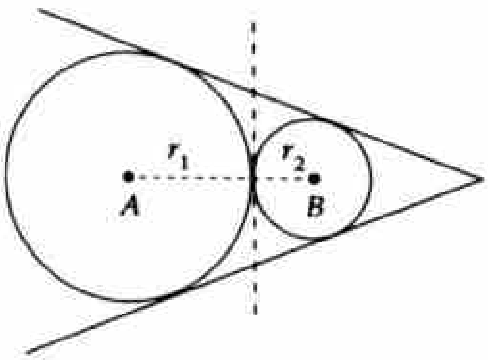
3.1. Four common tangents : If distance between centres of two given circles is greater than sum of their radii i.e. $AB > r_1 + r_2$ (see figure), then four common tangents can be drawn to the two circles.



See the given figure, T_1T_3 and T_2T_4 are direct common tangent while PQ and RS are transverse common tangents.

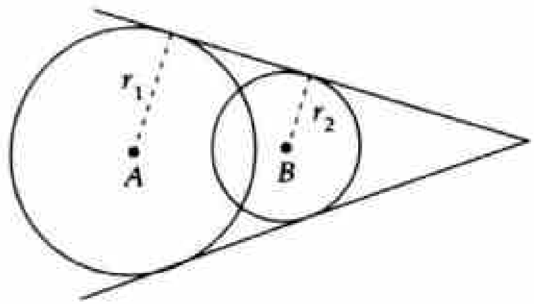
3.2. Three common tangents : When distance between centres of two circles is equal to sum of their radii ($AB = r_1 + r_2$) then maximum of three common tangents can be drawn to the circle. In this situation two circles touch externally.

(see the figure)



Two common tangents

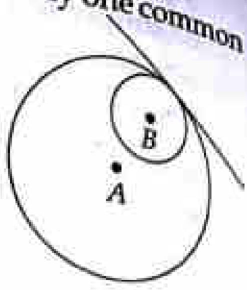
When two circles intersect each other at two distinct points then two common tangents can be drawn to the circles (see the figure). Hence distance between centres of two circles is less than sum of their radii but greater than difference of radii i.e.



$|r_1 - r_2| < AB < r_1 + r_2$

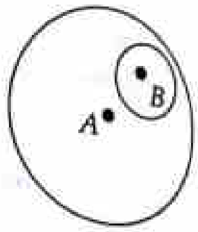
3.4. One common tangent :

When two circles touch each other internally then only one common tangent can be drawn to them (see the figure). In this situation distance between centres of the two circles is equal to difference of their radii i.e.



$AB = |r_1 - r_2|$

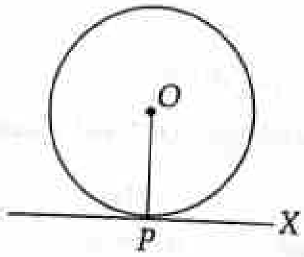
3.5. When one circle lies completely inside other circle, then common tangent cannot be drawn to the two circles (see the figure). Here distance between centres of two circles is less than difference between radii of two circles.



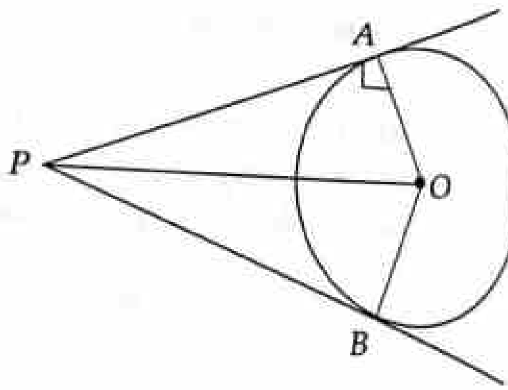
i.e. $AB < |r_1 - r_2|$

4. Some properties of tangents to a circle

4.1. Tangent drawn at any point to the circle is perpendicular to radius of the circle drawn through the point i.e. point of contact. In the given figure $\angle OPX = 90°$. Its converse is also true.



4.2. length of tangents drawn from an outside point to a given circle are equal. In the given figure, if PA and PB are tangents line then $PA = PB$



$\because$ PA is perpendicular to OA

$\therefore$ $PA^2 + OA^2 = OP^2$

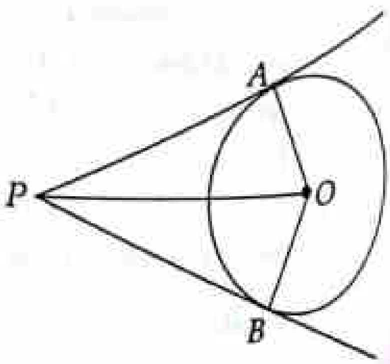
4.3. If PA and PB are tangents to a circle with centre O, then

$\angle APO = \angle BPO$

$\angle PAO = \angle PBO = 90°$

$\because$ Side PO is common

$\therefore$ $\Delta PAO \cong \Delta PBO$

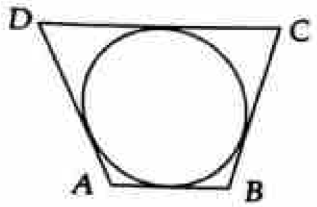


Also, $\angle AOB = 180° - \angle APB$

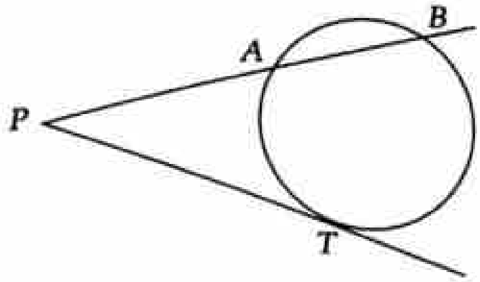
($\because$ sum of remaining two angles of quadrilateral $= 180°$)

4.4. If each side of a quadrilateral touches a given circle then sum of one pair of opposite side is equal to sum of another pair of opposite side.

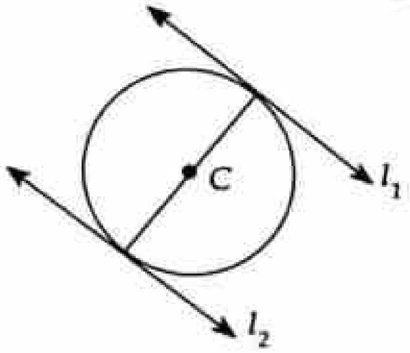
In the given figure $AB + CD = AD + BC$



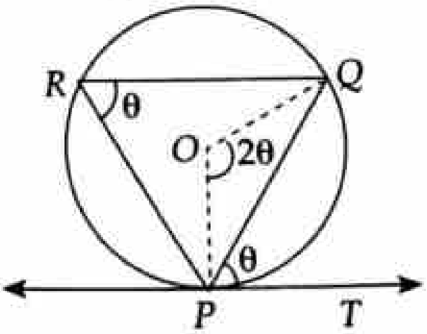
4.5. If PAB be a secant that intersects a given circle at A and B and PT is a tangent line then $PA \cdot PB = PT^2$



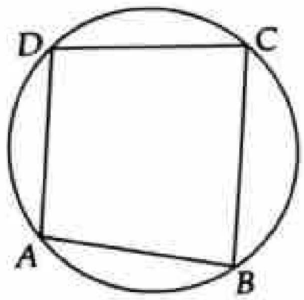
4.6. Tangents drawn at extrimities (end points) of a diameter of a given circle are parallel. In the given figure $l_1 \parallel l_2$



4.7. In the given figure, if PT is a tangent to the circle then $\angle PRQ = \angle TPQ = \theta$ and $\angle POQ = 2\theta$



5. Important properties of cyclic Quadrilateral



5.1. In the cyclic quadrilateral $ABCD$

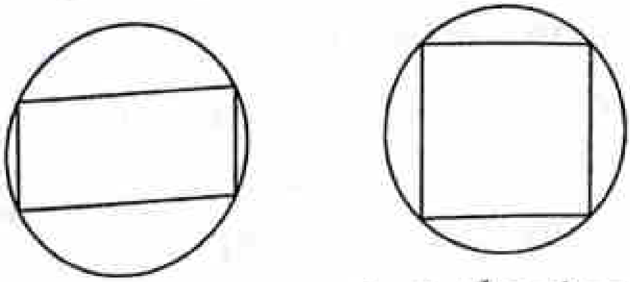
$\angle A + \angle C = 180^\circ$

and $\angle B + \angle D = 180^\circ$

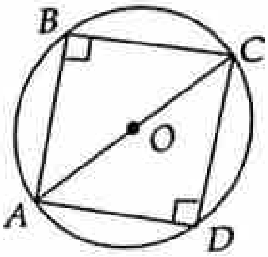
Its converse are also true i.e. in any quadrilateral if

$\angle A + \angle C = \angle B + \angle D = 180^\circ$ then $ABCD$ is a cyclic quadrilateral

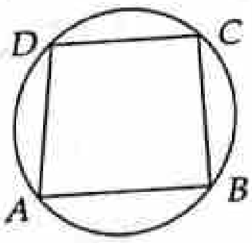
5.2. Every cyclic parallelogram is a rectangle. Every cyclic rhombus is a square



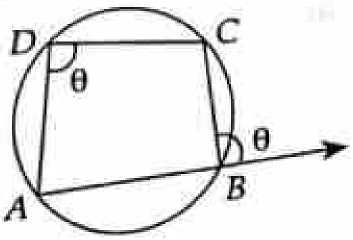
5.3. Angle in a semicircle is right angle. In the given figure if O is the centre then $\angle ABC = \angle ADC = 90^\circ$



5.4. If a trapezium $ABCD$, where $AB \parallel DC$, is inscribed in a circle then its non parallel sides are equal i.e. $BC = AD$. Thus we can say that a trapezium inscribed in a circle is always isosceles. Converse of the statement is also true.



5.5. If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.



... common tangents ... from point P to two

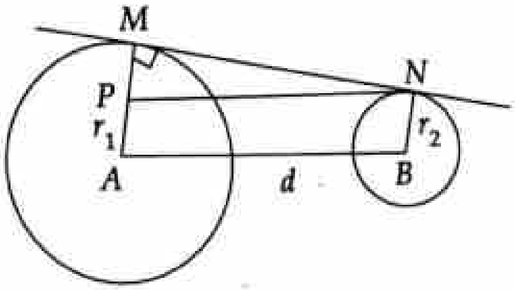
6.1.1 $\because \Delta PAM \sim \Delta PBN$ (P is common angle and $\angle AMP = \angle BNP = 90°$)

$\therefore \frac{PA}{PB} = \frac{PM}{PN} = \frac{AM}{BN}$

$\Rightarrow \boxed{\frac{PA}{PB} = \frac{r_1}{r_2}}$

i.e. Point P divides the line joining the centres which is AB in the ratio $r_1 : r_2$ (externally)

6.1.2 Length of direct common tangent (MN)



From point N draw a line parallel to AB that intersects AM at P. Since $ABNP$ is a parallelogram

$\therefore$ $PA = BN = r_2$

$\therefore$ $PM = r_1 - r_2$

In right angled ΔPNM

$$PN^2 = PM^2 + MN^2$$

or, $AB^2 = PM^2 + MN^2$ $\quad (\because PN = AB)$

$$MN^2 = AB^2 - (PM)^2$$

$$MN = \sqrt{(AB)^2 - (PM)^2} = \sqrt{d^2 - (r_1 - r_2)^2},$$

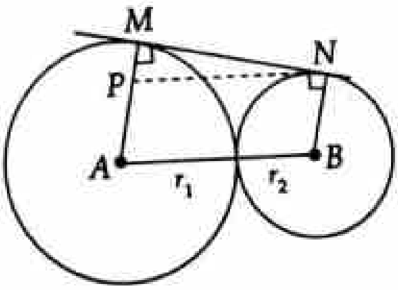
where d = distance between centres

Length of direct common tangents to two circles =

$$\sqrt{(\text{Distance between centre})^2 - (\text{Difference of radii})^2}$$

This result is also true if circles touch externally or intersect at two distinct points

6.1.3 Special case : If two circles touch externally then length of direct common tangent



$$MN = \sqrt{d^2 - (r_1 - r_2)^2}$$

$$= \sqrt{(r_1 + r_2)^2 - (r_1 - r_2)^2}$$

$$= \sqrt{4 r_1 \cdot r_2}$$

... centres.

or, $k = \dfrac{d}{r_1 - r_2}$

$\therefore \quad AP = r_1 k = \dfrac{r_1}{r_1 - r_2} d$

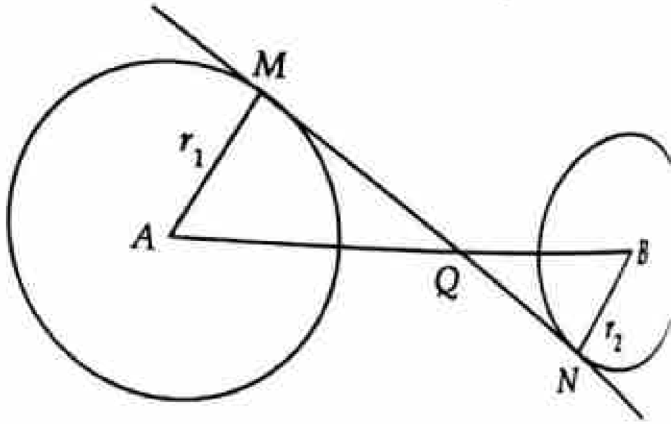
and $BP = r_2 k = \dfrac{r_2}{r_1 - r_2} d$

Thus Distance of centres from point P are respectively $\dfrac{r_1 d}{r_1 - r_2}$ and $\dfrac{r_2 d}{r_1 - r_2}$, where $r_1 > r_2$

6.1.5. Hence distance of P from N, $PN = \sqrt{BP^2 - BN^2} = \sqrt{BP^2 - r_2^2}$

Distance of P from M, $PM = \sqrt{AP^2 - AM^2} = \sqrt{AP^2 - r_1^2}$

7. Transverse Common Tangents :



In the adjacent figure MN is transverse common tangent. It touches the circle with centre A and radius r_1 at M while touches the circle with centre B and radius r_2 at N. MN and AB intersect at Q. Some important facts regarding them are as follows.

7.1. $\Delta AMQ \sim \Delta BNQ$ $\quad (\because \angle AMQ = \angle BNQ = 90^\circ$ and $\angle AQM = \angle BQN)$

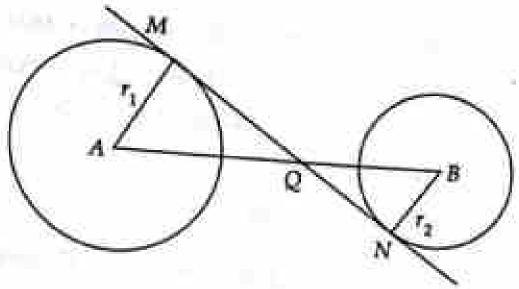
$\Rightarrow \dfrac{AM}{BN} = \dfrac{AQ}{BQ} = \dfrac{MQ}{NQ}$,

$\because \quad AM = r_1, BN = r_2$

$\therefore \quad \dfrac{AQ}{BQ} = \dfrac{r_1}{r_2}$

So, Q divides line AB joining the centres ... internally

7.2 $\because \quad \dfrac{AQ}{BQ} = \dfrac{r_1}{r_2}$



$\therefore \quad AQ + BQ = (r_1 + r_2)\,k$

$\Rightarrow \quad d = (r_1 + r_2)\,k$, where d is distance between centres

$\Rightarrow \quad k = \dfrac{d}{r_1 + r_2}$

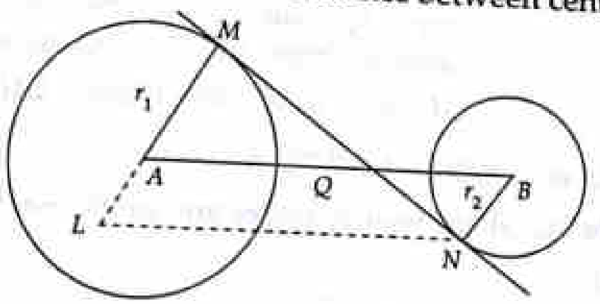
$\therefore \quad AQ = kr_1 = \dfrac{r_1 d}{r_1 + r_2}$ and $BQ = kr_2 = \dfrac{r_2 d}{r_1 + r_2}$

Thus

> Distance of centres from point Q are respectively $\dfrac{r_1 d}{r_1 + r_2}$ and $\dfrac{r_2 d}{r_1 + r_2}$

7.3. From N draw a line parallel to AB which intersects produced part of MA at L.

$\therefore \quad ML = r_1 + r_2$ and $LN = AB = d =$ distance between centres.



In triangle MNL

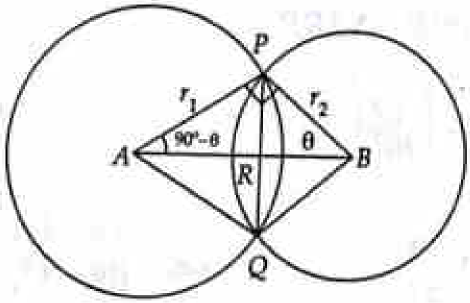
$\because \quad \angle LMN = 90°$

$\therefore \quad MN = \sqrt{LN^2 - LM^2} = \sqrt{d^2 - (r_1 + r_2)^2}$

Length of transverse common tangents to two circles =

$$\sqrt{(\text{Distance between centre})^2 - (\text{Sum of radii})^2}$$

8. Common chord :



Let two circle with centres A and B intersect each other at two di... points P and Q. Thus PQ is a common chord to the two circles. If ta... gents drawn from points P and Q to the two circles passes through the centres then

8.1. $\angle APB = 90^\circ$

8.2. If PQ, intersects AB at R then

$PR \perp r\ AB$ and $\Delta PAB \sim \Delta RPB \sim \Delta RAP$

Explanation : Since PA and PB are tangents and A and B are centr... thus $\angle APB = 90^\circ$

In right angled ΔPAB, Let $\angle PBA = \theta$ then $\angle PAB = 90^\circ - \theta$

In right angled ΔRPB, $\angle PRB = 90^\circ$, $\angle PBR = \theta$ and $\angle RPB = 90^\circ - \theta$

In right angled ΔRAP, $\angle PRA = 90^\circ$, $\angle RAP = 90^\circ - \theta$ and $\angle APR = \theta$

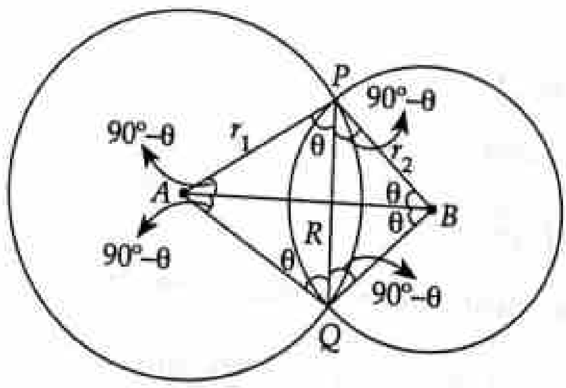
Hence $\Delta PAB \sim \Delta RPB \sim \Delta RAP$

8.3. $\Delta ARP \cong \Delta ARQ \sim \Delta PRB \cong QRB)$

Explanation : Since perpendicular drawn from centre to any chord bisects it, therefore $PL = LQ$, side AL is common and $\angle ALP = \angle ALQ = 90^\circ$. Hence $\Delta ALP \cong \Delta ALQ$

8.4. $\Delta ARP \cong \angle QRB$ and $\Delta ARQ \cong \Delta PRB$

(Note that all the four triangles are similar. See the figure and explain yourself)



8.5. $AR : RB = r_1^2 : r_2^2$

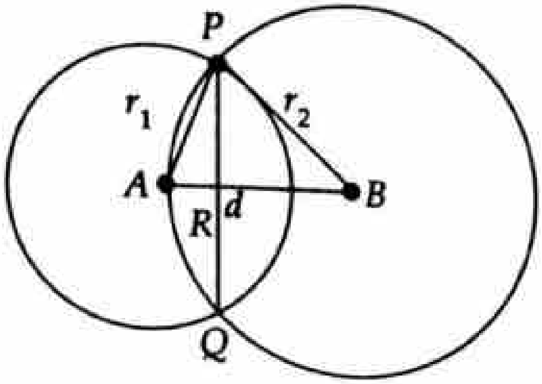
Explanation : $\because\ \Delta PRB \sim \Delta ARP$

$$\therefore \quad \frac{\text{area of } \Delta ARP}{\text{area of } \Delta BRP} = \left(\frac{AP}{BP}\right)^2$$

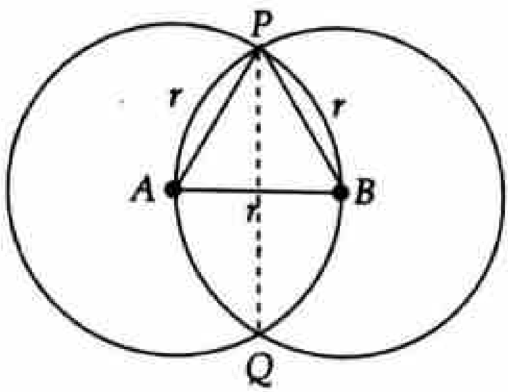
$$\Rightarrow \quad \frac{\frac{1}{2} \times AR \times PR}{\frac{1}{2} \times BR \times PR} = \left(\frac{r_1}{r_2}\right)^2 \qquad \text{or,} \quad \frac{AR}{BR} = \left(\frac{r_1}{r_2}\right)^2$$

9. **Some more important facts about common chord :**

9.1. If radii of two unequal circles are r_1 and r_2 and larger circle passes through centre of smaller one then $r_1^2 + r_2^2 = d^2$, where d is the distance between centres of the two circles.



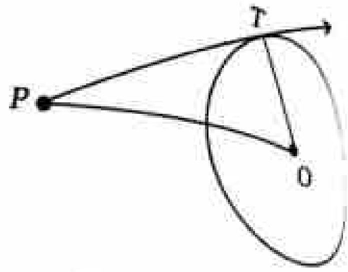
9.2. In two equal circles (circles with same radius) if one passes through the centre of the other then other must pass through centre of the former (see the figure). ΔAPB will be an equilateral triangle whose each side is equal to radius of the circle.



$\therefore$ $PQ = 2 \times$ altitude of the triangle $= 2 \times \frac{\sqrt{3}}{2} r$

$\boxed{\text{length of common chord} = \sqrt{3}\, r}$

2. In the figure ... of the circle. A tangent PT is drawn from an out side point P to the circle. If radius of circle is 5 cm and $OP = 13$ cm then find the length of tangent PT.



Solution : $PT \perp OT$

$\Rightarrow \angle OTP = 90^\circ$

$\therefore \quad PT^2 + OT^2 = OP^2$ [In right angled ΔOPT]

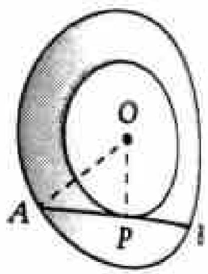
or, $PT^2 = OP^2 - OT^2$

$= 13^2 - 5^2$

$= 169 - 25 = 144 = 12^2$

$\therefore \quad PT = 12$ cm

3. Radius of two incentric circle are 5 cm and 3cm. Find out the length of arc of larger circle which touches to smaller circle ?



Solution : Let O be the common centre. AB is chord of larger circle that touches the smaller one.

Join $O - P$ then $\angle OPB = 90^\circ$

i.e. OP is perpendicular to AB

Since perpendicular drawn from centre of a circle to any of its chord bisect the chord

$\therefore \quad AP = PB$

Now in right angled ΔAPO

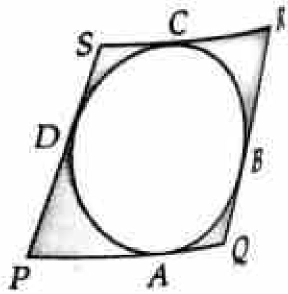
$OA^2 = AP^2 + OP^2$

$\Rightarrow \quad 5^2 = AP^2 + 3^2$

$\Rightarrow \quad AP = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$ cm

$\therefore$ Length of chord $AB = 2AP = 2 \times 4 = 8$ cm

4. In the adjacent figure lines PQ, QR, RS and SP are tangents drawn respectively at the points A, B, C, D to the circle If $PQ + SR = 16$ cm, then find the perimeter of the quadrilateral



Solution : If all sides of quadrilateral $PQRS$ touches a circle then $PQ + SR = PS + QR$

but $\quad PQ + SR = 16$ cm (given)

5. A circle is drawn circumscribing ... parallelogram ... its area. ... sides of

Solution : Since every cyclic parallelogram is a rectangle, therefore its sides are 4 cm and 3 cm

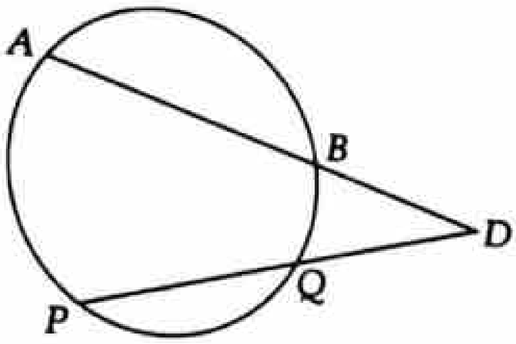
Hence required area $= 3 \times 4 = 12\ cm^2$

6. Two chords AB and PQ of a circle mutually intersect at an outside point D. If $AD = 12$ cm, $AB = 8$ cm, $DQ = 6$ cm then find PQ and PD.

Solution : $AD = AB + BD$

or, $12 = 8 + BD$

$\therefore$ $BD = 12 - 8 = 4$ cm



Now, $DB \cdot DA = DQ \cdot DP$

$\therefore$ $4 \times 12 = 6 \times DP$

or, $DP = \dfrac{4\times12}{6} = 8$ cm

But, $DP = DQ + QP$

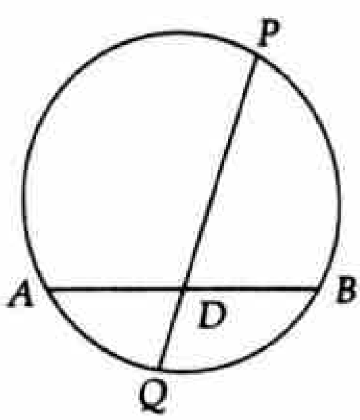
or, $8 = 6 + QP$

$\therefore$ $QP = 8 - 6 = 2$ cm

Hence, $PQ = 2$ cm and $PD = 8$ cm.

7. Two chord AB and PQ of a circle intersect at a point D inside the circle. If $AD = 4$ cm, $DB = 6$ cm, $QD = 3$ cm, then find PD and PQ.

Solution : $AD \cdot DB = QD \cdot DP$



or, $4 \times 6 = 3 \times DP$

or, $24 = 3 \times DP$

$\therefore$ $DP = \dfrac{24}{3} = 8$ cm

$\therefore$ $PQ = PD + DQ$

$= 8\text{ cm} + 3\text{ cm} = 11$ cm.

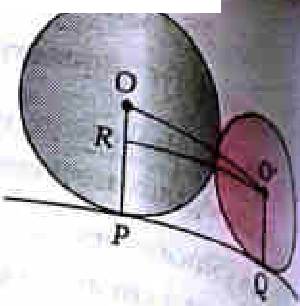
8. Radii of two circles are respectively 25 cm and 9 cm and their centres are 34 cm ... length of direct common tangent to the two

Draw $O'R \parallel PQ$

$\therefore \quad RP = O'Q = 9$ cm

$\therefore \quad OR = OP - RP$

$= 25$ cm $- 9$ cm $= 16$ cm



Now, in right angled $\Delta ORO'$

$$OO'^2 = OR^2 + O'R^2$$

or, $34^2 = 16^2 - RQ'^2$

$\therefore \quad RO'^2 = 34^2 - 16^2$

$= (34 + 16)(34 - 16)$

$= 50 \times 18 = 900 = 30^2$

$\therefore \quad RO' = 30$ cm

$\therefore \quad PQ = 30$ cm

Shortcut Mtd. : Length of direct common tangent $= \sqrt{d^2 - (r_1 - r_2)^2}$

$= \sqrt{34^2 - (25 - 9)^2}$

$= \sqrt{34^2 - 16^2}$

$= \sqrt{(34 + 16)(34 - 16)}$

$= \sqrt{50 \times 18}$

$= \sqrt{25 \times 36}$

$= 5 \times 6 = 30$ cm

9. Two circles of radii 5 cm and 3 cm intersect at two distinct points. Their centres are 4 cm apart. Find the length of their common chord.

Solution : In the given figure two intersecting circles of 5 cm and 3 cm are shown. Their centres are respectively O and C. AB is the common chord.

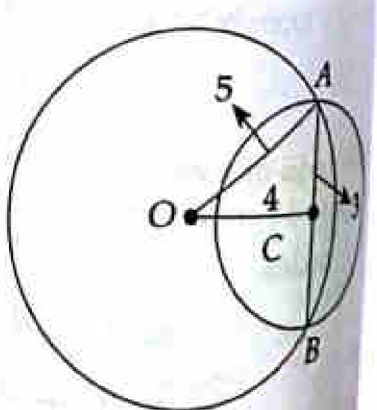
According to question, $OC = 4$ cm

$\because \quad 3^2 + 4^2 = 5^2$

$\therefore \quad AC^2 + OC^2 = OA^2,$

Hence in ΔOAC, $\angle ACO = 90°$

Similarly in triangle OCB, $\angle OCB = 90°$



Now, $\because$ $\angle OCA + \angle OCB = 90° + 90° = 180°$

Hence, AB is a straight line.

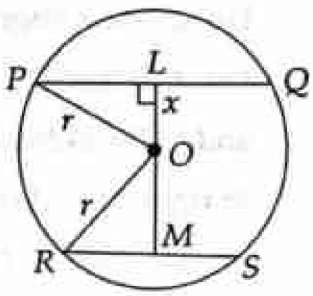
Since it is a straight line passing through centre of the smaller circle, hence it is diameter of this circle.

We conclude that common tangent is diameter of smaller circle

Hence, its length $= 3 \times 2 = 6$ cm

10. PQ and RS are two parallel chords of a circle. If $PQ = 30$ cm, $RS = 16$ cm and distance between PQ and RS is 23 cm, then find the radius of the circle.

Solution : See the figure, from centre O of the circle perpendicular OL is drawn to chord PQ and perpendicular OM is drawn to RS.



$\therefore PL = \frac{PQ}{2} = \frac{30}{2} = 15$ cm

and $RM = \frac{RS}{2} = \frac{16}{2} = 8$ cm

Let $OL = x$ cm,

then, $OM = (23 - x)$ cm

In ΔOLP, $OP^2 = PL^2 + LO^2$

or, $r^2 = 15^2 + x^2$ ... (i)

Again in ΔOMR, $OR^2 = OM^2 + RM^2$

or, $r^2 = (23 - x)^2 + 8^2$ ... (ii)

From equation (i) and (ii), $15^2 + x^2 = (23 - x)^2 + 8^2$

or, $225 + x^2 = 23^2 - 46x + x^2 + 64$

or, $225 = 529 - 46x + 64$

or, $225 = 593 - 46x$

$\therefore$ $46x = 368$

or, $x = \frac{368}{46} = 8$

Thus from (i) $r^2 = 15^2 + 8^2$

$= 225 + 64 = 289$

or, $r = \sqrt{289} = 17$ cm

radius $= 17$ cm

11. Length of one of the chord of a circle is 16 cm and it is 15 cm away from centre. Find the length of that chord of the circle which is 8 cm away from the centre.

Solution : In the given figure, O is the centre of the circle.

AB is a chord whose length is 16 cm

OM is perpendicular bisector of chord AB

$\therefore \quad MB = \frac{16}{2} = 8$ cm and $OM = 15$ cm (given)

In right angled ΔOMB, $OB^2 = OM^2 + MB^2$

or, $OB^2 = 15^2 + 8^2$

$= 225 + 64 = 289$

or, $OB = \sqrt{289} = 17$ cm

Thus radius of circle is 17 cm

Let CD be a chord of the circle at a distance of 8 cm from centre.

Let $ON \perp r\ CD$, then $ON = 8$ cm

and OD = radius of circle = 17 cm

In right angled triangle OND, $OD^2 = ON^2 + ND^2$

or, $17^2 = 8^2 + ND^2$

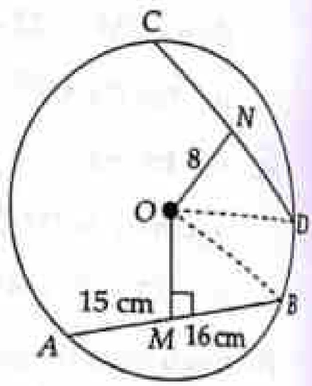
or, $ND^2 = 17^2 - 8^2 = 289 - 64 = 225$

or, $ND = \sqrt{225} = 15$ cm

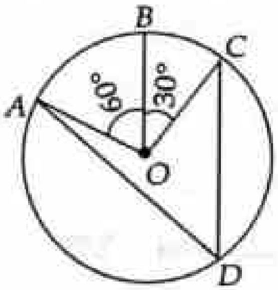
Since ON is perpendicular bisector of CD

$\therefore \quad CD = 2\ ND = 2 \times 15 = 30$ cm

Hence, length of chord which is 8 cm away from centre is 30 cm



12. In the given figure O is the centre of circle and $\angle BOC = 30°$, $\angle AOB = 60°$. If their is a point D on circle, not on arc ABC, then find $\angle ADC$.



Solution : $\angle AOC = \angle AOB + \angle BOC$

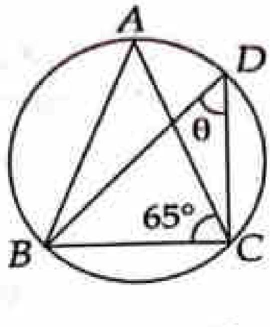
$= 60° + 30° = 90°$

∵ arc ABC subtends an angle ... at the centre

∴ arc ABC subtends an angle of $\frac{90°}{2} = 45°$ on point D.

Hence, $\angle ADC = \frac{1}{2}\angle AOC = 45°$.

13. In the given figure if $AB = AC$ then find θ.



Solution : In ΔABC, $AB = AC \Rightarrow \angle B = \angle C$

$\angle B = 65°$ $\quad$ $(\because \angle C = 65°)$

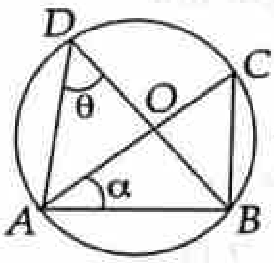
In ΔABC, $\angle A + 65° + 65° = 180°$

or, $\angle A = 180° - 65° - 65° = 50°$

Since, angle in the same segment are equal,

$\therefore \theta = \angle A = 50°$

14. In the figure given below O is the centre of the circle, if $\theta = 60°$, then find angle α



Solution : $\because$ AC and BD are passing through centre.

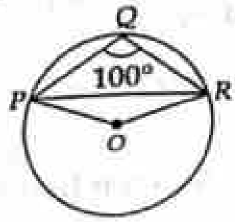
$\therefore$ ΔBAD and ΔABC are right angle triangle with $\angle BAD = 90°$ and $\angle ABC = 90°$

$\because$ θ and $\angle ACB$ are angle of the same segment

$\therefore \angle ACB = \theta = 60°$

Now, in ΔABC, $\angle ACB + \angle ABC + \alpha = 180°$

15. In the given figure $\angle PQR = 100°$, where P, Q, R are points on a circle with centre O. Find the measure of $\angle OPR$



Solution : Since angle subtended by arc of a circle at the centre twice the angle subtended by it at the circumference

$\therefore$ reflex $\angle POR = 2\angle PQR = 2 \times 100° = 200°$

and $\angle POR = 360° - 200° = 160°$

Now, in ΔOPR

$OP = OR$ (radii of the circle)

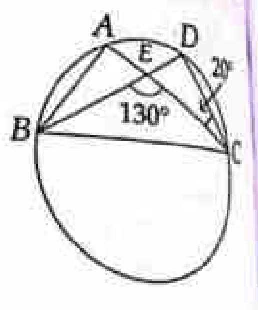
or, $\angle OPR = \angle ORP$ (angle opposite to equal sides are equal)

$\therefore$ $\angle POR + \angle OPR + \angle ORP = 180°$ (Sum of angles of a triangle)

or, $160° + 2\angle OPR = 180°$ ($\because \angle OPR = \angle ORP$)

or, $\angle OPR = \dfrac{180° - 160°}{2} = 10°$

16. In the given figure A, B, C, D are four points on a circle. AC and BD intersect at point E such that $\angle BEC = 130°$ and $\angle ECD = 20°$. Find the measure of $\angle BAC$



Solution : Since BD is a straight line,

$\therefore$ $\angle BEC + \angle CED = 180°$

or, $\angle CED = 180° - \angle BEC$

$= 180° - 130° = 50°$

Now, In ΔECD, $\angle EDC + \angle CED + \angle DCE = 180°$

or, $\angle EDC + 50° + 20° = 180°$

or, $\angle EDC = 180° - 50° - 20° = 110°$

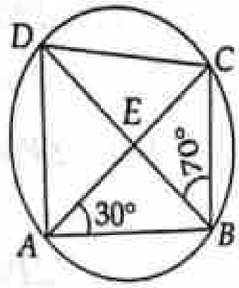
Since angles in the same segment are equal

(here see the segment above base BC)

$\therefore$ $\angle BAC = \angle BDC = 110°$

17. $ABCD$ is a cyclic quadrilateral whose diagonals intersect at E. If $\angle DBC = 70°$ and $\angle BAC = 30°$ then find $\angle BCD$. Again if $AB = BC$ then find $\angle ECD$.

Solution : In the given figure $\angle BDC = \angle BAC$



$\angle BAC = 30°$ is given)

$\therefore$ $\angle BDC = 30°$

In ΔBCD, $\angle BDC + \angle DBC + \angle BCD = 180°$

or, $30° + 70° + \angle BCD = 180°$

($\because$ $\angle DBC = 70°$ is given, $\angle BDC = 30°$ is evaluated)

or, $\angle BCD = 180° - 30° - 70° = 80°$

Again if $AB = BC$ then

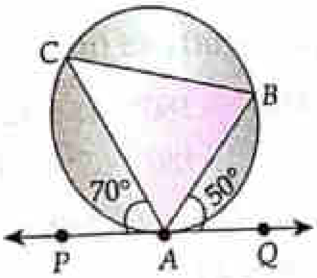
$\angle BCA = \angle BAC = 30°$

(Angles opposite to equal sides of a triangle are equal)

$\therefore$ $\angle ECD = \angle BCD - \angle BCE$

$= 80° - 30° = 50°$

18. In the figure given below, find each angle of ΔABC.



Solution : $\because$ PQ touches circle at A.

$\therefore$ $\angle BAQ = \angle ACB = 50°$ *[Angle in the alternate segment]*

Similarly, $\angle PAC = \angle ABC = 70°$

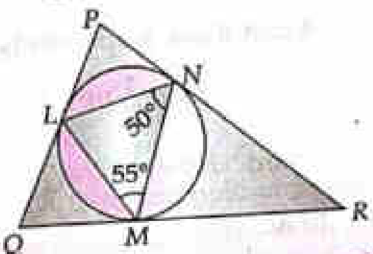
but, $\angle PAC + \angle CAB + \angle BAQ = 180°$ *[$\because$ P, A, Q are collinear]*

$\therefore$ $70° + \angle CAB + 50° = 180°$

$\therefore$ $CAB = 180° - (70° + 50°) = 60°$

Hence, angles of ΔABC are, $\angle A = 60°$, $\angle B = 70°$ and $\angle C = 50°$.

19. In the given figure PQ, QR and RP touches a given circle respectively at point L, M and N. If $\angle LMN = 55°$ and $\angle MNL = 50°$, then find $\angle P$, $\angle Q$ and $\angle R$



Solution : In ΔLMN

$\angle MLN + \angle LMN + \angle MNL = 180°$

$\therefore$ $\angle MLN + 55° + 50° = 180°$

[Given that $\angle LMN = 55°$ and $\angle MNL = 50°$] ... (i)

$\therefore$ $\angle MLN = 180° - (55° + 50°) = 75°$

In ΔPNL, $PN = PL$ [length of tangent from P to the circ

$\therefore \angle PNL = \angle PLN$

but, $\angle PNL = \angle NML = 55°$

[angle in the alternate segment on chord

$\therefore \angle PNL = \angle PLN = 55°$

Hence, $\angle LPN = 180° - (55 + 55°) = 70°$

[Sum of angles of ΔPLN = 180

Again, in ΔRMN, $RN = RM$ [length of tangent from R to the circ

$\therefore \angle RNM = \angle RMN$

but, $\angle PNL + \angle LNM + \angle MNR = 180°$

[P, N, R are collinear

or, $55° + 30° + \angle MNR = 180°$

$\therefore \angle MNR = 180° - (50° + 55°) = 75°$

$\Rightarrow \angle RMN = 75°$ (from (iii) and (iv))

Now in ΔRMN, $\angle MRN = 180° - (\angle RMN + \angle RNM)$

$= 180° - (75° + 75°) = 30°$

Now, In ΔPQR, $\angle P + \angle Q + \angle R = 180°$

$\therefore \angle Q = 180° - (\angle P + \angle R)$

$= 180° - (70° + 30°) = 180° - 100° = 80°$ [from (ii) and

$\therefore \angle P = 70°, \angle Q = 80°$ and $\angle R = 30°$;

20. PQ is a line segment and R is its midpoint. Semicircles are drawn at the same side of PQ taking PR, RQ and PQ as diameters. A circle of radius r and centre O is drawn touching all the three semi circles.

Prove that $r = \frac{1}{6}PQ$

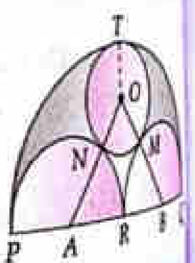
Solution : Let, $PQ = x$

$\therefore PR = RQ = \frac{1}{2}PQ = \frac{x}{2}$ ... (i)

A and B are respectively midpoints of PR and RQ.

$\therefore AR = \frac{1}{2}PR = \frac{x}{4}$ [from (i)]

and $RB = \frac{1}{2}RQ = \frac{x}{4}$



In ΔOAB

$OA = ON + NA = r + \frac{x}{4}$

$OB = OM + MB = r + \frac{x}{4}$

$\therefore \quad OA = OB$

Thus ΔOAB is an isosceles triangle and R is midpoint of its base AB.

$\therefore \quad OR \perp AB$

Now from right angled ΔORA,

$$OA^2 = OR^2 + AR^2$$

or, $(ON + NA)^2 = OR^2 + AR^2$

or, $\left(\frac{x}{4}+r\right)^2 = (RT - TO)^2 + \left(\frac{x}{4}\right)^2$ $\quad$ $[\because \; OR = RT - TO]$

or, $\frac{x^2}{16}+r^2+\frac{1}{2}x \cdot r = \left(\frac{x}{2}-r\right)^2 + \frac{x^2}{16}$ $\quad$ $[\because \; RT = RQ = \frac{x}{2}]$

or, $\frac{x^2}{16}+r^2+\frac{xr}{2} = \frac{x^2}{4} - rx + r^2 + \frac{x^2}{16}$

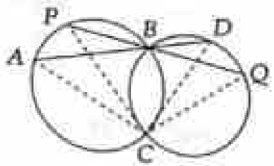
or, $\frac{3}{2}xr = \frac{x^2}{4}$

or, $3r = \frac{x}{2}$

or, $r = \frac{x}{6}$

$\therefore \quad r = \frac{1}{6} PQ$ **Proved**

21. In the given figure two circles intersect at two points B and C. Two line segments ABD and PBQ passing through point B, intersects circles respectively at A, D and P, Q. Prove that $\angle ACP = \angle QCD$



Solution : Since angle in the same segment are equal

$\therefore \quad \angle ACP = \angle ABP,$ $\quad$ (take AP as base) $\quad$ ... (i)

$\angle QCD = \angle QBD,$ $\quad$ (take QD as base) $\quad$ ... (ii)

and $\angle ABP = \angle QBD$ $\quad$ (vertically opposite angle) $\quad$ ... (iii)

from Adding (i), (ii) and (iii)

$$\angle ACP = \angle QCD$$

22. If two circles are drawn taking any two sides to the triangle as diameter then prove that point of intersection of two circles lies on the third side.

Solution : Let ABC be a triangle. Two circles are taking AB and AC as diameters. Both circles intersect

Prove To : Point D lies on line BC.

Join $A-D$

Since AB and AC are diameter of two circles and angle in a semi circle is right angle.

$\therefore \angle ADB = 90°$ and $\angle ADC = 90°$

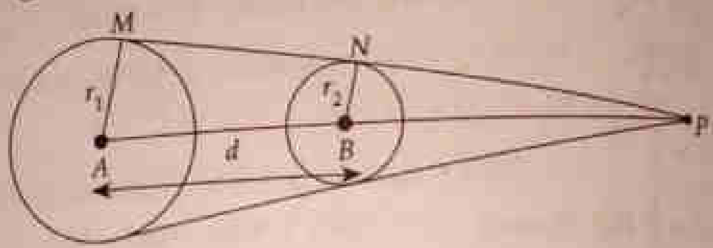
Adding $\angle ADB + \angle ADC = 90° + 90° = 180°$

i.e. BDC is a straight line

Thus point D lies on line BC. Proved.

Exercise—8A

Instruction (1 – 6) : Answer the questions given below on the basis following figure.



A and B are centres of the circles whose radii are respectively r_1 and r_2. PNM is a direct common tangent touching the circles respectively at M and N.

1. Length of MN is

(a) $\sqrt{d^2-(r_1-r_2)^2}$ (b) $\sqrt{d^2+(r_1-r_2)^2}$

(c) $d^2-(r_1-r_2)^2$ (d) $d+\sqrt{(r_1-r_2)^2}$

2. Ratio $PA : PB$ equals

(a) $r_1 : r_2$ (internal) (b) $r_1 : r_2$ (external)

(c) $r_2 : r_1$ (internal) (d) $r_2 : r_1$ (external)

3. Length of AP is

(a) $\frac{r_1 d}{r_1+r_2}$ (b) $\frac{r_2 d}{r_1+r_2}$ (c) $\frac{r_1 d}{r_1-r_2}$ (d) $\frac{r_2 d}{r_1-r_2}$

4. Length of BP is

(a) $\frac{r_1 d}{r_1+r_2}$ (b) $\frac{r_2 d}{r_1+r_2}$ (c) $\frac{r_1 d}{r_1-r_2}$ (d) $\frac{r_2 d}{r_1-r_2}$

Distance between P and N is

(a) $\frac{r_2}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$ (b) $\frac{r_1}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$

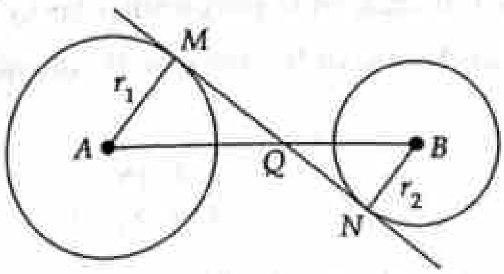
(c) $\frac{r_2}{r_1+r_2}\sqrt{d^2-(r_1-r_2)^2}$ (d) $\frac{r_1}{r_1+r_2}\sqrt{d^2-(r_1-r_2)^2}$

Distance between P and M is

(a) $\frac{r_2}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$ (b) $\frac{r_1}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$

(c) $\frac{r_2}{r_1+r_2}\sqrt{d^2-(r_1-r_2)^2}$ (d) $\frac{r_1}{r_1+r_2}\sqrt{d^2-(r_1-r_2)^2}$

Instruction (7 – 12) : Answer the questions given below on the basis of following figure



MN is a transverse common tangent. A and B are centres of the circles whose radii are respectively r_1 and r_2. Length of AB is d.

Length of MN is

(a) $\sqrt{d^2-(r_1+r_2)^2}$ (b) $\sqrt{d^2-(r_1-r_2)^2}$

(c) $\sqrt{d^2+(r_1-r_2)^2}$ (d) $\sqrt{d^2+(r_1+r_2)^2}$

Ratio $AQ : QB$ equals

(a) $r_1 : r_2$ (external) (b) $r_1 : r_2$ (internal)

(c) $r_2 : r_1$ (internal) (d) $r_2 : r_1$ (external)

Length of AQ is

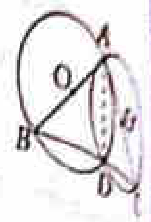
(a) $\frac{r_1 d}{r_1-r_2}$ (b) $\frac{r_2 d}{r_1-r_2}$ (c) $\frac{r_1 d}{r_1+r_2}$ (d) $\frac{r_2 d}{r_1+r_2}$

Length of BQ is

(a) $\frac{r_1 d}{r_1-r_2}$ (b) $\frac{r_2 d}{r_1-r_2}$

(c) $\frac{r_1 d}{r_1+r_2}$ (d) $\frac{r_2 d}{r_1+r_2}$

Prove To : Point D lies on line BC.



Join $A-D$

Since AB and AC are diameter of two circles and angle in a semi circle is right angle.

$\therefore \quad \angle ADB = 90^\circ$ and $\angle ADC = 90^\circ$

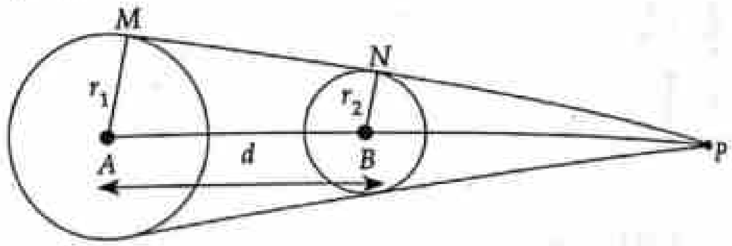
Adding $\angle ADB + \angle ADC = 90^\circ + 90^\circ = 180^\circ$

i.e. BDC is a straight line

Thus point D lies on line BC. Proved.

Exercise—8A

Instruction (1 – 6) : Answer the questions given below on the basis of following figure.



A and B are centres of the circles whose radii are respectively r_1 and r_2. PNM is a direct common tangent touching the circles respectively at M and N.

1. Length of MN is

(a) $\sqrt{d^2-(r_1-r_2)^2}$ (b) $\sqrt{d^2+(r_1-r_2)^2}$

(c) $d^2-(r_1-r_2)^2$ (d) $d+\sqrt{(r_1-r_2)^2}$

2. Ratio $PA : PB$ equals

(a) $r_1 : r_2$ (internal) (b) $r_1 : r_2$ (external)

(c) $r_2 : r_1$ (internal) (d) $r_2 : r_1$ (external)

3. Length of AP is

(a) $\frac{r_1 d}{r_1+r_2}$ (b) $\frac{r_2 d}{r_1+r_2}$ (c) $\frac{r_1 d}{r_1-r_2}$ (d) $\frac{r_2 d}{r_1-r_2}$

4. Length of BP is

(a) $\frac{r_1 d}{r_1+r_2}$ (b) $\frac{r_2 d}{r_1+r_2}$ (c) $\frac{r_1 d}{r_1-r_2}$ (d) $\frac{r_2 d}{r_1-r_2}$

5. Distance between P and N is

(a) $\frac{r_2}{r_1 - r_2}\sqrt{d^2 - (r_1 - r_2)^2}$

(b) $\frac{r_1}{r_1 - r_2}\sqrt{d^2 - (r_1 - r_2)^2}$

(c) $\frac{r_2}{r_1 + r_2}\sqrt{d^2(r_1 - r_2)^2}$

(d) $\frac{r_1}{r_1 + r_2}\sqrt{d^2 - (r_1 - r_2)^2}$

6. Distance between P and M is

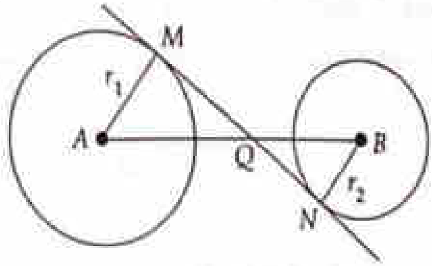
(a) $\frac{r_2}{r_1 - r_2}\sqrt{d^2 - (r_1 - r_2)^2}$

(b) $\frac{r_1}{r_1 - r_2}\sqrt{d^2 - (r_1 - r_2)^2}$

(c) $\frac{r_2}{r_1 + r_2}\sqrt{d^2 - (r_1 - r_2)^2}$

(d) $\frac{r_1}{r_1 + r_2}\sqrt{d^2 - (r_1 - r_2)^2}$

Instruction (7 – 12) : Answer the questions given below on the basis of following figure



MN is a transverse common tangent. A and B are centres of the circles whose radii are respectively r_1 and r_2. Length of AB is d.

7. Length of MN is

(a) $\sqrt{d^2 - (r_1 + r_2)^2}$

(b) $\sqrt{d^2 - (r_1 - r_2)^2}$

(c) $\sqrt{d^2 + (r_1 - r_2)^2}$

(d) $\sqrt{d^2 + (r_1 + r_2)^2}$

8. Ratio $AQ : QB$ equals

(a) $r_1 : r_2$ (external)

(b) $r_1 : r_2$ (internal)

(c) $r_2 : r_1$ (internal)

(d) $r_2 : r_1$ (external)

9. Length of AQ is

(a) $\frac{r_1 d}{r_1 - r_2}$ (b) $\frac{r_2 d}{r_1 - r_2}$ (c) $\frac{r_1 d}{r_1 + r_2}$ (d) $\frac{r_2 d}{r_1 + r_2}$

10. Length of BQ is

(a) $\frac{r_1 d}{r_1 - r_2}$

(b) $\frac{r_2 d}{r_1 - r_2}$

(c) $\frac{r_1 d}{r_1 + r_2}$

(d) $\frac{r_2 d}{r_1 + r_2}$

11. Distance between point Q and M is

(a) $\frac{r_1}{r_1+r_2}\sqrt{d^2-(r_1+r_2)^2}$
(b) $\frac{r_2}{r_1+r_2}\sqrt{d^2-(r_1+r_2)^2}$
(c) $\frac{r_1}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$
(d) $\frac{r_2}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$

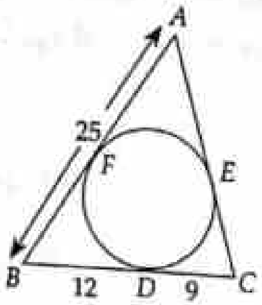
12. Distance between point Q and N is

(a) $\frac{r_1}{r_1+r_2}\sqrt{d^2-(r_1+r_2)^2}$
(b) $\frac{r_2}{r_1+r_2}\sqrt{d^2-(r_1+r_2)^2}$
(c) $\frac{r_1}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$
(d) $\frac{r_2}{r_1-r_2}\sqrt{d^2-(r_1-r_2)^2}$

13. Two circles cut each other at points P and Q. Centres of two circles are respectively A and B and PA is perpendicular to PB. If AB intersects segment PQ at R and ratio of the two circles are respectively 16 : 9 then $AR : BR$ is

(a) 4 : 3
(b) 16 : 9
(c) 256 : 81
(d) $2 : \sqrt{3}$

14. In the given figure length of side AC is



(a) 20
(b) 22
(c) 21
(d) 18

15. PQ is a line segment of 12 cm whose midpoint is R. Taking PR, RQ and PQ as diameters semicircles are drawn at the same side of PQ. The area of the circle that touches all the three circles is

(a) 2π sq. cm
(b) 4π sq. cm
(c) 6π sq. cm
(d) $\frac{9}{4}$ sq. cm

16. The difference in lengths of parallel sides of a trapezium inscribed in a circle is 6 cm; if distance between parallel sides is 4 cm then difference in lengths of its non parallel side is

(a) 10 cm
(b) 5 cm
(c) 0 cm
(d) Cannto be determined

17. The length of two perpendicular chords of a circle are respectively $2a$ and $2b$. If distance of its point on intersection from the centre is c, then what is the radius of circle ?

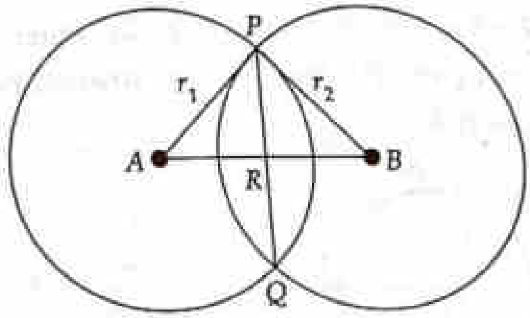
(a) $\frac{\sqrt{a^2+b^2+c^2}}{2}$

(b) $\sqrt{\frac{a^2+b^2+c^2}{2}}$

(c) $\frac{\sqrt{a^2+b^2-c^2}}{2}$

(d) $\sqrt{\frac{a^2+b^2-c^2}{2}}$

18. In the given figure A and B are centres of the circles. If $AR = a$, $RB = b$ then which of the following is equal to $a - b$?



(a) $\frac{r_1^2 - r_2^2}{\sqrt{r_1^2+r_2^2}}$

(b) $\frac{r_1^2 + r_2^2}{\sqrt{r_1^2-r_2^2}}$

(c) $\frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}}$

(d) None of these

19. A circle with radius r has a chord PQ whose length is $2a$. The tangents drawn at points P and Q to the circle meet at T, what is the length of TP ?

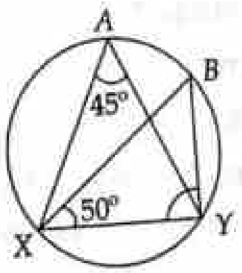
(a) $\frac{ar}{\sqrt{r^2-a^2}}$

(b) $\frac{2ar}{\sqrt{r^2-a^2}}$

(c) $\frac{r^2+a^2}{\sqrt{r^2-a^2}}$

(d) $\frac{ar}{r-a}$

20. In the figure given below what is the measure of $\angle BYX$?



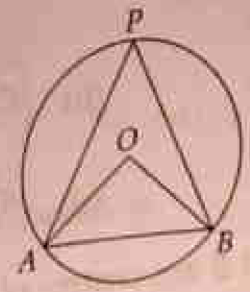
(a) 85°

(b) 50°

(c) 45°

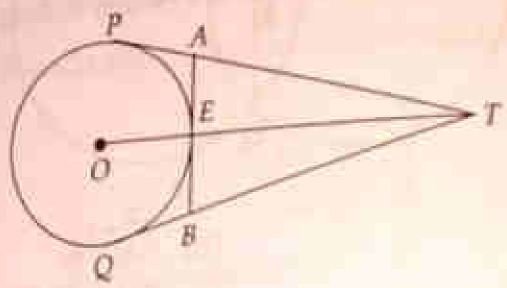
(d) 90°

21. In the figure given below, radius OA is equal to chord AB. What is the measure of $\angle APB$?



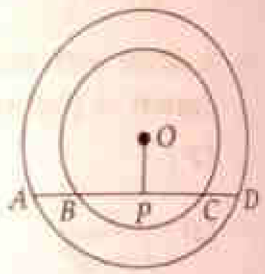
(a) $30°$ (b) $60°$ (c) $15°$ (d) $45°$

22. From a point T which is 13 cm away from center O of a circle whose radius is 5 cm, tangents PT and QT are drawn. What is the length of AB in figure given below.



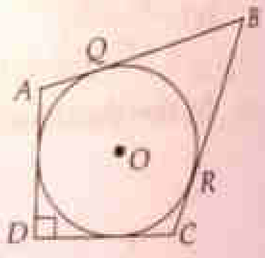
(a) $\frac{19}{3}$ cm (b) $\frac{20}{3}$ cm (c) $\frac{40}{13}$ cm (d) $\frac{22}{3}$ cm

23. In the adjacent figure AD is a straight line. OP is perpendicular to AD and O is centre of both circles. If $OA = 20$ cm, $OB = 15$ cm and $OP = 12$ cm then what is the length of AB ?



(a) 7 cm (b) 8 cm
(c) 10 cm (d) 12 cm

24. In the adjacent figure, a circle is inscribed in the quadrilateral $ABCD$. If $BC = 38$ cm, $QB = 27$ cm, $DC = 25$ cm and AD is perpendicular to DC then what is the radius of the circle ?

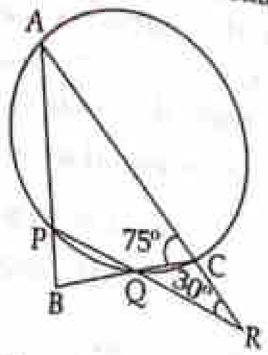


(a) 11 cm (b) 14 cm
(c) 15 cm (d) 16 cm

25. Each side of a quadrilateral touches a circle. If length of its three consecutive sides are 6 cm, 7 cm and 5 cm then what is length of its fourth side ?

(a) 3 cm (b) 4 cm
(c) 5 cm (d) 8 cm

26. In the figure given below, what is the measure of $\angle CBA$?



(a) 30° (b) 45° (c) 50° (d) 60°

27. A, B, C, D are four distinct points on a circle whose centre is O. If $\angle OBD - \angle CDB = \angle CBD - \angle ODB$ then what is the measure of $\angle A$?

(a) 45° (b) 60° (c) 120° (d) 135°

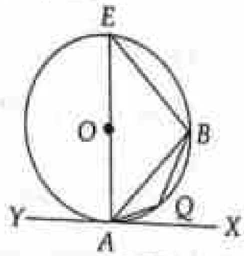
28. PQ is a common chord of the two circles. APB is a secant line joining points A and B respectively on the two circles. Two equal tangents AC and BC are drawn. If $\angle ACB = 45°$ then which is equal to $\angle AQB$?

(a) 75° (b) 90° (c) 120° (d) 135°

29. $ABCD$ is a cyclic quadrilateral. Tangents at A and C intersect at P. If $\angle ABC = 100°$ then what is the measure of $\angle APC$?

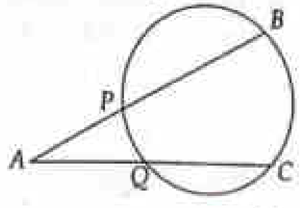
(a) 10° (b) 20° (c) 30° (d) 40°

30. In the adjacent figure, YAX is a tangent to the circle with centre O. If $\angle BAX = 70°$ and $\angle BAQ = 40°$ then what is $\angle ABQ$?



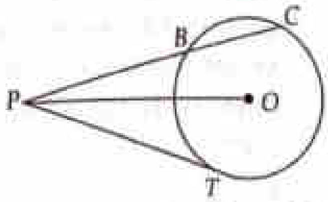
(a) 20° (b) 30°
(c) 35° (d) 40°

31. In the adjacent figure, $AP = 3$ cm, $PB = 5$ cm, $AQ = 2$ cm and $QC = x$. What is the value of x ?



(a) 6 cm (b) 8 cm
(c) 10 cm (d) 12 cm

32. In the adjacent figure PT is tangent to the circle with radius 6 cm. If distance between point P and centre O is 10 cm and $PB = 5$ cm, then is the length of chord BC ?

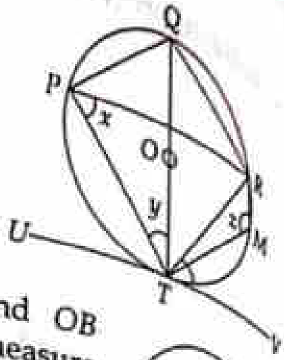


(a) 7.8 cm (b) 8.0 cm
(c) 8.4 cm (d) 9.0 cm

33. A point moves such that its distance from two fixed points A and B always remains same. What is the locus of point P ?

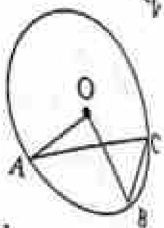
(a) a straight line which is perpendicular bisector to AB
(b) a circle whose centre is A
(c) a circle whose centre is B
(d) a straight line passing thorough either A or B.

34. In the adjacent figure O is the centre of the circle. At a point T on the circle tangent $\angle UTV$ is draw. If $\angle VTR = 52°$ and triangle PTR is an isosceles triangle such that $TP = TR$ then $\angle x + \angle y + \angle z$ is equal to—



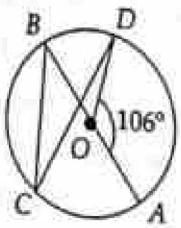
(a) 175° (b) 208°
(c) 218° (d) 250°

35. In the adjacent figure, $\angle AOB = 46°$; AC and OB mutually intersect at right angle. What is the measure of $\angle OBC$ where O is the centre of the circle.



(a) 44° (b) 46°
(c) 67° (d) 78.5°

36. In the figure given below O is the centre of the circle and $\angle AOD = 106°$. $\angle BCD$ is equal to

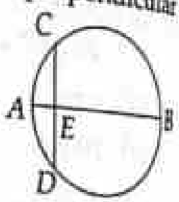


(a) 53° (b) 43° (c) 40° (d) 37°

37. How many circles can pass through a given pair of points ?

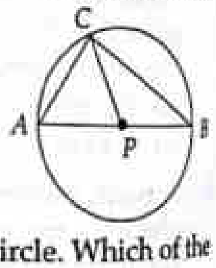
(a) one (b) only two
(c) more than two but finite (d) infinitely many

38. In the given figure AB is a diameter of the circle and CD is perpendicular to AB. If $AB = 10$ cm and $AE = 2$ cm then what is the length of ED ?



(a) 5 cm (b) 4 cm
(c) $\sqrt{10}$ cm (d) $\sqrt{20}$ cm

39. In the given figure A and B are extremities of a diameter of a circle with centre P and C is a point on the circumference of the circle such that $\angle ABC = 35°$. What is the measure of $\angle PCA$?



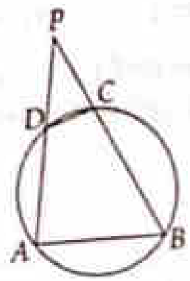
(a) 25° (b) 30°
(c) 35° (d) 55°

40. $ABCD$ is a quadrilateral whose sides touch a given circle. Which of the following is true regarding above statement ?

(a) $AB + AD = CB + CD$ (b) $AB : CD = AD : BC$
(c) $AB + CD = AD + BC$ (d) $AB : AD = CB : CD$

41. Suppose PAB is a secant to a circle which intersects circle at A and B and PC is a tangent. Which of the following is true ?
(a) Area of the rectangle with PA, PB as adjacent sides is equal to area of square whose side is PC
(b) Area of the rectangle with PA, PC as adjacent sides is equal to area of square whose each side is PB
(c) Area of the rectangle with PC, PB as adjacent sides is equal to area of square whose each side is PA
(d) Perimeter of the rectangle with PA, PB as adjacent side is equal to perimeter of the square whose each side is PC

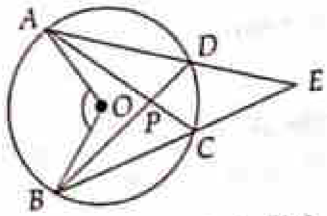
42. In the figure given below if $\angle BAD = 60^\circ$, $\angle ADC = 105^\circ$ then $\angle DPC$ is equal to



(a) 40° (b) 45° (c) 50° (d) 60°

43. In the given figure O is centre and PQ is a diameter of the circle. If $\angle ROS = 44^\circ$ and OR is bisector of $\angle PRQ$ then measure of $\angle RTS$ is
(a) 46° (b) 64°
(c) 69° (d) None of these

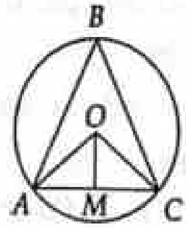
44. In the figure given below O is the centre of the circle while AC and BD intersect at P. If $\angle AOB = 100^\circ$ and $\angle DAP = 30^\circ$ then what is the measure of $\angle APB$?



(a) 77° (b) 80° (c) 85° (d) 90°

45. Suppose A and B are two fixed points. What is the locus of P if angle $APB = 90^\circ$?
(a) line AB itself
(b) Point P itself
(c) circumference of the circle having AB as diameter
(d) perpendicular bisector to line AB

46. In the figure given below O is centre of the circle, $OA = 3cm$, $AC = 3cm$ and OM is perpendicular to AC. What is the measure of $\angle ABC$?



(a) $60°$ (b) $45°$
(c) $30°$ (d) None of these

47. Two circles touch each other internally. Their radii are respectively 4 cm and 6 cm. What is the maximum length of chord of outer circle which lies out side the inner circle.

(a) $4\sqrt{2}$ cm (b) $4\sqrt{3}$ cm (c) $6\sqrt{3}$ cm (d) $8\sqrt{2}$ cm

48. Centres of two circles whose radii are respectively 4.5 cm and 3.5 cm are 10 cm apart. What is the length of transverse common tangent to the two circle ?

(a) 8 cm (b) 7 cm (c) 6 cm (d) None of these

49. If radii of two circles are respectively 6 cm and 3 cm and length of transverse common tangent to the two circles is 8 cm, then what is the distance between centres of the two circles ?

(a) 14 cm (b) $\sqrt{145}$ cm (c) $\sqrt{155}$ cm (d) 13 cm

50. ABC is an equilateral triangle inscribed in a circle with $AB = 5$ cm. Suppose bisector of angle A meets BC at X and circle at Y, then what is the value of $3\ AX.\ AY$?

(a) $16\ cm^2$ (b) $20\ cm^2$ (c) $25\ cm^2$ (d) $30\ cm^2$

51. Two unequal circles touch each other externally at point P. If APB and CPD are two secants intersecting circles at A, B, C and D then which of the following is true ?

(a) $ACBD$ is a parallelogram (b) $ACBD$ is a trapezium
(c) $ACBD$ is a rhombus (d) None of the above

52. Suppose C is a given circle. A variable point P moves such that tangents drawn from P to the circle C always subtends an angle $60°$. What is the locus of P ?

(a) a straight line
(b) a circle concentric with circle C
(c) a circle touching circle C
(d) a circle intersecting circle C at two distinct points

53. $ABCD$ is a cyclic quadrilateral and $A + B = 2(C + D)$. If $\angle C > 30°$, then which of the following is true

(a) $\angle D \geq 90°$ (b) $\angle D < 90°$ (c) $\angle D \leq 90°$ (d) $\angle D > 90°$

54. If diameters of two circles are 6 units and 10 units and their centres are 8 unit apart, how many tangents line can be drawn to the circle ?

(a) 1 (b) 2 (c) 3 (d) 4

55. Point A is situated at a distance of 6.5 cm from the centre of a circle. The length of tangent drawn from point A to the circle is 6 cm. What is the radius of the circle ?

(a) 5 cm (b) 4 cm (c) 3.5 cm (d) 2.5 cm

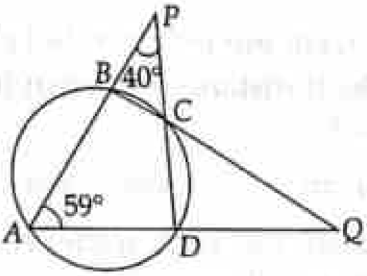
56. A circle with centre O is given and C is a point on its minor arc AB. If $\angle AOB = 100°$ then $\angle ACB$ is equal to which of the following ?

(a) 80° (b) 90° (c) 100° (d) 130°

57. ΔABC is a triangle with $AB = AC$. A circle passing through point P touches AC at D and cuts AB at P. If D is the midpoint of AC then which one of the following is true ?

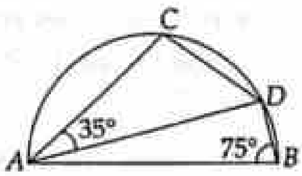
(a) $AB = 2AP$ (b) $AB = 3AP$ (c) $AB = 4AP$ (d) $2AB = 5AP$

58. In the figure given below $\angle PAQ = 59°$, $\angle APD = 40°$ then $\angle AQB$ is equal to which of following ?



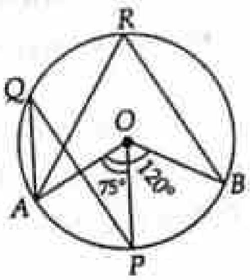
(a) 19° (b) 20° (c) 22° (d) 27°

59. In the adjacent figure C and D are two points on circumference of a semicircle having AB as diameter. Given that $\angle ABD = 75°$ and $\angle DAC = 35°$. What is the measure of $\angle BDC$?



(a) 130° (b) 110°
(c) 90° (d) 100°

60. In the figure given below, if $\angle AOP = 75°$ and $\angle AOB = 120°$, then what is $\angle AQP$?



(a) 45° (b) 37.5° (c) 30° (d) 22.5°

61. Length of two chords AB and AC of a circle are respectively 8 cm and 6 cm and $\angle BAC = 90°$. What is the radius of the circle ?
(a) 25 cm (b) 20 cm (c) 4 cm (d) 5 cm

62. The chord of circle whose radius is 5 cm touches another circle whose radius is 3 cm. If two circles are concentric, then what is length of the chord ?
(a) 10 cm (b) 12.5 cm (c) 8 cm (d) 7 cm

63. A chord of a circle is equal to its radius. Angle subtends by the chord on major arc of the circles is
(a) 30° (b) 45° (c) 60° (d) 90°

64. Radii of two concentric circles are, respectively 9 cm and 15 cm. If chord of larger circle touches the smaller one then length of the chord is
(a) 24 cm (b) 12 cm (c) 30 cm (d) 18 cm

65. Two chords AB and CD of a circle with centre O meet at an out side point P. If $\angle AOC = 50°$, $\angle BOD = 40°$ then what is the measure of $\angle BPD$?
(a) 60° (b) 40° (c) 45° (d) 5°

66. $AB = 8$ cm and $CD = 6$ cm are two parallel chords lie on same side of centre of a given circle. If distance between them is 1 cm then what is the radius of the circle ?
(a) 5 cm (b) 4 cm (c) 3 cm (d) 2 cm

67. What is the distance between two parallel chords each having length 8 cm of a circle of diameter 10 cm ?
(a) 6 cm (b) 7 cm (c) 8 cm (d) 5.5 cm

68. On the centres of two circles are of same length respectively subtend angle 60° and 75°. What is the ratio of radii of the two circles.
(a) 5 : 2 (b) 5 : 4 (c) 3 : 2 (d) 2 : 1

69. Tangents are drawn at extremities AB of a diameter of a circle centred at P. If tangents drawn at a point C on the circle meet the other two tangents respectively at Q and R then measure of $\angle QPR$ is
(a) 45° (b) 60° (c) 90° (d) 180°

70. AB is a chord to a given circle and PAT is a tangent to the circle at point A. If $\angle BAT = 75°$ and $\angle BAC = 45°$ and C is a point on the circle, then $\angle ABC$ is
(a) 40° (b) 45° (c) 60° (d) 70°

71. Consider a circle centred at O. Tangents at A and B to the circle intersects at P. In the quadrilateral $PAOB$ if $\angle AOB : \angle APB = 5 : 1$, then measure of $\angle APB$ is
(a) 30° (b) 60° (c) 45° (d) 15°

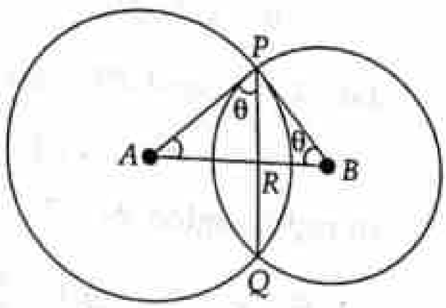
Answers—8A

1. (a)	2. (b)	3. (c)	4. (d)	5. (a)	6. (b)	7. (a)	8. (b)
9. (c)	10. (d)	11. (a)	12. (b)	13. (b)	14. (b)	15. (b)	16. (c)
17. (b)	18. (a)	19. (a)	20. (a)	21. (a)	22. (b)	23. (a)	24. (b)
25. (b)	26. (d)	27. (b)	28. (d)	29. (b)	30. (b)	31. (c)	32. (a)
33. (a)	34. (c)	35. (c)	36. (d)	37. (d)	38. (b)	39. (d)	40. (c)
41. (a)	42. (b)	43. (d)	44. (b)	45. (c)	46. (c)	47. (d)	48. (c)
49. (b)	50. (c)	51. (b)	52. (b)	53. (b)	54. (c)	55. (d)	56. (d)
57. (c)	58. (c)	59. (a)	60. (b)	61. (d)	62. (c)	63. (a)	64. (a)
65. (d)	66. (a)	67. (a)	68. (b)	69. (d)	70. (c)	71. (a)	

Explanation

For 1 to 12. See theory portion carefully.

13. (b) $\because \Delta ARP \sim PRB$



$$\frac{\text{area of }(\Delta ARP)}{\text{area of }(\Delta PRB)} = \left(\frac{AP}{PB}\right)^2$$

or, $$\frac{\frac{1}{2}\cdot AR\cdot PR}{\frac{1}{2}\cdot RB\cdot PR} = \frac{r_1^{\,2}}{r_2^{\,2}}$$

$$\Rightarrow \frac{AR}{RB} = \frac{r_1^2}{r_2^2} = \frac{16}{9}$$

14. (b) $BF = BD = 12$

$\therefore\ AF = 25 - 12 = 13 = AE$

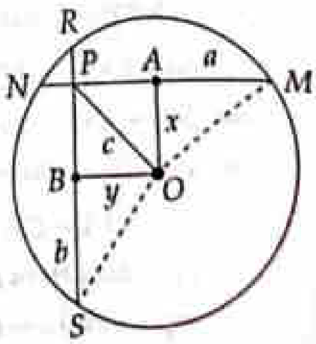
Hence, $AC = AE + EC = 13 + 9 = 22$

15. (b) See solved example 20

$r = \frac{PQ}{6} = \frac{12}{6} = 2$ cm $\qquad \therefore$ Area $= \pi r^2 = 4\pi\ \text{cm}^2$.

16. (c) We know that non parallel sides of a trapezium inscribed in a circle are equal. Thus required difference = 0 cm

17. (b) In figure, chord $MN = 2a$, chord $RS = 2b$



$OA \perp r\ MN \quad \Rightarrow \quad AM = a$

$OB \perp r\ RS \quad \Rightarrow \quad BS = b$

Let $OA = x$, $OB = y$

$\therefore\ PB = x$ and $PA = y$

In ΔOAM, $a^2 + x^2 = r^2$

In ΔOBS, $b^2 + y^2 = r^2$

adding $a^2 + b^2 + x^2 + y^2 = 2r^2 \qquad \ldots$ (i)

but in ΔOPA, $x^2 + y^2 = c^2$

$\therefore$ from (i), $a^2 + b^2 + c^2 = 2r^2$

or, $r = \sqrt{\dfrac{a^2+b^2+c^2}{2}}$

18. (a) $r_1^2 = PR^2 + a^2$ and $r_2^2 = PR^2 + b^2$

$r_1^2 - r_2^2 = a^2 - b^2$

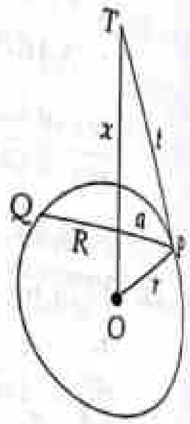
or, $(a-b)(a+b) = r_1^2 - r_2^2$

or, $(a-b) = \dfrac{r_1^2 - r_2^2}{a+b} = \dfrac{r_1^2 - r_2^2}{\sqrt{r_1^2 + r_2^2}}$ $\quad (\because a + b = AB = \sqrt{AP^2 + BP^2})$

19. (a) In the given figure,

$OR = \sqrt{r^2 - a^2}$

Let $RT = x$ and $PT = t$, then in right angle ΔTRP



$t^2 = x^2 + a^2 \quad \ldots$ (i)

In right angled ΔOPT,

$t^2 + r^2 = (x + OR)^2 = \left(x + \sqrt{r^2 - a^2}\right)^2$

or, $t^2 + r^2 = x^2 + r^2 - a^2 + 2x\sqrt{r^2 - a^2}$

or, $r^2 + a^2 + r^2 = r^2 + r^2 - a^2 + 2x\sqrt{r^2 - a^2}$

from (i), $t^2 = x^2 + a^2$)

or, $2a^2 = 2x\sqrt{r^2 - a^2}$

or,, $x = \dfrac{a^2}{\sqrt{r^2 - a^2}}$ $\qquad \therefore\ t^2 = x^2 + a^2 = \dfrac{a^4}{r^2 - a^2} + a^2 = \dfrac{a^4 + a^2 r^2 - a^4}{r^2 - a^2}$

or, $t = \dfrac{ar}{\sqrt{r^2 - a^2}}$

20. (a) We know that angles in the same segment of a circle are equal.

$\therefore$ $\angle XBY = \angle XAY = 45°$

In ΔBXY, $\angle BXY + \angle XBY + \angle BYX = 180°$

$\Rightarrow$ $50° + 45° + \angle BYX = 180°$

$\Rightarrow$ $\angle BYX = 180° - 95° = 85°$ $\quad (\because \angle BXY = 50°)$

21. (a) $\because$ $OA = OB = AB$ $\quad$ (given)

$\therefore$ ΔAOB is an equilateral triangle.

$\Rightarrow$ $\angle AOB = 60°$

We know that the angle at the centre in a circle is double the angle at circumference.

$\therefore$ $\angle AOB = 2\angle APB \quad \Rightarrow \quad \angle APB = \frac{60°}{2} = 30°$

22. (b) Given, $OT = 13$, $OP = 5$

$\therefore$ $PT = \sqrt{13^2 - 5^2} = 12$, $TE = OT - OE = 13 - 5 = 8$

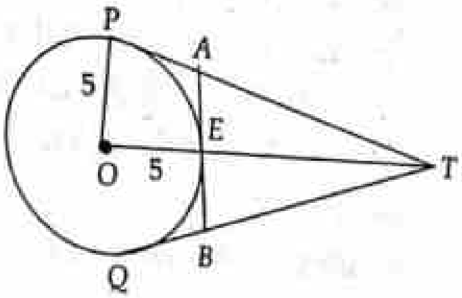
In ΔTOP and ΔTAE, angle at point T is common

and $\angle OPT = \angle AET = 90°$ (angle made by tangent line)

$\therefore$ $\Delta TOP \sim \Delta TAE \Rightarrow \frac{TO}{TA} = \frac{TP}{TE} = \frac{OP}{AE}$

$\Rightarrow \frac{13}{TA} = \frac{12}{8} = \frac{5}{AE}$

$\Rightarrow AE = \frac{5 \times 8}{12} = \frac{10}{3}$



$\therefore$ By Symmetry, $AB = 2AE = \frac{20}{3}$

23. (a) In ΔOPB,

$OB^2 = OP^2 + BP^2$

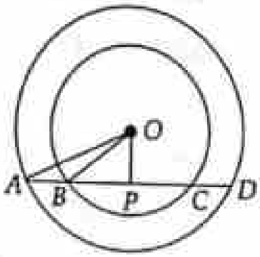
$\Rightarrow (15)^2 = (12)^2 + BP^2$

$\Rightarrow BP^2 = \sqrt{15^2 - 12^2} = 9$

and In ΔAOP,

$OA^2 = OP^2 + AP^2$

$\Rightarrow (20)^2 = (12)^2 + AP^2 \Rightarrow AP = \sqrt{20^2 - 12^2} = 16$



Hence $AB = AP - BP = 16 - 9 = 7$ cm

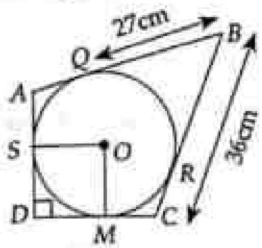
24. (b) $\because$ Length of tangents from an out side point are equal

$\therefore$ $BQ = BR = 27$

$\Rightarrow RC = 38 - 27 = 11$cm

$\therefore$ $RC = CM = 11$ cm

Now, $DM = 25 - 11 = 14$ cm = radius of circle



25 (b) See the figure

Let, $AP = AS = a$

$BP = BQ = b$

$CQ = CR = c$

and $DR = DS = d$

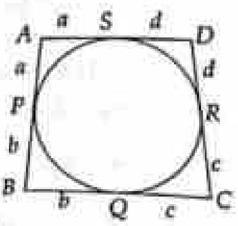
According to question,

$a + b = 6, b + c = 7, c + d = 5$

$\therefore$ $(a + b) + (c + d) - (b + c) = 6 + 5 - 7$

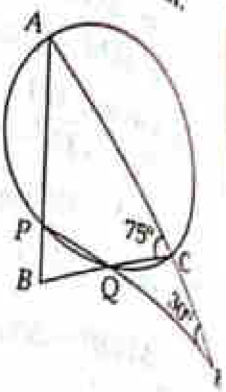
or, $a + d = 4$

or, $AD = 4$



26. (d) $\because$ Sum of opposite angles of a cyclic quadrilateral are equal.

$\therefore \quad \angle ACQ + \angle APQ = 180^\circ$



$\Rightarrow \quad 75^\circ + \angle APQ = 180^\circ$

$\Rightarrow \quad \angle APQ = 105^\circ$

$\because \quad \angle APQ + \angle BPQ = 180^\circ$

$\therefore \quad 105^\circ + \angle BPQ = 180^\circ$

or, $\angle BPQ = 180^\circ - 105^\circ = 75^\circ$

$\because \quad \angle ACQ$ is an external angle of ΔRCQ

$\therefore \quad \angle ACQ = \angle CRQ + \angle COR$

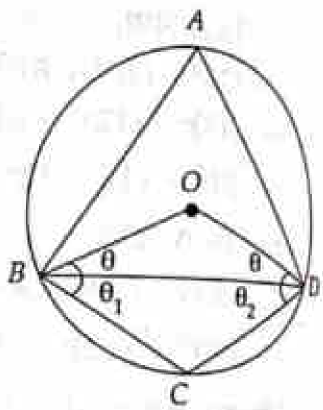
$\Rightarrow \quad 75^\circ = 30^\circ + \angle COR$

$\Rightarrow \quad \angle COR = 45^\circ$

In ΔBPQ, $\angle B = 180^\circ - 75^\circ - 45^\circ = 60^\circ$

27. (b) Given, $\angle OBD + \angle ODB = \angle CBD + \angle CDB$

Let $\angle OBD = \angle ODB = \theta$



and $\angle DBC = \theta_1, \angle BDC = \theta_2$

$\therefore \quad \theta + \theta = \theta_1 + \theta_2$

$\Rightarrow \quad 2\theta = \theta_1 + \theta_2 \quad \ldots$ (i)

$\therefore \quad \angle BOD = 180^\circ - 2\theta$

$\Rightarrow \quad \angle BCD = \dfrac{360^\circ - (180^\circ - 2\theta)}{2}$

$\Rightarrow \quad 180^\circ - (\theta_1 + \theta_2) = 90^\circ + \theta$

$\Rightarrow \quad 180^\circ - 2\theta = 90^\circ + \theta \quad \Rightarrow \quad \theta = 30^\circ$

$\therefore \quad \angle BOD = 120^\circ \quad \Rightarrow \quad \angle BAD = 60^\circ$

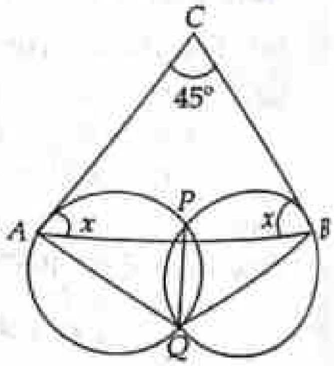
28. (d) Since AC and BC are equal, therefore $\angle CAB = \angle CBA$

let $\angle CAB = \angle CBA = x$

$\therefore \quad 45^\circ + x + x = 180^\circ$

$\Rightarrow \quad 2x = 180^\circ - 45^\circ$

$\Rightarrow \quad x = 67\frac{1}{2}^\circ$



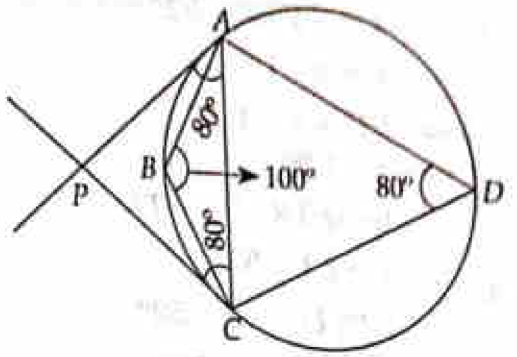
$\angle AQP = \angle x = \angle BQP = 67\frac{1}{2}^\circ$

(Angle at the alternate segment)

$\Rightarrow \quad \angle AQB = \angle AQP + \angle BQP$

$= 67\frac{1}{2}^\circ + 67\frac{1}{2}^\circ = 135^\circ$

(a) We know that sum of opposite angles of a cyclic quadrilateral is 180°



$\therefore \angle B + \angle D = 180^\circ$

$\Rightarrow 100 + \angle D = 180^\circ$

$\Rightarrow \angle D = 80$

$\therefore \angle ACP = \angle PAC = 80^\circ$ *(Angle at the alternate segment)*

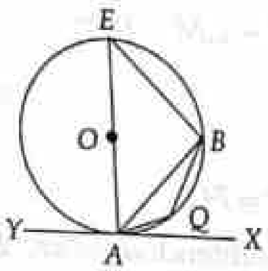
In ΔPAC,

$\angle P + \angle PAC + \angle PCA = 180^\circ$

$\Rightarrow \angle P + 80^\circ + 80^\circ = 180^\circ$

$\Rightarrow \angle P = 180^\circ - 160^\circ = 20^\circ$

(b) $\angle QAX = 70^\circ - 40^\circ = 30^\circ$



$\therefore \angle QAX = \angle ABQ = 30^\circ$ *[From theorem]*

11. (c) See result of article 1.10, we have

$AB \times AP = AC \times AQ$

$\Rightarrow 8 \times 3 = (2 + x) \times 2$

$\Rightarrow \frac{8 \times 3}{2} = 2 + x$

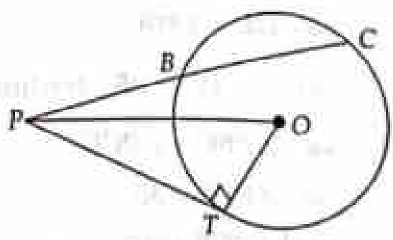
$\Rightarrow x = 10$ cm

12. (a) $PO = 10$ cm, radius $OT = 6$ cm, $PB = 5$ cm

In ΔOTP, $OP^2 = PT^2 + OT^2$

$\Rightarrow 10^2 = PT^2 + 6^2$

$\Rightarrow PT = 8$ cm



But, $PT^2 = PB \times PC$

$\Rightarrow 8^2 = 5 \times (BC + PB)$

$\Rightarrow 64 = 5\,(BC + 5)$

$\Rightarrow 5BC = 39 \quad \Rightarrow \quad BC = 7.8$ cm

13. (a) P will be perpendicular bisector of AB.

34. (c) $x = \angle VTR = 52°$ $\quad$ ($\because$ *PTMR is a cyclic quadrilateral*)

$x + z = 180°$

$\Rightarrow 52° + z = 180°$

$\Rightarrow z = 128°$

In ΔPTR, $PT = TR$

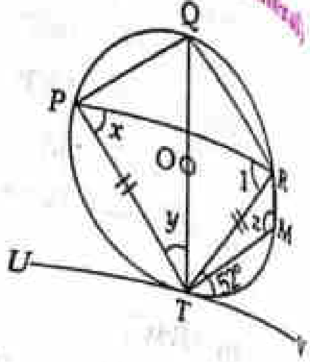
$\therefore x = \angle 1 = 52°$

$\angle PTU = \angle 1 = 52°$

$\angle QTU = y + 52°$

$\Rightarrow 90° = y + 52° \Rightarrow y = 38°$

$\therefore x + y + z = 52° + 38° + 128° = 218°$



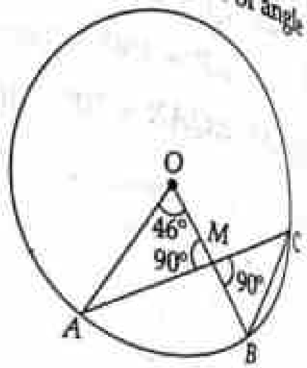
35. (c) Since angle subtended by are at circumference is half that of angle subtended at centre.

$\because \angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 46° = 23°$

In ΔMCB, $\angle C + \angle B + \angle M = 180°$

$\Rightarrow 23 + \angle B + 90° = 180°$

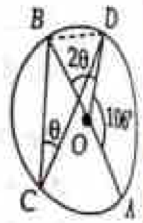
$\Rightarrow \angle B = 67°$



36. (d) $\angle BOD = 180° - 106° = 74°$

$\because$ $\angle BOD$, is the angle subtended by arc BD at centre and $\angle BCD$, is the angle subtended by arc BD at circumference

$\therefore \angle BCD = \frac{1}{2} \angle BOD = \frac{1}{2} \times 74° = 37°$



37. (d) For two given points, infinite circles can pass

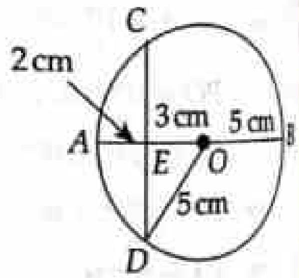
38. (b) In ΔOED,

$(OD)^2 = (DE)^2 + (EO)^2$

$\Rightarrow (5)^2 = (DE)^2 + (3)^2$

$\Rightarrow (DE)^2 = 25 - 9 = 16$

$\Rightarrow DE = 4$ cm



39. (d) $\because PC = PB$ (radius)

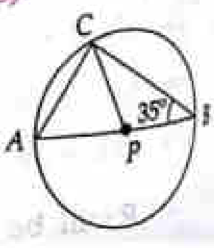
$\Rightarrow \angle PBC = \angle PCB$ $\quad$ *(angle opposite to equal sides)*

$\Rightarrow \angle PCB = 35°$

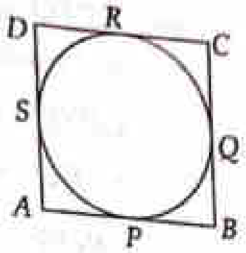
and $\angle ACB = 90°$ $\quad$ *(angle in a semicircle)*

$\Rightarrow \angle PCA + \angle PCB = 90°$

$\Rightarrow \angle PCA = 90° - 35° = 55°$



(c) We know that length of tangents drawn from an outside point to a given circle are equal.



$\therefore$ $AP = AS$

$BP = BQ$

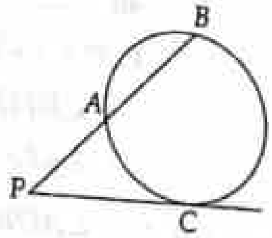
$CR = CQ$

$DR = DS$

$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow AB + CD = AD + BC$

(a) $\therefore PC^2 = PA \times PB$



Area of rectangle whose adjacent sides are PA and PB is equal to area of square whose each side is PC.

(b) $\because \angle BAD = 60^\circ, \quad \angle ADC = 105^\circ$

In cyclic quadrilateral $ABCD$,

$\angle BAD + \angle BCD = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 60^\circ = 120^\circ$

Now, $\angle BCD + \angle DCP = 180^\circ$ *(linear pair of angles)*

$\Rightarrow \angle DCP = 180^\circ - 120^\circ = 60^\circ$

and $\angle ADC + \angle CDP = 180^\circ$ *(linear pair of angles)*

$\Rightarrow 105^\circ + \angle CDP = 180^\circ \quad \therefore \angle CDP = 75^\circ$

Hence in ΔCPD, $\angle DCP + \angle CDP + \angle DPC = 180^\circ$

$\Rightarrow 60^\circ + 75^\circ + \angle DPC = 180^\circ$

$\Rightarrow \angle DPC = 180^\circ - 135^\circ = 45^\circ$

8. (d) $\because$ Line OR is bisector of $\angle PRQ$

$\therefore \angle PRO = \angle ORQ = 45^\circ$

Also $OP = OR$ *(radius)*

$\therefore \angle OPR = 45^\circ$

In ΔORS, $OR = OS$

$\Rightarrow \angle ORS = \angle OSR = \dfrac{180^\circ - 44^\circ}{2} = 68^\circ$

$\therefore \angle MRS = 68^\circ - 45^\circ = 23^\circ$

$\Rightarrow \angle PRS = 90^\circ + 23^\circ = 113^\circ$

Since sum of opposite angle of a cyclic quadrilateral is 180°

$\angle PRS + \angle PQS = 180^\circ$

$\Rightarrow \angle PQS = 180^\circ - 113^\circ = 67^\circ$

In ΔPTQ, $\angle QPT + \angle PQT + \angle PTQ = 180^\circ$

$\Rightarrow \angle PTQ = 180^\circ - 45^\circ - 67^\circ = 68^\circ$

Second method—

$\angle PRQ = 90^\circ$ (Angle in a semi

$\angle PRQ = \angle QRT = 90^\circ$

$\angle RQS = \frac{1}{2} \angle ROS = \frac{1}{2} \times 44^\circ = 22^\circ$

In ΔRTQ, $\angle QRT + \angle RQT + \angle RTQ = 180^\circ$

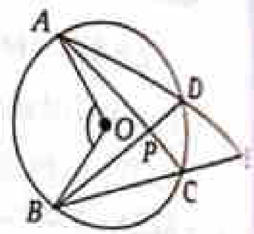
$90^\circ + 22^\circ + \angle RTQ = 180^\circ$

$112^\circ + \angle RTQ = 180^\circ$

$\angle RTQ = 68^\circ$

$\angle RTS = 68^\circ$

44. (b) $\because \angle ADB = \frac{1}{2} \angle AOB = 50^\circ$



In ΔDPA, $\angle DAP + \angle ADP + \angle DPA = 180^\circ$

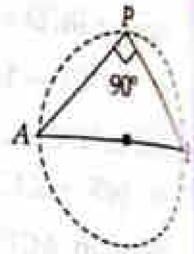
$\Rightarrow 30^\circ + 50^\circ + \angle DPA = 180^\circ$

$\Rightarrow \angle DPA = 100^\circ$

and $\angle DPA + \angle APB = 180^\circ$

$\Rightarrow \angle APB = 180^\circ - 100 = 80^\circ$

45. (c) Locus of point P is a circle with AB as one of the diameter because when P moves on the circle $\angle APB$ remain 90°. (Due to the fact that angle of a semicircle is 90°)



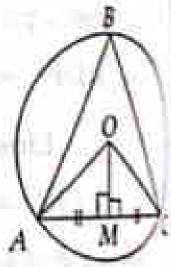
46. (c) $OA = OC = 3$ cm *(Radus of the circle)*

$\therefore$ $OA = OC = AC = 3$ cm

$\therefore$ ΔAOC is an equilatral triangle.

$\angle AOC = 60^\circ$

$\angle ABC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 60^\circ = 30^\circ$



47. (d) Since PQ is tangent to the internal circle, therefore $PQ \perp AB$. Clearly $OP = OQ$ and two chords of outer circle are AB and PQ.

$\therefore$ $OA \times OB = OP \times OQ$

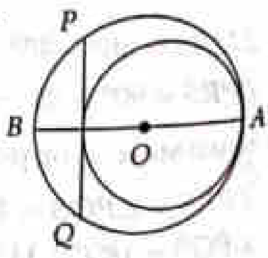
$\because$ OA = diameter of internal circle = 8 cm.

$OB = AB - OA = 12 - 8 = 4$ cm

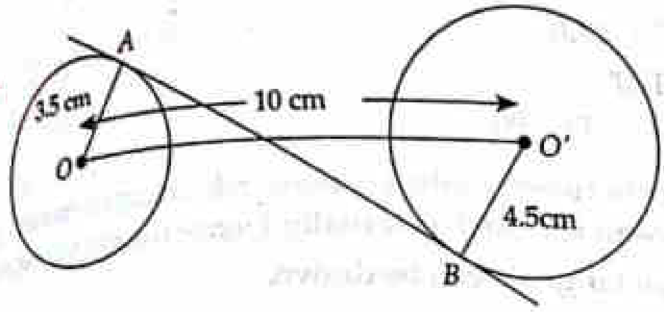
$\therefore$ $4 \times 8 = OP^2$

$\Rightarrow$ $OP = \sqrt{32} = 4\sqrt{2}$ cm

$\therefore$ $PQ = 2 \times 4\sqrt{2} = 8\sqrt{2}$ cm



(a)



length of transverse common tangent

$= \sqrt{(\text{distance between centres of circles})^2 - (\text{sum of radii})^2}$

$= \sqrt{10^2 - (4.5+3.5)^2} = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$ cm

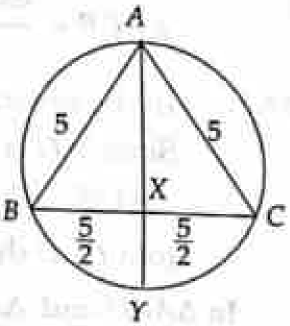
(b) length of transverse common tangent $= \sqrt{d^2 - (r_1 + r_2)^2}$

or, $8 = \sqrt{d^2 - (6+3)^2}$

or, $64 = d^2 - 81$

$\Rightarrow d^2 = 81 + 64 = 145 \quad \Rightarrow d = \sqrt{145}$

(c) In ΔABC,



$BX = \frac{5}{2}$ cm, $CX = \frac{5}{2}$ cm

and $AX = \frac{\sqrt{3}}{2} \times 5 = \frac{5\sqrt{3}}{2}$ cm

$\because$ AY and BC are two chords of circle

$\therefore \; AX \times XY = BX \times CX$

$\frac{5\sqrt{3}}{2} \times XY = \frac{5}{2} \times \frac{5}{2}$

$\Rightarrow XY = \frac{5}{2\sqrt{3}}$

$\therefore \; AX \cdot AY = \left(\frac{5\sqrt{3}}{2} + \frac{5}{2\sqrt{3}}\right) \times \frac{5}{2\sqrt{3}} = \frac{25}{3}$ cm^2

(b) $ABCD$ will be a trapezium

(b) Locus of P is a circle which is concentric with circle 'C'.

(b) In cyclic quadrilateral $ABCD$,

$\angle A + \angle C = 180° \Rightarrow \angle A = 180° - \angle C$ ... (i)

and $\angle B + \angle D = 180° \Rightarrow \angle B = 180° - \angle D$ ... (ii)

It is given that

$\angle A + \angle B = 2(\angle C + \angle D)$

$\Rightarrow 180° - \angle C + 180° - \angle D = 2(\angle C + \angle D)$ (from equation (i) and (ii))

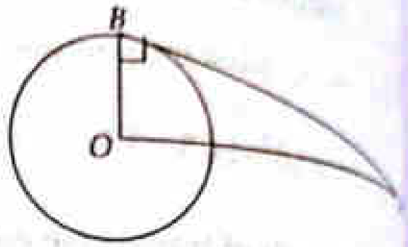
$\Rightarrow$ $360° = 3(\angle C + \angle D)$

$\Rightarrow$ $\angle C + \angle D = 120°$

But, $C > 30°$ $\therefore$ $D < 90°$

54. (c) Since distance between centres of two circles is equal to sum of the radii, the two circles touch externally. Hence maximum number of three common tangents can be drawn.

55. (d) $\because$ $AB = 6$ cm and $OA = 6.5$ cm



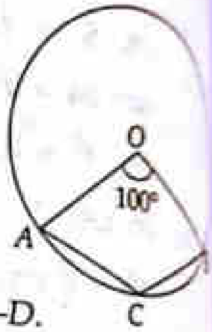
$\therefore$ In ΔOAB,

$$OB = \sqrt{OA^2 - AB^2}$$

$$= \sqrt{(6.5)^2 - (6)^2}$$

$$= \sqrt{42.25 - 36} = \sqrt{6.25} = 2.5 \text{ cm}$$

56. (d) $\because$ $\angle AOB = 100°$



$\therefore$ external $\angle AOB = 360° - \angle AOB$

$$= 360° - 100° = 260°$$

$$\angle ACB = \frac{\text{external } \angle AOB}{2} = \frac{260°}{2} = 130°$$

57. (c) In the given figure D is midpoint of AC. Join B–D.

Since AD is tangent line and BD is diameter

$\therefore$ $\angle ADB = 90°$

Join P–D then $\angle APD = 90°$ *(Angle in a semicircle is right angle)*

In ΔAPD and ΔADB, $\angle A =$ common,

$\angle ADB = 90° = \angle APD$

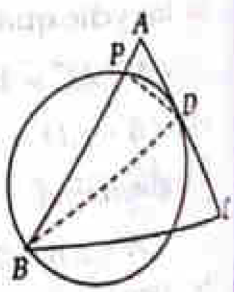
$\therefore$ $\Delta APD \sim \Delta ADB$

$$\Rightarrow \frac{AP}{AD} = \frac{AD}{AB}$$

$$\Rightarrow \frac{AP}{AB} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$\therefore$ ($\because$ $AD = \frac{1}{2}AC = \frac{1}{2}AB$) $\Rightarrow 4AP = AB$

58. (c) Given, $\angle PAD = 59°$ and $\angle APD = 40°$



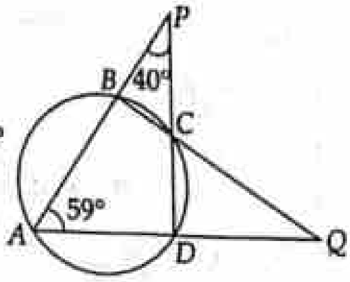
In ΔAPD, $\angle PAD + \angle APD + \angle ADP = 180°$

$\Rightarrow$ $59° + 40° + \angle ADP = 180°$

$\Rightarrow$ $\angle ADP = 81°$

and $\angle ABC + \angle ADC = 180°$

$\Rightarrow$ $\angle ABC = 180° - 81° = 99°$



$\therefore$ $\angle CDQ = \angle ABC = 99°$

and $\angle QCD = \angle BAD = 59°$

In ΔCQD, $\angle CQD + \angle CDQ + \angle QCD = 180°$

$\Rightarrow$ $\angle CQD = 180° - 59° - 99°$

$= 180° - 158° = 22°$

$\Rightarrow$ $\angle AQB = \angle CQD = 22°$

59. (a) $\angle ADB = 90°$ *(Angle in a semicircle)*

$\angle DAB = 15°$

$\therefore$ $\angle CAB = 35° + 15° = 50°$

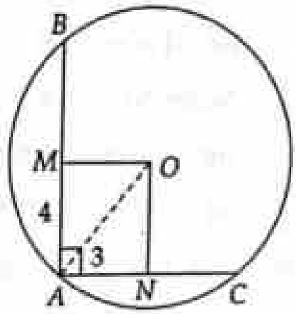
Hence, $\angle BDC = 180° - 50° = 130°$

(opposite angle of a cyclic quadrilateral)

60. (b) $\angle AQP = \frac{1}{2} \times \angle AOP$

$= \frac{75°}{2} = 37.5°$

61. (d) See the figure



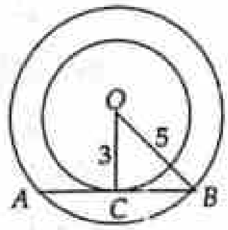
$AM = ON = \frac{8}{2} = 4$ cm

$AN = OM = \frac{6}{2} = 3$ cm

$\therefore$ radius of circle, $AO = \sqrt{(AN)^2 + (ON)^2}$

$= \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$ cm

62. (c) See the figure,

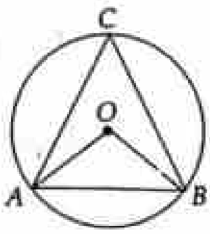


$BC = \sqrt{(OB)^2 - (OC)^2}$

$= \sqrt{(5)^2 - (3)^2} = \sqrt{25 - 9} = 4$

Hence, $AB = 2 \times BC = 2 \times 4 = 8$ cm

63. (a) See the figure,

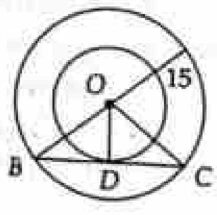


$\because$ $AO = OB = AB =$ radius

$\therefore$ $\angle AOB = 60°$

$\therefore$ $\angle ACB = 30°$

64. (a) $\because$ $BO = OC = 15$ cm

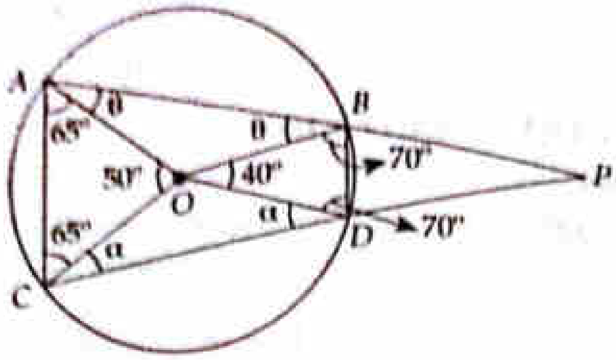


and $OD = 9$ cm

$\therefore$ $BD = \sqrt{15^2 - 9^2} = 12$ cm

$\therefore$ $BC = 2 \times 12 = 24$ cm

65. (d)



$\because$ ΔAOC is an isosceles triangle $\therefore$ $\angle OAC = \angle OCB = 65°$

$\because$ ΔBOD is an isosceles triangle $\therefore$ $\angle OBD = \angle ODB = 70°$

Let $\angle OAB = \angle OBA = \theta$ and $\angle OCD = \angle ODC = \alpha$

$\therefore$ In quadrilateral $ABCD$,

$2(\theta + 65° + \alpha + 70°) = 360°$

$\theta + \alpha + 135° = 180°$

or, $\theta + \alpha = 45°$

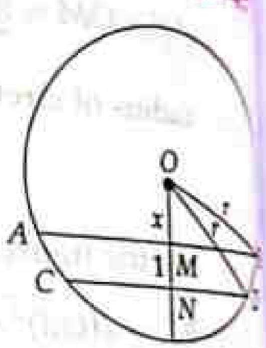
Now, In ΔAPC, $\angle APC + \theta + 65° + 65° + \alpha = 180°$

or, $\angle APC + 130° + 45° = 180°$ $(\because \theta + \alpha = 45°)$

or, $\angle APC = 180° - 175° = 5° = \angle BPD$.

66. (a) Let larger chord is x. cm away from centre.

see the figure



In ΔOMB,

$r^2 = x^2 + 4^2$

In ΔOND,

$r^2 = (x+1)^2 + 3^2$

$\therefore$ $x^2 + 4^2 = (x+1)^2 + 3^2$

$\therefore$ $x^2 + 4^2 = (x+1)^2 + 3^2$

solving $x = 3$, Hence radius $= \sqrt{3^2 + 4^2} = 5$

67. (a) Distance of 8 cm chord from centre $= \sqrt{r^2 - \left(\frac{8}{2}\right)^2}$

$= \sqrt{5^2 - 4^2} = 3$

$\therefore$ Required distance $= 3 \times 2 = 6$ cm

68. (b) Trick : Ratio of radii is inversely proportional to corresponding angles

$\therefore$ $\frac{r_1}{r_2} = \frac{75°}{60°} = 5 : 4$

(d) Shortcut: Note that QP is bisector of $\angle AQC$ and $\angle PR$ is bisector of $\angle BRC$

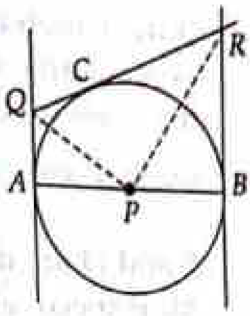
But $\angle AQC + \angle BRC = 180°$

(∵ In quadrilateral $ABQC$, $\angle QAB = \angle ABR = 90°$)

Hence in ΔQPC,

$\angle QPC = 180° - (\angle AQC + \angle BRC)$

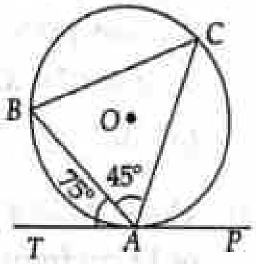
$= 180° - 90° = 90°$



(c) Since angle in the alternate segment are equal

∴ $\angle ACB = \angle BAT = 75°$

∴ $\angle ABC = 180° - 45° - 75° = 60°$

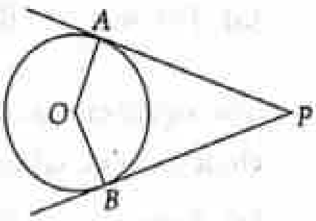


(d) ∵ $\angle AOB + \angle APB = 360° - \angle OAB - \angle OBA$

$= 360° - 90° - 90° = 180°$

∵ $\angle AOB : \angle APB = 5 : 1$

Hence, $\angle APB = \frac{180}{6} \times 1 = 30°$



Exercise—8B

1. Two tangents are drawn from a point P to a circle at A and B. O is the centre of the circle. If $\angle AOP = 60°$, then $\angle APB$ is
   (a) 60° (b) 30° (c) 120° (d) 90°
   [SSC Tier-I 2012]

2. In the following figure, O is the centre of the circle and XO is perpendicular to OY. If the area of the triangle XOY is 32, then the area of the circle is

   (a) 16π (b) 32π
   (c) 64π (d) 256π

3. Two circles of radii 4 cm and 9 cm respectively touch each other externally at a point and a common tangent touches them at the points P and Q respectively. Then the area of a square with one side PQ, is
   (a) 72 sq. cm (b) 144 sq. cm (c) 97 sq. cm (d) 194 sq. cm
   [SSC Tier-I 2012]

4. The tangents drawn at the points A and B of a circle centred at O meet at P. If $\angle AOB = 120°$ then $\angle APB : \angle APO$ is :
   (a) 2 : 5 (b) 3 : 2 (c) 4 : 1 (d) 2 : 1
   [SSC Tier-I 2012]

5. If the length of a chord of a circle, which makes an angle 45° with the tangent drawn at one end point of the chord, is 6 cm, then the radius of the circle is :
(a) $6\sqrt{2}$ cm (b) 5 cm (c) $3\sqrt{2}$ cm (d) 6 cm
[SSC Tier-I 2012]

6. P and Q are the middle points of two chords (not diameters) AB and AC respectively of a circle with centre at a point O. The lines OP and OQ are produced to meet the circle respectively at the points R and S. T is any point on the major arc between the points R and S of the circle. If $\angle BAC = 32°$, $\angle RTS = ?$
(a) 32° (b) 74° (c) 106° (d) 64°
[SSC Tier-I 2012]

7. The radius of a circle is 13 cm and AB is a chord which is at a distance of 12 cm from the centre. The length of the chord is :
(a) 15 cm (b) 12 cm (c) 10 cm (d) 20 cm
[SSC Tier-I 2012]

8. Two equal circles pass through each other's centre. If the radius of each circle is 5 cm, what is the length of the common chord ?
(a) 5 cm (b) $5\sqrt{3}$ cm (c) $10\sqrt{3}$ cm (d) $\frac{5\sqrt{3}}{2}$ cm
[SSC Tier-I 2012]

9. ABC is a triangle. The internal bisector of the angles $\angle A$, $\angle B$ and $\angle C$ intersect the circumcircle at X, Y and Z respectively. If $\angle A = 50°$, $\angle CZY = 30°$, then $\angle BYZ$ will be
(a) 45° (b) 55° (c) 35° (d) 30°
[SSC Tier-I 2012]

10. If a circle with radius of 10 cm has two parallel chords 16 cm and 12 cm and they are one the same side of the centre of the circle, then the distance between the two parallel chords is
(a) 2 cm (b) 3 cm (c) 5 cm (d) 8 cm
[SSC Tier-I 2012]

11. Two circles of radii 8 cm and 2 cm respectively touch each other externally at the point A. PQ is the direct common tangent of those two circles of centres O_1 and O_2 respectively. Then length of PQ is equal to
(a) 2 cm (b) 3 cm (c) 4 cm (d) 8 cm
[SSC Tier-I 2012]

12. $ABCD$ is a cyclic quadrilateral. Sides AB and DC, when produced meet at the point P and sides AD and BC, when produced meet at the point Q. If $\angle ADC = 85°$ and $\angle BPC = 40°$, then $\angle CQD$ is equal to
(a) 30° (b) 40° (c) 55° (d) 85°
[SSC Tier-I 2012]

3. If a square is inscribed in a circle whose area is 314 sq. cm, then the length of each side of the square is [Given $\pi = 3.14$]
(a) $5\sqrt{2}$ cm (b) $20\sqrt{2}$ cm (c) 10 cm (d) $10\sqrt{2}$ cm

4. Two circles with same radius r intersect each other and one passes through the centre of the other. Then the length of the common chord is
(a) r (b) $\sqrt{3}r$ (c) $\frac{\sqrt{3}}{2}r$ (d) $\sqrt{5}r$

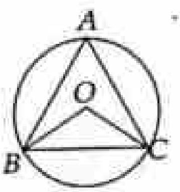
5. Two circles intersect each other at P and Q. PA and PB are two diameter. Then $\angle AQB$ is
(a) 120° (b) 135° (c) 160° (d) 180°
[SSC Tier-I 2012]

6. A and B are centres of the two circles whose radii are 5 cm and 2 cm respectively. The direct common tangents to the circles meet AB extended at P. Then P divides AB.
(a) externally in the ratio 5 : 2 (b) internally in the ratio 2 : 5
(c) internally in the ratio 5 : 2 (d) externally in the ratio 7 : 2
[SSC Tier-I 2012]

7. AC and BC are two equal chords of a circle. BA is produced to any point P and CP, when joined cuts the circle at T. Then
(a) $CT : TP = AB : CA$ (b) $CT : TP = CA : AB$
(c) $CT : CB = CA : CP$ (d) $CT : CB = CP : CA$

8. PQ is a direct common tangent of two circles of radii r_1 and r_2 touching each other externally at A. Then the value of PQ^2 is
(a) r_1r_2 (b) $2r_1r_2$ (c) $3r_1r_2$ (d) $4r_1r_2$
[SSC Tier-I 2012]

9. BC is the chord of a circle with centre O. A is a point on major arc BC as shown in the figure. What is the value of $\angle BAC + \angle OBC$?



(a) 120° (b) 60°
(c) 90° (d) 180°
[SSC Tier-I 2012]

10. Two circles with radii 5 cm and 8 cm touch each other externally at a point A. If a straight line through the point A cuts the circles at points P and Q respectively, then $AP : AQ$ is
(a) 8 : 5 (b) 5 : 8 (c) 3 : 4 (d) 4 : 3
[SSC Tier-I 2012]

11. Perimeter of a semi-circular bow is 72 cm. Diameter of the bow (in cm) is
(a) 7 cm (b) 14 cm (c) 28 cm (d) 21 cm
[SSC Tier-I 2012]

12. R and r are the radius of two circles ($R > r$). If the distance between the centre of the two circles be d, then length of common tangent of two circles is

(a) $\sqrt{d^2-(R-r)^2}$ (b) $\sqrt{(R-r)^2-d^2}$

(c) $\sqrt{R^2-d^2}$ (d) $\sqrt{r^2-d^2}$

[SSC Tier-I 20

23. AB is a diameter of circle with centre O. CD is a chord equal to the radius of the circles. AC and BD are produced to meet at P. Then the measure of $\angle APB$ is

(a) $30°$ (b) $60°$ (c) $90°$ (d) $120°$

[SSC Tier-I 20

24. P is a point outside a circle and is 13 cm away from its centre. A secant drawn from the point P intersects the circle at points A and B in such a way that $PA = 9$ cm and $AB = 7$ cm. The radius of the circle is

(a) 5 cm (b) 4 cm (c) 4.5 cm (d) 5.5 cm

[SSC Tier-I 20

25. The area of the largest triangle that can be inscribed in a semi circle of radius x in square unit is.

(a) x^2 (b) $2x^2$ (c) $3x^2$ (d) $4x^2$

[SSC Tier-I 20

26. The length of the common chord of two circles of radii 15 cm and 20 cm whose centres are 25 cm apart is (in cm).

(a) 24 (b) 25 (c) 15 (d) 20

[SSC Tier-I 20

27. SR is transverse common tangent of two circles whose radii are respectively 8 cm and 3 cm and centres are 13 cm apart. If S and R are points of contact, then the length of SR is

(a) 17 cm (b) 10 cm (c) 12 cm (d) 11 cm

[SSC Tier-I 20

Answers—8B

1. (a)	2. (c)	3. (b)	4. (d)	5. (c)	6. (b)	7. (c)	8. (b)
9. (c)	10. (a)	11. (d)	12. (a)	13. (d)	14. (b)	15. (d)	16. (a)
17. (c)	18. (d)	19. (c)	20. (b)	21. (c)	22. (a)	23. (b)	24. (a)
25. (a)	26. (a)	27. (c)					

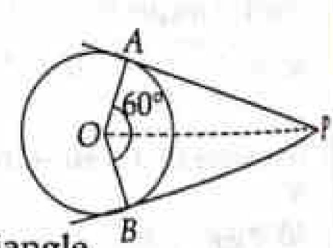
Explanation

1. (a) $\angle APO = 180° - \angle OAP - \angle AOP$

$= 180° - 90° - 60° = 30°$

$\therefore$ $\angle APB = 2\angle APO$

$= 2 \times 30° = 60°$



2. (c) $OX \perp OY \Rightarrow \Delta XOY$ is a right angled triangle

If $OX = OY = r$ = radius of circle

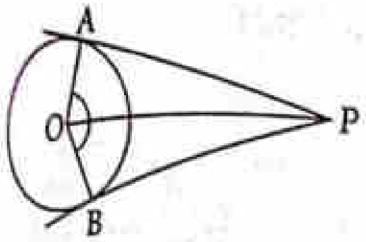
Area of triangle $= \frac{1}{2} \times r \times r = 32 \quad \Rightarrow \quad r = \sqrt{64} = 8$

$\therefore$ Area of circle $= \pi r^2 = 64\pi$

(a) Using $PQ = \sqrt{d^2 - (R-r)^2}$

$\therefore$ Area of square $= PQ^2 = d^2 - (R-r)^2$

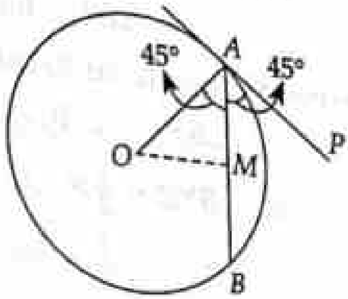
$= 13^2 - (9-4)^2 = 144$ sq. cm^2

(d) 

$\because$ OP is bisector of $\angle APB$

$\therefore$ $\angle APB : \angle APO = 2:1$

(c) AB is a chord which makes $45°$ with tangent line AP.

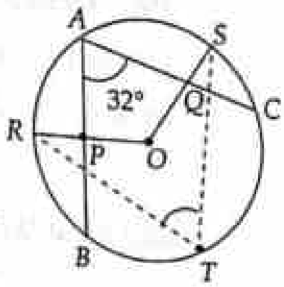


If O is centre of circle and M is midpoint of chord then $\angle OAM = 90° - 45° = 45°$

In ΔAOM, $\cos 45° = \frac{AM}{OA} = \frac{3}{r} \Rightarrow r = 3\sqrt{2}$ cm

(b) Since OP and OQ are respectively bisector of chord AB and AC therefore,

$\angle APO = 90°$, $\angle AQO = 90°$



Hence in quadrilateral $APOQ$

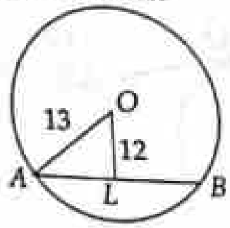
$\angle POQ = 180° - 32° = 148°$

Hence we cay say the chord RS (not drawn in figure, draw yourself) subtends angle $148°$ on centre O.

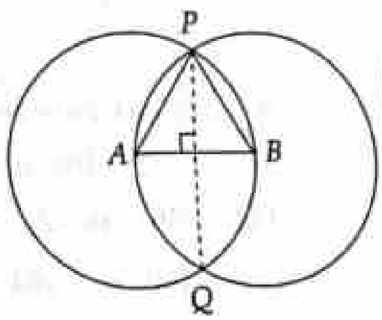
$\therefore$ Chord RS subtands $\frac{148°}{2} = 74°$ on circumference

(c) See the figure, $AL = \sqrt{13^2 - 12^2}$

$\therefore AB = 5 \times 2 = 10$ cm



(b) A and B are respectively centres of two circles and PQ is the common chord.



cleraly $AB = 5$ radius of circle $= 5$

$AP = BP =$ radius of circle $= 5$

Hence ΔAPB is an equilateral triangle

Altitude of triangle $= \frac{\sqrt{3}}{2} \times \text{side}$

$= \frac{\sqrt{3}}{2} \times 5$

$\therefore$ Common chord $PQ = 2 \times \text{altitude} = 5\sqrt{3}$

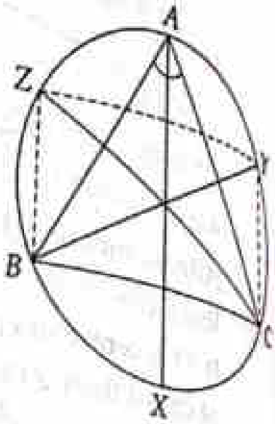
9. (c) Considering CY as base,

$\angle CBY = \angle CZY = 30°$

$\therefore \angle B = 2 \times \angle CBY = 2 \times 30° = 60°$

and $\angle C = 180° - \angle A - \angle B$

$= 180° - 60° - 50° = 70°$



considering chord BZ as base

$\angle BYZ = \angle BCZ$

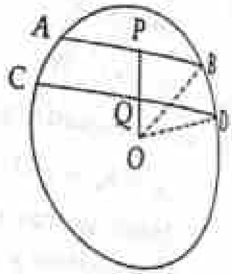
or, $\angle BYZ = \frac{1}{2} \times \angle C$

$= \frac{1}{2} \times 70° = 35°$

10. (a) In fig. $AB = 12$ cm and $CD = 16$ cm

$\therefore PB = 6$ cm and $QD = 8$ cm

Let $PQ = x$ cm then



In ΔOQD, $OQ = \sqrt{OD^2 - QD^2}$

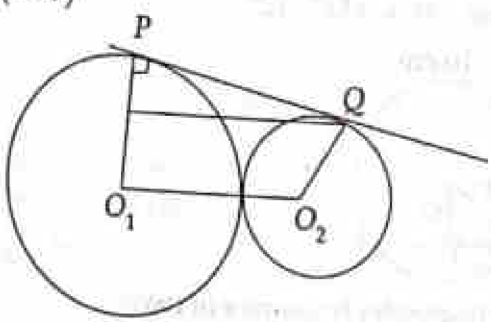
$= \sqrt{100 - 64} = \sqrt{36} = 6$ cm

and In ΔOBP, $OP = \sqrt{OB^2 - PB^2}$

$= \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = 8$ cm

$\therefore x = PQ = OP - OQ = 8 - 6 = 2$ cm

11. (d) $PQ = \sqrt{d^2 - (R-r)^2}$



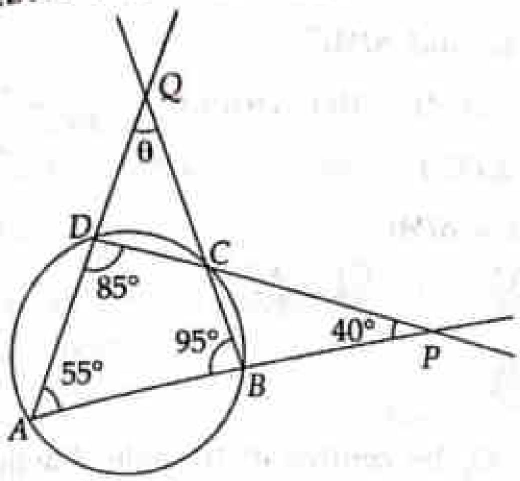
d = distance between two centres $= 8 + 2 = 10$ cm

R = Radius of bigger circle and r = radius of smaller circle.

$\therefore PQ = \sqrt{10^2 - (8-2)^2}$

$= \sqrt{100 - 36} = \sqrt{64} = 8$ cm

13. (a) In ΔAPD, $\angle DAB + 85^\circ + 40^\circ = 180^\circ$



$\Rightarrow \angle DAB = 55^\circ$

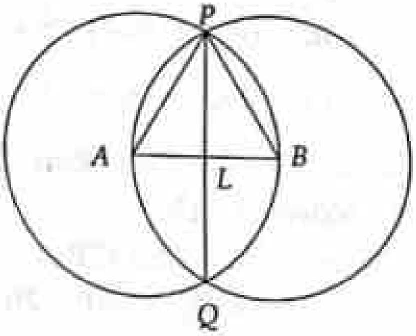
In cyclic quadrilateral, $\angle ABC = 180^\circ - 85^\circ = 95^\circ$

Let $\angle CQD = \theta$

In ΔABQ, $\theta + 55^\circ + 95^\circ = 180^\circ$

$\Rightarrow \theta = 180^\circ - 150^\circ = 30^\circ$

14. (b) See the figure $AB = AP = BP =$ radius



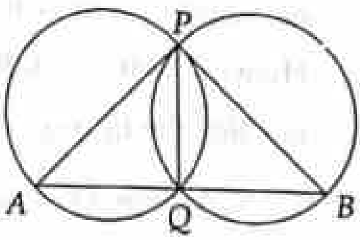
$\therefore$ ΔAPB is an quadrilateral triangle

$PQ =$ common chord $= 2 \times AL$

$= 2 \times$ altitade

$= 2 \times \frac{\sqrt{3}}{2} r = \sqrt{3}\, r$ (Learn)

15. (d) See the figure, Since angle in a semicircle is right angled

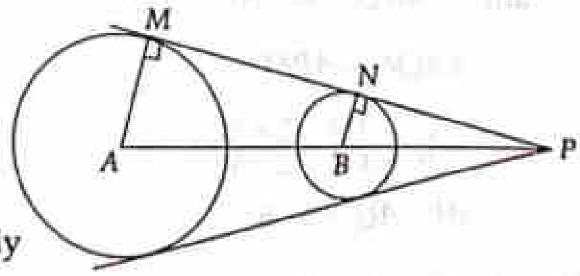


$\therefore \angle AQP = 90^\circ$ and $\angle BQP = 90^\circ$

Hence $\angle AQB = 90^\circ + 90^\circ = 180^\circ$

16. (a) $\because$ $\Delta AMP \sim \Delta BNP$

$\therefore$ $\frac{AP}{BP} = \frac{AM}{BN}$



or, $\frac{AP}{BP} = \frac{5}{2}$

$\because$ P is out side AB

$\therefore$ If divides AB externally

17. (c) $\because$ $BC = AC$

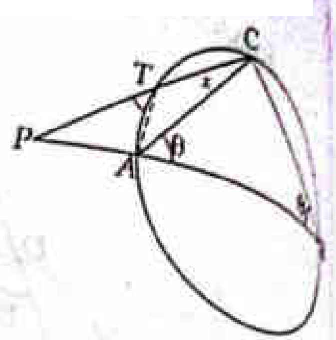
$\therefore$ $\angle CBA = \angle CAB = \theta$ (Say)

Join $A - T$

$ABCT$ is a cyclic quadrilateral

$\therefore\ \angle ATC = 180°$

Now, In ΔATC and ΔPAC 2



$$\angle ATC = \angle CAP = 180° - \theta \text{ and}$$

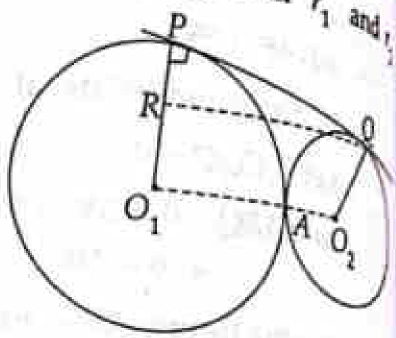
$$\angle TCA = \angle PCA \text{ (common angle)}$$

Hence, $\Delta ATC \sim \Delta PAC$

or, $\dfrac{AC}{TC} = \dfrac{PC}{AC} \quad \therefore\ \dfrac{CT}{AC} = \dfrac{AC}{PC}$

or, $\dfrac{CT}{BC} = \dfrac{AC}{CP}$ $(\because AC = b$

18. **(d)** Let O_1 and O_2 be centres of triangles having radii r_1 and r_2 respectively



Draw $QR \parallel O_1O_2$

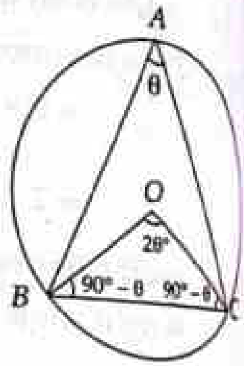
In right angled ΔRPQ 2

$$QR^2 = PQ^2 + PR^2$$

or, $(r_1 + r_2)^2 = PQ^2 + (r_1 - r_2)^2$

or, $PQ^2 = (r_1 + r_2)^2 - (r_1 - r_2)^2 = 4r_1r_2$

19. **(c)** If $\angle BAC = \theta$ then $\angle BOC = 2\theta$



Now, $B = OC$ 2

$\Rightarrow\ \angle OBC = \angle OCB$

$\therefore\ 2\angle OBC = 180° - 2\theta$

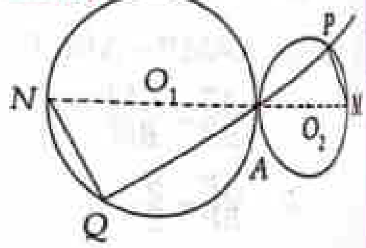
or,, $\angle OBC = 90° - \theta$

Hence $\angle BAC + \angle OBC = \theta + 90° - \theta = 90°$

20. **(b)** See the figure

$$\angle AQN = 90° = \angle APM \text{ (angle in a semicircle)}$$

and $\angle NAQ = \angle MAP$ (opposite angle)



$\therefore\ \Delta AQN \sim APM$

$\Rightarrow\ \dfrac{AQ}{AP} = \dfrac{AN}{AM} = \dfrac{2\times 8}{2\times 5}$

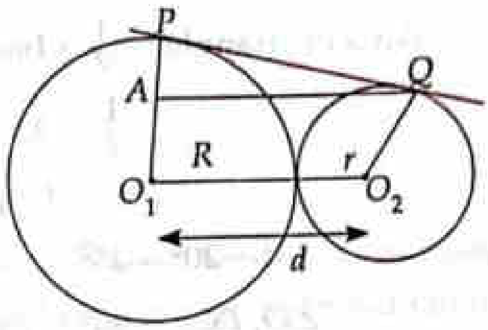
$\therefore\ AP : AQ = 5 : 8$

21. **(c)** $\pi r + 2r = 72$ 2

$\Rightarrow\ \dfrac{22}{7}r + 2r = 72 \Rightarrow \dfrac{36r}{7} = 72$

$\Rightarrow\ r = 2 \times 7 = 14$ cm

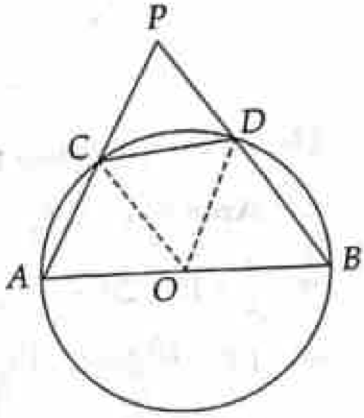
$\therefore$ Diameter $= 2 \times 14 = 28$ cm



22. (b) $\because$ $O_1P \perp r\ PQ$, $O_2Q \perp r\ PQ$
and $QA \perp r\ OP_1$
$\therefore$ AO_1O_2Q is a parallelogram
$\therefore$ $PA = O_1P - OA_1$
$= O_1P - O_2Q = R - r$
ΔAPQ is a right angled triangle with $\angle APQ = 90°$
$\therefore$ $AQ^2 = AP^2 + PQ^2$
ΔAOC and ΔBOD also equilateral triangle

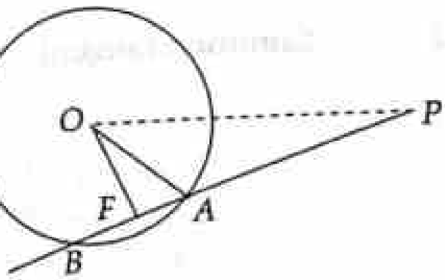
In ΔABP,

$d^2 = (R - r)^2 + PQ^2 \quad PQ = \sqrt{d^2 - (R - r)^2}$



23. (b) ΔOCD is an equilateral triangle as length of CD is equal to radius
OCD is an equilateral triangle
$\Rightarrow$ all its angle are $60°$
In ΔABP,
$\angle APB = 180° - 60° - 60° = 60°$

24. (a) Draw $OF \perp r\ AB$



ΔOFP is an equilateral triangle $(\angle F = 90°)$
in which $OP = 12$, $PF = PA + AF$

$$= 9 + \frac{7}{2} = \frac{25}{2}$$

$\therefore$ $OF^2 = OP^2 - PF^2$

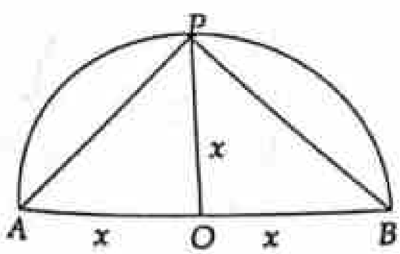
$$= 169 - \left(\frac{25}{2}\right)^2$$

$$= \frac{676 - 625}{4} = \frac{51}{4}$$

In ΔOFA, $r^2 = OF^2 + FA^2$

$$= \frac{51}{4} + \left(\frac{7}{2}\right)^2 = \frac{51}{4} + \frac{49}{4} = \frac{100}{4} = 25 \quad \therefore r = 5$$

25. (a)



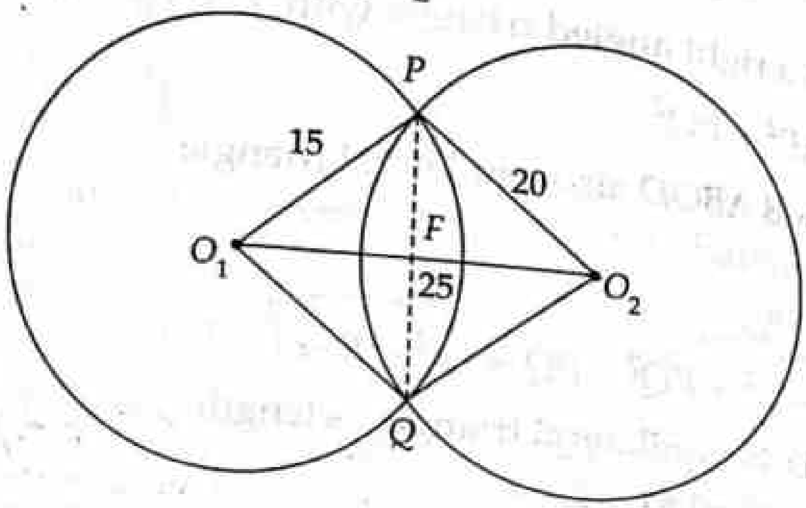
Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 2x \times x = x^2$

($\because$ greatest base $= AB =$ greatest height $OP = r$)

26. (a) $\because \ 15^2 + 20^2 = 25^2$

$\therefore \ \angle O_1PO_2 = 90° = \angle O_1QO_2$



Hence ΔO_1PO_2 is a right angled triangle

$\therefore$ Area of $O_1PO_2 = \frac{1}{2} \times OP_1 \times OP_2 = \frac{1}{2} O_1O_2 \times PF$

$\Rightarrow \ \frac{1}{2} \times 15 \times 20 = \frac{1}{2} \times 25 \times PF$

$\Rightarrow \ PF = \frac{15 \times 20}{25} = \frac{15 \times 4}{5} = 12$ cm

$\therefore \ PQ = 12 \times 2 = 24$ cm

27. (c) Common tangent $= \sqrt{d^2 - (R - r)^2}$

$= \sqrt{13^2 - (8-3)^2} = \sqrt{169 - 25} = \sqrt{144} = 12$

★★★

09

Measurement of Angle : Radian and Degree

1. **To measure an angle in degree** : The angle between two perpendicular lines is called a right angle. A right angle is equal to 90 degree, it is written as 90°.

Thus, if a right angle is divided into 90 equal parts then one part is called one degree. It is written as 1°.

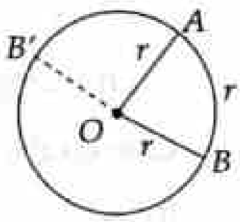
If 1° is divided into 60 equal parts, each part is called 1 minute. It is denote by 1′

$\frac{1}{60}$th part of 1′ is called one second. It is written as 1″.

Hence 1 right angle $= 90°$
$= 90 \times 60' = 5400' = 5400$ minute
$= 90 \times 60 \times 60'' = 324000'' = 324000$ seconds

Again, $\boxed{1° = 60' = 60 \times 60'' = 3600''}$

2. **To measure an angle in radian** : Let AB be an arc of a given circle whose length is equal to radius of the circle. The angle subtended by arc AB at the centre O of the circle is measured as 1 radian i.e., $\angle AOB = 1$ radian. It is denoted by 1 or 1 rad. In the given figure $\angle AOB = 1$ rad. It is also written as 1^c.



3. **Relation between degree measure and radian measure :**

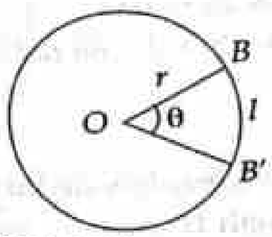
$\because \pi \text{ rad} = 180°$ $\therefore$ $\boxed{x° = \frac{\pi x}{180} \, rad}$

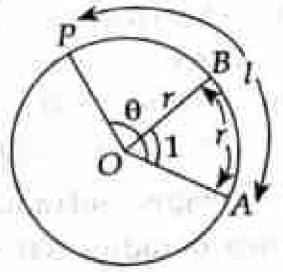
and $\boxed{x \, rad = \frac{180}{\pi} x°}$ $\boxed{1\text{rad} = \frac{180}{3.14} = 57°16'22''}$

Thus to change degree into radian, multiply by $\frac{\pi}{180}$ and to change radian into degree multiply by $\frac{180}{\pi}$.

If mentioned, take $\pi = \frac{22}{7}$ or 3.14.

4. **Relation between length of arc (l), radius (r) and angle (θ) :**





If an arc of length l of a circle ... the circle is r then $\theta = \frac{l}{r}$.

Hence, (i) when $\theta = \frac{l}{r}$ and r is constant then $\theta \propto l$ i.e., $\theta_1 : \theta_2 = l_1 : l_2$

(ii) when $\theta = \frac{l}{r}$ and θ is constant then $l \propto r$ i.e., $l_1 : l_2 = r_1 : r_2$

(iii) when $\theta = \frac{l}{r}$ and l is constant then $\theta \propto \frac{1}{r}$ or $r \propto \frac{1}{\theta}$

i.e., $\theta_1 : \theta_2 = r_2 : r_1$ *(reverse order)*

5. If θ is in radian and is very-very small then $\sin\theta = \theta = \tan\theta$ *(approximate)*

Solved Example

1. Convert the following degree measures in the radian measure.

(i) $42°30'$ (ii) $-520°$

Solution : We know that $x° = \frac{\pi x}{180}$ rad

$\therefore$ (i) $42°30' = 42\frac{1°}{2} = \frac{85°}{2}$ $\left(\because 30' = \frac{1°}{2}\right)$

$$= \frac{\pi}{180} \times \frac{85}{2} = \frac{\pi \times 17 \times 5}{5 \times 36 \times 2} = \frac{17\pi}{72}$$

(ii) $-520° = -520 \times \frac{\pi}{180} = \frac{-52\pi}{18} = \frac{-26\pi}{9}$

2. Convert the following radian measure in degree measurers $\left(\text{Use } \pi = \frac{22}{7}\right)$

(i) 4 (ii) $-\frac{5\pi}{3}$

Solution : $\because$ π radian $= 180°$

$\therefore$ 1 radian $= \frac{180°}{\pi} = 180° \times \frac{7}{22}$

or, 4 radian $= \frac{180° \times 7 \times 4}{22} = \frac{90° \times 7 \times 4}{11} = \frac{2520°}{11} = 229\frac{1}{11}$ degree.

(ii) $-\frac{5\pi}{3} = -\frac{5}{3} \times 180° = -5 \times 60° = -300°$

3. A wheel makes 180 revolutions in one minute. Through how many radians does it turn in one second ? Also find its degree measure.

Solution : $\because$ Wheel makes 180 revolution in 60 seconds

$\therefore$ Wheel makes $\frac{180}{60} = 3$ revolutions in 1 second.

Now, $\because$ One complete revolution measures 2π radian.

$\therefore$ Three complete revolutions measure $2\pi \times 3 = 6\pi$ radian

Again, $\because \pi$ rad $= 180°$

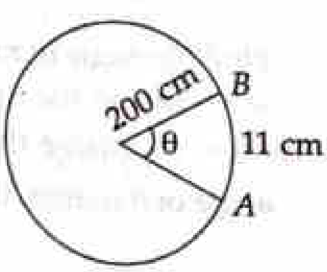
$\therefore$ 6π rad $= 6 \times 180° = 1080°$

4. Find the degree and radian measure of the angle subtended at the centre of a circle of radius 200 cm by an arc of length 11 cm. $\left(\text{use } \pi = \frac{22}{7}\right)$

Solution : Given $r = 200$ cm, l = Arc AB = 11cm

Suppose angle subtended at the centre of circle be θ radian

then, $\theta = \frac{l}{r} = \frac{11}{200}$ rad



$\because$ π rad = 180°

$\therefore$ 1 rad = $\frac{180°}{\pi}$

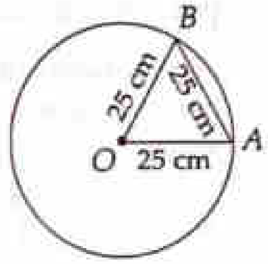
or, 1 rad = $\frac{7}{22} \times 180°$

$$\therefore \frac{11}{200} \text{ rad} = \frac{11}{200} \times \frac{7}{22} \times 180° = \frac{7 \times 180°}{200 \times 2} = \frac{7 \times 45°}{100} = \frac{7 \times 9°}{20}$$

$$= \frac{63°}{20} = 3\frac{3°}{20} = 3 \text{ degree } \frac{3}{20} \times 60 \text{ seconds} = 3°9'$$

5. In a circle of diameter 50 cm, the length of a chord is 25 cm. Find the length of minor arc and major arc of the chord.

Solution : See the figure



Given that radius of the circle = $\frac{50 \text{ cm}}{2}$ = 25 cm

and chord AB of the circle = 25 cm

Clearly ΔOAB is an equilateral triangle,

therefore $\angle AOB = 60° = \frac{\pi}{3} = \theta$ (say)

If minor arc $AB = l$ then from $\theta = \frac{l}{r}$

$$l = r\theta = \frac{25\pi}{3}$$

and major arc = $25\left(2\pi - \frac{\pi}{3}\right) = 25\left(\frac{5\pi}{3}\right) = \frac{125\pi}{3}$

6. If in two circles, arcs of the same length subtend angle 60° and 75° at the centre, find the ratio of their radii.

Solution : Let the radii of two circles be r_1 and r_2 respectively.

According to the question, arc $AB = l$ (say) in the two circle.

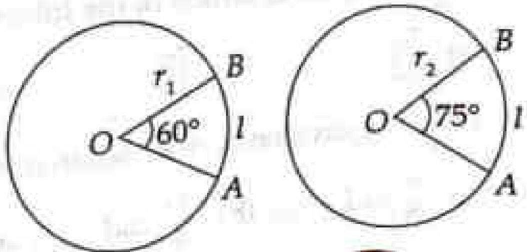
Given that $\theta_1 = 60° = 60 \times \frac{\pi}{180} = \frac{60\pi}{180}$ radian

And $\theta_2 = 75° = \frac{75\pi}{180}$ radian

$\because$ $\theta = \frac{l}{r} \therefore \theta_1 = \frac{l}{r_1}$ and $\theta_2 = \frac{l}{r_2}$

or, $l = r_1\theta_1 = r_2\theta_2$

or, $\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\frac{75\pi}{180}}{\frac{60\pi}{180}} = \frac{75}{60} = \frac{5}{4}$



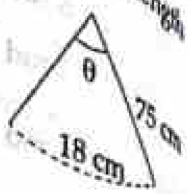
Shortcut : since l is constant,

$$\text{therefore } r_1 : r_2 = \theta_2 : \theta_1 = 75^\circ : 60^\circ = 5 : 4$$

7. Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc length 18 cm.

Solution : Suppose the pendulum swings through an angle of θ radian. then, $\theta = \frac{l}{r} = \frac{18}{75}$ rad (see figure)

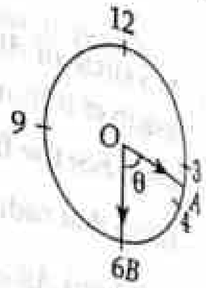
$$= \frac{6}{25} \text{ rad}$$



8. Find the angle in radians between the hands of a clock at half past three.

Solution: In 60 minutes the minute hand of a watch completes one revolution i.e., moves through an angle of 2π radian (360°)

Also, at three past half, the hour hand is exactly at the midway between 3 and 4, (shown by point A is figure) and minute hand is exactly at 6 (shown by point B in figure).



Hence there is a difference of $2 \times 5 + \frac{5}{2} = \frac{25}{2}$ minute between A and B.

Now, $\because$ 60 minute revolution = 2π rad

$\therefore$ $\frac{25}{2}$ minute revolution $= \frac{25}{2 \times 60}(2\pi) = \frac{5\pi}{12}$ rad

Hence the two hands of the clock makes an angle of $\frac{5\pi}{12}$ rad at half past three.

Shortcut : If two hands of a clock are respectively at H hour and M minute and angle between them is $A°$ then $\frac{11}{2}\ M = 30H \pm A$.

here, $H = 3, M = 30$

$\therefore$ $\frac{11}{2} \times 30 = 30 \times 3 \pm A$

or, $165^\circ - 90^\circ = \pm A \Rightarrow A = 75^\circ$

Exercise–9A

1. In radian measure 120° equals

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{3\pi}{4}$ (d) $\frac{4\pi}{3}$

2. $37\frac{1}{2}^\circ$ is equal to which of the following radian measure ?

(a) $\frac{5\pi}{12}$ (b) $\frac{7\pi}{12}$ (c) $\frac{5\pi}{24}$ (d) $\frac{7\pi}{24}$

3. $11\frac{1}{4}^\circ$ is equivalent to the radian measure

(a) $\frac{\pi}{8}$ rad (b) $\frac{3\pi}{8}$ rad (c) $\frac{3\pi}{16}$ rad (d) $\frac{\pi}{16}$ rad

4. $\frac{5}{6}$ right angle in radian equals
(a) $\frac{5\pi}{24}$ (b) $\frac{5\pi}{12}$ (c) $\frac{7\pi}{12}$ (d) $\frac{\pi}{12}$

5. Degree equivalent of $\frac{7\pi}{12}$ radian is
(a) 105° (b) 75° (c) 135° (d) 165°

6. Radian value of $35^\circ 30'$ is
(a) $\frac{81\pi}{360}$ rad (b) $\frac{71\pi}{360}$ rad (c) $\frac{61\pi}{360}$ rad (d) $\frac{143\pi}{360}$ rad

7. Radian value of $560^\circ 20'$ is
(a) $\frac{1481\pi}{540}$ rad (b) $\frac{1481\pi}{360}$ rad (c) $\frac{1681\pi}{540}$ rad (d) $\frac{1681\pi}{360}$ rad

8. Radian measure of $72^\circ 40'$ is
(a) $\frac{109\pi}{270}$ (b) $\frac{109\pi}{180}$ (c) $\frac{219\pi}{540}$ (d) $\frac{219\pi}{360}$

9. One radian is equal to following degree measure,
(a) $57^\circ 14' 21''$ (b) $57^\circ 16' 22''$ (c) $58^\circ 14' 21''$ (d) $58^\circ 16' 22''$

10. Value of $\frac{3}{5}$ rad is
(a) $34^\circ 23' 19''$ (b) $36^\circ 23' 19''$ (c) $36^\circ 21' 49''$ (d) $34^\circ 21' 49''$

11. Measure of 6 rad is
(a) $343^\circ 18' 11''$ (b) $341^\circ 18' 11''$ (c) $341^\circ 38^\circ 11''$ (d) $343^\circ 38' 11''$

12. If 1 rad = $57^\circ 16' 21''$ then 10 rad equals
(a) $570^\circ 16' 21''$ (b) $573^\circ 43' 10''$ (c) $571^\circ 43' 40''$ (d) $572^\circ 43' 30''$

13. If one unit of an angle is $29^\circ 46' 55''$ then five units of the angle equals
(a) $148^\circ 54' 35''$ (b) $146^\circ 54' 35''$ (c) $149^\circ 34' 25''$ (d) $147^\circ 44' 35''$

14. If one unit of an angle is $15^\circ 49' 50''$ then measure of 100 unit of the angle equals
(a) $1580^\circ 30' 20''$ (b) $1582^\circ 3' 20''$
(c) $1583^\circ 3' 20''$ (d) $1581^\circ 30' 20''$

15. A wheel makes 90 revolutions in half hour. Through how many degree does it turn in one minute ?
(a) 120° (b) 720° (c) 1080° (d) 540°

16. A wheel makes 720 revolutions in one hour. Through how many radian does it turn in one second ?
(a) $\frac{\pi}{5}$ rad (b) $\frac{2\pi}{5}$ rad (c) $\frac{3\pi}{5}$ rad (d) $\frac{4\pi}{5}$ rad

17. The diameter of a circle is 2 meter. The angle subtended at the centre by an arc of 22 cm is
(a) $12^\circ 60''$ (b) $12^\circ 36'$ (c) $24^\circ 60'$ (d) $6^\circ 36'$

18. The diameter of a circle is 60 cm. The length of minor arc created by a chord of 30 cm is (take $\pi = \frac{22}{7}$)
(a) $31\frac{3}{7}$ cm (b) 34 cm (c) $32\frac{2}{7}$ cm (d) $32\frac{4}{7}$ cm

19. In a circle of radius 50 cm the length of a chord is $50\sqrt{2}$ cm. The length of major arc of the chord is (take $\pi = 3.14$)
(a) 245.5 cm (b) 235.5 cm (c) 255.5 cm (d) None of these

20. The angle in degree through which a pendulum of length 100 cm swings and the tip describes an arc length of 10 cm is
(a) 5° 43' 38" (b) 7° 43' 38" (c) 5° 34' 18" (d) 7° 34' 18"

21. The minute hand of a watch is 5 cm. How far does the tip move in 20 minutes ? (take $\pi = \frac{22}{7}$)
(a) 10 cm (b) 9.53 cm (c) 11 cm (d) 10.47 cm

22. The tip of a pendulum swings. It covers an arc of 50 cm and subtends 60° at the fixed point. The length of pendulum is (take $\pi = \frac{22}{7}$)
(a) 43.72 cm (b) 45.72 cm (c) 47.72 cm (d) 45.27 cm

23. If the arc of same length in two circle subtends angles 75° and 120° at their respective centres, then ratio of their diameter is
(a) 8 : 5 (b) 5 : 8 (c) 3 : 5 (d) 5 : 3

24. The angle between the hands of a clock at 4 hour 45 minute is
(a) $112\frac{1}{2}^\circ$ (b) $122\frac{1}{2}^\circ$ (c) 125° (d) $127\frac{1}{2}^\circ$

25. The angle between the hands of a clock at half past one is
(a) $\frac{3\pi}{4}$ rad (b) $\frac{2\pi}{3}$ rad (c) $\frac{5\pi}{12}$ rad (d) $\frac{5\pi}{6}$ rad

26. Two angles of a triangle are $\frac{3}{2}$ rad and $\frac{4}{3}$ rad. The triangle
(a) is an acute angled triangle (b) is an obtuse angled triangle
(c) is a right angled triangle (d) dosenot form

27. If two angle of a triangle are 2 rad and $\frac{1}{2}$ rad then its third angle in degree is
(a) $105\frac{3}{7}^\circ$ (b) $15\frac{3}{7}^\circ$ (c) $105\frac{5}{7}^\circ$ (d) $36\frac{9}{11}^\circ$

28. A wheel revolves 24 times in 10 seconds. How many time does it take in revolving an angle of 110 rad ?
(a) 5 sec (b) 7.5 sec (c) 10 sec (d) None of these

29. Radian measure of 40° 20' 50" is
(a) $\frac{481}{1196}\pi$ rad (b) $\frac{681}{1296}\pi$ rad (c) $\frac{581}{2592}\pi$ rad (d) $\frac{581}{1296}\pi$ rad

30. A pendulum of length 60 cm swings and creates an arc of 18 cm. The angle at the fixed point of the pendulum is

(a) 15° (b) $17\frac{1}{2}^\circ$ (c) 20° (d) $22\frac{1}{2}^\circ$

31. Radius of a circle is 54 cm. If an arc of circle subtends an angle of 20° at centre then length of the arc is (take $\pi = \frac{22}{7}$)

(a) $19\frac{1}{7}$ cm (b) $17\frac{4}{7}$ cm (c) $18\frac{6}{7}$ cm (d) None of these

32. An arc of length 40 cm subtends $22\frac{1}{2}^\circ$ at the centre of the circle. Radius of the circle is

(a) 92 cm (b) 102 cm (c) 96 cm (d) 108 cm

33. Ananta's (A) and Shailvi's (S) house are situated at a circular road and subtends 90° at a fixed point. If fixed point is at a distance of 100 meter from each house, the distance travelled between the both house on the road is

(a) 628 meter (b) 314 meter (c) 157 meter (d) 235.5 meter

34. The angle covered by minute hand of a watch during 1 hour 15 minutes noon to half past three noon is

(a) 4.5π (b) 5π (c) 4.25π (d) None of these

35. The angle covered by hour hand of a clock from half past six in the morning to three o'clock in the noon is

(a) 270° (b) 245° (c) 255° (d) 265°

36. Assuming that the Moon's diameter subtends an angle $\left(\frac{1}{2}\right)^\circ$ at the eye of an observer, find how far from the eye of a coin of 1 cm diameter must be held so as just to hide Moon ? (take $\pi = \frac{22}{7}$)

(a) $112\frac{5}{11}$ cm (b) $110\frac{6}{11}$ cm (c) $116\frac{5}{11}$ cn (d) $114\frac{6}{11}$ cm

37. The earth revolves in its axis in 24 hours. How much angle does it move in 4 hours and 12 minutes ?

(a) 63° (b) 64° (c) 65° (d) 70°

38. If angle of a triangle are in AP, then the middle one is

(a) always 60° (b) less than 60°

(c) move than 60° (d) less than 90°

Answer–9A

1. (b)	2. (c)	3. (d)	4. (b)	5. (a)	6. (b)	7. (c)	8. (a)
9. (b)	10. (c)	11. (d)	12. (d)	13. (a)	14. (c)	15. (c)	16. (b)
17. (b)	18. (a)	19. (b)	20. (a)	21. (d)	22. (c)	23. (a)	24. (d)
25. (a)	26. (a)	27. (d)	28. (b)	29. (c)	30. (b)	31. (c)	32. (b)
33. (c)	34. (a)	35. (c)	36. (d)	37. (a)	38. (a)		

1. (b) $120^\circ = \frac{120}{180} \times \pi \text{ rad} = \frac{2\pi}{3} \text{ rad}$

2. (c) $37\frac{1}{2}^\circ = \left(\frac{75}{2}\right)^\circ$

$$= \frac{75}{2 \times 180} \times \pi = \frac{5\pi}{2 \times 12} = \frac{5\pi}{24} \text{ rad}$$

3. (d) $11\frac{1}{4}^\circ = \left(\frac{45}{4}\right)^\circ$

$$= \frac{\left(\frac{45}{4}\right)}{180} \times \pi \text{ rad} = \frac{\pi}{16} \text{ rad}$$

4. (b) $\frac{5}{6}$ right angle $= \frac{5}{6} \times \frac{\pi}{2} \text{ rad} = \frac{5\pi}{12} \text{ rad}$

5. (a) $\frac{7\pi}{12} = \frac{7 \times 180^\circ}{12} = 7 \times 15^\circ = 105^\circ$

6. (b) $35^\circ 30' = 35\frac{1}{2}^\circ = \left(\frac{71}{2}\right)^\circ$

$$= \frac{71}{2} \times \frac{\pi}{180} = \frac{71\pi}{360} \text{ rad}$$

7. (c) $560^\circ 20' = \left(560\frac{20}{60}\right)^\circ = \left(560\frac{1}{3}\right)^\circ$

$$= \left(\frac{1681}{3}\right)^\circ = \frac{1681}{3} \times \frac{\pi}{180} = \frac{1681}{540} \pi \text{ rad}$$

8. (a) $72^\circ 40' = \left(72\frac{40}{60}\right)^\circ = \left(72\frac{2}{3}\right)^\circ$

$$= \left(\frac{218}{3}\right)^\circ = \left(\frac{218}{3} \times \frac{\pi}{180}\right)$$

$$= \left(\frac{109\pi}{3 \times 90}\right) = \frac{109\pi}{270} \text{ rad}$$

9. (b) $\because \pi \text{ rad} = 180^\circ$

$$\therefore 1 \text{ rad} = \frac{180^\circ}{\pi} \text{ rad} = \left(\frac{180}{22} \times 7\right)^\circ$$

$$= \left(\frac{90 \times 7}{11}\right)^\circ = \left(\frac{630}{11}\right)^\circ = \left(57\frac{3}{11}\right)^\circ$$

$$= 57^\circ \left(\frac{3}{11} \times 60'\right) = 57^\circ \left(\frac{180}{11}\right)'$$

$$= 57^\circ \left(16\frac{4}{11}\right)' = 57^\circ 16' \left(\frac{4}{11} \times 60''\right)$$

$$= 57^\circ 16' \frac{240''}{11} = 57^\circ 16' 22'' \text{ (approximate)}$$

10. (c) $\because \pi \text{ rad} = 180^\circ$

$\therefore \frac{3}{5} \text{ rad} = \frac{3}{5} \times \frac{180^\circ}{\pi}$

$= \frac{3}{5} \times \frac{7}{22} \times 180^\circ = \frac{7 \times 3 \times 18^\circ}{11}$

$= \frac{378^\circ}{11} = \left(34\frac{4}{11}\right)^\circ$

$= 34^\circ\left(\frac{4}{11} \times 60'\right) = 34^\circ\left(\frac{240'}{11}\right)$

$= 34^\circ\left(21\frac{9}{11}\right)' = 34^\circ 21'\left(\frac{9}{11} \times 60'\right)$

$= 34^\circ 21'\left(\frac{540}{11}\right)'' = 34^\circ 21' 49''$

11. (d) $\because \pi \text{ rad} = 180^\circ$

$\therefore 1 \text{ rad} = \frac{180^\circ}{\pi}$

$6 \text{ rad} = \frac{6 \times 180^\circ}{\pi} = \frac{6 \times 180^\circ \times 7}{22} = \frac{21 \times 180^\circ}{11} = \frac{3780^\circ}{11}$

$\left(343\frac{7}{11}\right)^\circ = 343^\circ\left(\frac{420}{11}\right)' = 343^\circ\left(38\frac{2}{11}\right)'$

$= 343^\circ 38'\left(\frac{120}{11}\right)'' = 343^\circ 38' 11''$ (approximate)

Second Method,

$\because 1 \text{ rad} = 57^\circ 16' 22''$

$\therefore 6 \text{ rad} = 57^\circ \times 6 + 16' \times 6 + 22'' \times 6$

$= 342^\circ + 96' + 132''$

$= 342^\circ + (1^\circ + 36') + (2' + 12'')$

$= (\because 1^\circ = 60' \text{ and } 1' = 60'') = 343^\circ 38' 12''$ (approximate)

12. (d) $10 \text{ rad} = (57^\circ 16' 21'') \times 10$

$= 570^\circ + 160' + 210'' = 570^\circ + 2^\circ 40' + 3' 30'' = 572^\circ 43' 30''$

13. (a) $5 \text{ unit} = (29^\circ 46' 55'') \times 5$

$= 145^\circ 230' 275'' = 145^\circ + 3^\circ 50' + 4' 35'' = 148^\circ 54' 35''$

14. (c) $100 \text{ unit} = (15^\circ 49' 50'') \times 100$

$= 1500^\circ 4900' 5000'' = 1500^\circ + (81^\circ 40') + 83' 20''$

$\left(\because \frac{4900}{60} = 81\frac{40}{60}, \frac{5000}{60} = 83\frac{20}{60}\right)$

$= 1500^\circ + 81^\circ 40' + 1^\circ 23' 20''$

$= 1582^\circ 63' 20'' = 1583^\circ 3' 20''$

15. (c) Wheel revolves $= \frac{90}{30} = 3$ turn in one minute

$\because 1 \text{ turn} = 360^\circ$

$\therefore 3 \text{ turn} = 1080^\circ$

16. (b) Wheel covers $\frac{720}{3600} = \frac{1}{5}$ turn in 1 second

$\because$ 1 turn = 2π rad

$\therefore \frac{1}{5}$ turn = $\frac{2\pi}{5}$ rad

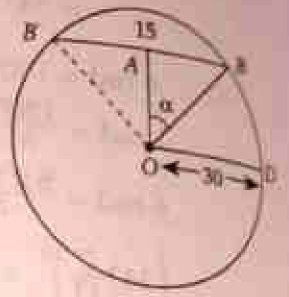
17. (b) Radius = 1 meter = 100 cm = r

$\because \theta = \frac{l}{r}$, here $l = 22$ cm.

$\therefore \theta = \frac{22}{100}$ rad $= \frac{22}{100} \times \frac{180}{\pi}$ degree $= \frac{22}{100} \times \frac{180}{22} \times 7$ degree

$= \frac{180 \times 7}{100} = \frac{126}{10}$ degree $= \left(12\frac{6}{10}\right)^\circ = 12^\circ\left(\frac{6}{10} \times 60\right)' = 12^\circ 36'$

18. (a) In ΔOAB,



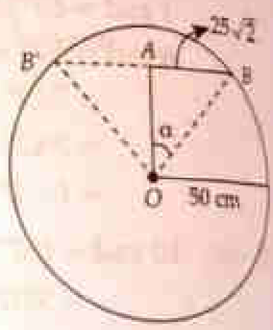
$\text{Sin}\alpha = \frac{15}{30} = \frac{1}{2} \Rightarrow \alpha = 30^\circ = \frac{\pi}{6}$

In figure chord BB' is 30 cm that subtends $2\theta = 2 \times \frac{\pi}{6} = \frac{\pi}{3}$ at the centre.

Hence, from $\theta = \frac{l}{r}$, $\frac{\pi}{3} = \frac{l}{30} \Rightarrow l = 10\pi$

$= \frac{10 \times 22}{7} = \frac{220}{7}$ cm $= 31\frac{3}{7}$ cm

19. (b) $\sin\alpha = \frac{25\sqrt{2}}{50} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$



$\therefore \ 2\alpha = 90^\circ$

Hence major arc of chord BB' subtends $360^\circ - 90^\circ = 270^\circ = \frac{3\pi}{2}$ at centre.

$\therefore$ using $\theta = \frac{l}{r}$

major arc $l = \theta \times r = \frac{3\pi}{2} \times 50$

$= 75\pi = 75 \times 3.14$ cm $= 235.50$ cm

20. (a) From $\theta = \frac{l}{r}$, $\theta = \frac{10}{100} = \frac{1}{10}$ rad

$= \frac{180^\circ}{\pi \times 10} = \left(\frac{18 \times 7}{22}\right)^\circ$

$= \left(\frac{63}{11}\right)^\circ = \left(5\frac{8}{11}\right)^\circ = 5^\circ\left(\frac{8}{11} \times 60'\right) = 5^\circ\left(\frac{480'}{11}\right)$

$= 5^\circ\left(43\frac{7}{11}\right)' = 5^\circ 43'\left(\frac{7}{11} \times 60''\right)$

$= 5^\circ 43' 38''$ (approximate)

21. (d) In 20 minute, hand covers $\frac{20}{60} \times 2\pi = \frac{2\pi}{3}$ rad distance.

from $\theta = \frac{l}{r}$, $l = \theta r$

$$= \frac{2\pi}{3} \times 5 = \frac{10\pi}{3} = \frac{10}{3} \times \frac{22}{7} = \frac{220}{21} = 10.47 \text{ cm}$$

22. (c) $\theta = 60^\circ = \frac{\pi}{3}$ and $l = 50$ cm

$\therefore$ using $\theta = \frac{l}{r}$,

$$r = \frac{l}{\theta} = \frac{50\text{ cm}}{\frac{\pi}{3}} = \frac{150}{\pi}\text{ cm} = \frac{150}{\frac{22}{7}} = \frac{150 \times 7}{22} = 47.72 \text{ cm}$$

23. (a) $\theta = \frac{l}{r} \Rightarrow r = \frac{l}{\theta} \Rightarrow \frac{r_1}{r_2} = \frac{\frac{l}{75^\circ}}{\frac{l}{120^\circ}} = \frac{120^\circ}{75^\circ} = 8:5$

24. (d) Using, $\frac{11}{2} M = 30H \pm A$

here, $H = 4$, $M = 45$

$\because \quad \frac{11}{2} \times 45 = 30 \times 4 \pm A$

$247.5 = 120^\circ \pm A$

$\therefore \quad A = 247.5^\circ - 120^\circ = 127.5^\circ$

25. (a) using $\frac{11}{2} M = 30 H \pm A$

$\frac{11}{2} \times 30 = 30 \times 1 \pm A$

$\Rightarrow A = 11 \times 15 - 30 = 135^\circ = \frac{135}{180} \times \pi \text{ rad} = \frac{3\pi}{4}$ rad

26. (a) 1 right angle $= \frac{\pi}{2}$ rad $= 1.57$ rad (approximate)

$\frac{3}{2} = 1.5$ rad, which is an acute angle.

$\frac{4}{3} = 1.33$ rad, which is an acute angle

Third angle $= \pi$ rad $- 1.5$ rad $- 1.33$ rad

$= (3.14 - 1.5 - 1.33)$

$= 0.31$ rad which is also an acute angle.

27. (d) Third angle $= \pi \text{ rad} - 2 \text{ rad} - \frac{1}{2}$ rad

$$= \left(\frac{22}{7} - \frac{5}{2}\right) \text{rad} = \frac{9}{14} \text{ rad}$$

$$= \frac{9}{14} \times \frac{180^\circ}{\pi} = \frac{9}{14} \times \frac{180}{22} \times 7 = \frac{45 \times 9}{11} = \frac{405}{11} = 36\frac{9^\circ}{11}$$

$\therefore$ It covers $\frac{24}{10} \times 2\pi$ rad angle in 1 second.

Hence in covering 110 rad, wheel takes

$$\frac{110}{\frac{24}{10}\times 2\pi} = \frac{110\times 10\times 7}{24\times 2\times 22} = 7.3 \text{ second.}$$

29. (c) $40^\circ\, 20'\, 50'' = 40^\circ \left(20\frac{50}{60}\right)'$

$$= 40^\circ \left(\frac{125}{6}\right)' = 40^\circ \left(\frac{125}{6\times 60}\right)^\circ$$

$$= \left(40\frac{25}{72}\right)^\circ = \left(\frac{2905}{72}\right)^\circ$$

$$= \frac{2905}{72}\times\frac{\pi}{180} \text{ rad}$$

$$= \frac{581\pi}{72\times 36} = \frac{581\pi}{2592} \text{ rad}$$

30. (b) From $\theta = \frac{l}{r}$, $\theta = \frac{18}{60} = \frac{3}{10}$ rad $= 0.3$ rad

$\because$ 1 rad $= 57^\circ\, 16''\, 22'$ (approximate)

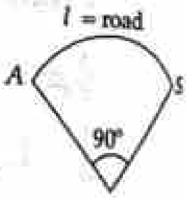
$\therefore$ 0.3 rad $= 5.7^\circ \times 3 =$ more than 17° and less than 18°

31. (c) $20^\circ = \frac{20^\circ}{180^\circ}\pi = \frac{\pi}{9}$ rad

From, $\theta = \frac{l}{r}$ $\qquad l = \theta\, r = \frac{\pi}{9}\times 54 = \frac{22}{7\times 9}\times 54$ cm

$$= \frac{22\times 6}{7} = \frac{132}{7} = 18\frac{6}{7} \text{ cm}$$

32. (b) $22\frac{1^\circ}{2} = \frac{\left(\frac{45^\circ}{2}\right)}{180}\times\pi \text{ rad} = \frac{\pi}{8}$ rad



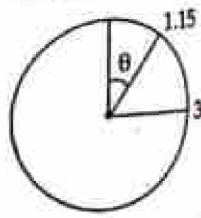
$\because\ \theta = \frac{l}{r} \Rightarrow r = \frac{l}{\theta} = \frac{(40)}{\left(\frac{\pi}{8}\right)} = \frac{320}{\pi} = \frac{320\times 7}{22} = 101.8$ cm

33. (c) $90^\circ = \frac{\pi}{2}$

From, $\theta = \frac{l}{r}$ $\ l = \theta\, r = \frac{\pi}{2}\times 100$ meter

$$= \frac{3.14}{2}\times 100 = 157 \text{ meter}$$

34. (a) From 1 hour 15 minutes to half past three, minute hand covers 2 hours 15 minutes i.e., $2\frac{1}{4}$ rotations.

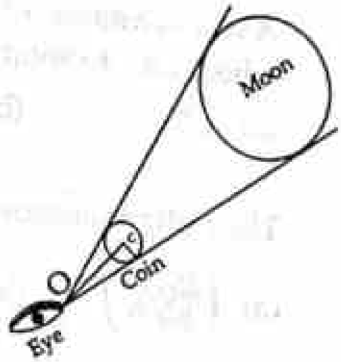


$\therefore$ It covers $2\frac{1}{4}\times 2\pi = (4.5)\,\pi$ rad distance.

(d) From half past six in the morning to 3 o'clock at noon, time elapsed is 8 hours 30 minutes.

Since hour hand covers 30° in 5 minute therefore it covers $30° \times 8\frac{1}{2}$ $= 255°$ in $8\frac{1}{2}$ hours.

(d) See the figure, let coin is placed at a distance l cm from the eye of the observer



i.e. $OC = l$ cm

In ΔOCF, $\angle COF = \frac{1}{2} \times 30' = 15' = \left(\frac{1}{4}\right)^{\circ}$

$$\sin(\angle COF) = \frac{CF}{OC} = \frac{\left(\frac{1}{2}\right)}{l}$$

($\because$ diamter = 1 cm $\therefore$ CF = $\frac{1}{2}$ cm)

$\Rightarrow \sin\left(\frac{1}{4}\right)^{\circ} = \frac{1}{2l}$

When θ is very-very small then sinθ = θ

(Learn that it is true when θ is in radian)

$\therefore \left(\frac{1}{4}\right)^{\circ} = \frac{1}{2l}$

$\Rightarrow \left(\frac{1}{4} \times \frac{180}{\pi}\right) \text{rad} = \frac{1}{2l}$

$\Rightarrow l = \frac{4 \times 180}{2 \times \pi} = \frac{360}{\pi}$

$= \frac{360 \times 7}{22} = \frac{1260}{11} = 114\frac{6}{11}$ cm

(a) Revolution in 24 hours = 360°

$\therefore$ Revolution in 1 hours = $\frac{360°}{24} = 15°$

Revolution in 4 hours = $15° \times 4 = 60°$

$\because$ Revolution in 60 minutes = 15°

Revolution in 12 minutes = $\frac{15° \times 12°}{60°} = 3°$

$\therefore$ Revolution in 4 hours 12 minutes = $60° + 3° = 63°$

Exercise–9B

1. The angle formed by the hour-hand and the minute-hand of a clock at 2 : 15 p.m. is

(a) $22\frac{1}{2}°$ (b) 30° (c) $27\frac{1}{2}°$ (d) 45°

[SSC Tier-I 2012]

2. Two angles of triangle are $\frac{1}{2}$ and $\frac{1}{3}$ radian. The measure of the third angle in degree $\left(\text{taking } \pi = \frac{22}{7}\right)$ is

(a) $132\frac{1}{11}^\circ$ (b) $132\frac{2}{11}^\circ$ (c) $132\frac{3}{11}^\circ$ (d) 132°

[SSC Tier-I 2012]

3. A wheel rotates 3.5 times in one second. What time (in second) does the wheel take to rotate 55 radian of angle ?

(a) 1.5 (b) 2.5 (c) 3.5 (d) 4.5

[SSC Tier-I 2012]

4. The radian measure of 63°14'51" is

(a) $\left(\frac{2811\pi}{8000}\right)^c$ (b) $\left(\frac{3811\pi}{8000}\right)^c$ (c) $\left(\frac{4811\pi}{8000}\right)^c$ (d) $\left(\frac{5811\pi}{8000}\right)^c$

[SSC Tier-I 2012]

5. When a pendulum of length 50 cm oscillates, it produce an arc of 16 cm. The angle so formed in degree measure is (approx)

(a) 18°25' (b) 18°35' (c) 18°20' (d) 18°08'

[SSC Tier-I 2012]

6. A rail road curve is to be laid out on a circle. What radius should be used if the track is to change direction by 25° in a distance of 40 metres?

(a) 91.64 metres (b) 90.46 metres (c) 89.64 metres (d) 93.64 metres

[SSC Tier-I 2012]

7. An arc of a circle of radius 42 cm subtends an angle of 15° at the centre. Taking $\pi = \frac{22}{7}$, the length of the arc is :

(a) $\frac{88}{5}$ cm (b) 11 cm (c) 12 cm (d) $\frac{44}{5}$ cm

[SSC Tier-I 2012]

Answer–9B

1. (a) 2. (c) 3. (b) 4. (a) 5. (b) 6. (a) 7. (b)

Explanation

1. (a) From Trick,

using, $\frac{11}{2}M = 30H \pm A$

Here, $M = 15, H = 2$

Hence, $\frac{11}{2}\times 15 = 30 \times 2 \pm A$

or, $A = \frac{165}{2} - 60 = 82\frac{1}{2} - 60 = 22\frac{1}{2}$.

Therefore, angle will be $22\frac{1}{2}^\circ$

Second Method : Minute hand forms an angle of 30° when moves from 2 to 3. During this time, hour hand forms an angle of $\frac{15}{60} \times 30^\circ = 7\frac{1^\circ}{2}$

Therefore, angle will be $= 30^\circ - 7\frac{1^\circ}{2} = 22\frac{1^\circ}{2}$

Sum of two angle $= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ rad

$\frac{22}{7}$ rad $= 180^\circ$ $\quad (\because \pi = \frac{22}{7})$

$\therefore \frac{5}{6}$ rad $= \frac{180^\circ}{22} \times 7 \times \frac{5}{6} = \frac{30^\circ \times 35}{22} = \frac{15^\circ \times 35}{11}$

$\therefore$ Remain angle $= 180^\circ - \frac{15^\circ \times 35}{11} = 180^\circ - \frac{525^\circ}{11}$

$= 180^\circ - 47\frac{8^\circ}{11} = 132\frac{3^\circ}{11}$

(a) 1 rotation $\equiv 2\pi$ radian

$\therefore$ 3.5 rotation $= 3.5 \times 2\pi$ radian

$= 3.5 \times 2 \times \frac{22}{7} = 22$ radian

$\therefore$ Wheel rotation in one second is 22 radian.

$\therefore$ Wheel rotation in 55 radian $\frac{55}{22} = 2.5$ second.

(a) $63^\circ 14' 51'' = 63\left(14\frac{51}{60}\right)' = 63\left(14\frac{17}{20}\right)'$

$= 63\left(\frac{297}{20}\right)' = \left(63\frac{297}{20 \times 60}\right)^\circ$

$= \left(63\frac{99}{20 \times 20}\right)^\circ = \left(63\frac{99}{400}\right)^\circ$

$= \left(\frac{25299}{400}\right)^\circ = \left(\frac{25299}{400} \times \frac{\pi}{180}\right)$ rad

$= \left(\frac{2811\pi}{400 \times 20}\right) = \left(\frac{2811\pi}{8000}\right)$

Trick, Value of $63^\circ 14' 51''$ is 60° (approximate)

$\therefore$ Value of $63^\circ 14' 51''$ should be more than $\frac{\pi}{3} = 0{\cdot}33\pi$

From option (a),

$\left(\frac{2811\pi}{8000}\right) = \left(\frac{2800\pi}{8000}\right)$ (approximate)

$= \left(\frac{28}{80}\pi\right) = 0{\cdot}35\pi$ (approximate)

$\frac{38}{80}\pi = 0{\cdot}47\pi$ (approximate)

So, option (a) is correct.

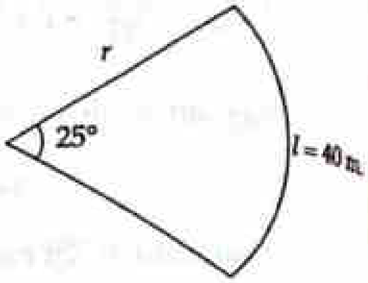
5. (b) $\theta = \frac{l}{r} \Rightarrow \theta = \frac{16}{50}$ rad

$$= \left(\frac{16}{50} \times \frac{180}{\pi}\right)^\circ = \left(\frac{16 \times 18}{5} \times \frac{7}{22}\right)^\circ$$

$$\left(\frac{16 \times 9 \times 7}{5 \times 11}\right)^\circ = \left(\frac{1008}{55}\right)^\circ$$

$$= \left(18\frac{18}{55}\right)^\circ = 18^\circ 35' \text{ (approximate)}$$

6. (a) $25^\circ = 25 \times \frac{\pi}{180}$ rad $= \frac{5\pi}{36}$ rad

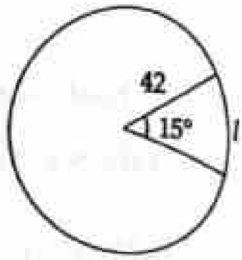


From $\theta = \frac{l}{r}$, $\frac{5\pi}{36} = \frac{40}{r}$

$\therefore\ r = \frac{40 \times 36}{5\pi}$

$$= \frac{8 \times 36 \times 7}{22} = \frac{2016}{22} = 91{\cdot}64 \text{ meter}$$

7. (b) $15^\circ = \frac{15}{180} \times \pi = \frac{\pi}{12}$



From $\theta = \frac{l}{r}$, $l = \theta r$

or, $l = \frac{\pi}{12} \times 42 = \frac{22}{7} \times \frac{1}{12} \times 42 = 11$ cm

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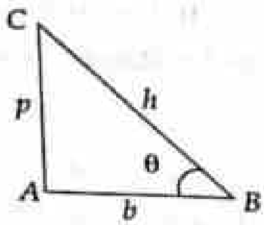
10

Trigonometric Ratio of Specific Angles

1. Trigonometric Ratio :

To study different trigonometric ratio functions we will consider a right angled triangle. Suppose ABC is a right angled triangle with $\angle A = 90°$.



We can obtain six different trigonometric ratio from the sides of these triangle. They are respectively $\frac{AC}{BC}, \frac{AB}{BC}, \frac{AC}{AB}, \frac{AB}{AC}, \frac{BC}{AB}$ and $\frac{BC}{AC}$. If $\angle B = \theta$ then these ratio are respectively called sinθ, cos θ, tan θ, cot θ, sec θ and cosec θ. Clearly for the given angle θ, AC (p) is perpendicular, AB (b) is base and BC (h) is hypotenuse. Hence six different trigonometric ratios are as follows (see the given figure)

$$\sin\theta = \frac{p}{h} = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}} = \frac{AC}{BC}$$

$$\cos\theta = \frac{b}{h} = \frac{\text{base}}{\text{hypotenuse}} = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}} = \frac{AB}{BC}$$

$$\tan\theta = \frac{p}{b} = \frac{\text{perpendicular}}{\text{base}} = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta} = \frac{AC}{AB}$$

$$\cot\theta = \frac{b}{p} = \frac{\text{base}}{\text{perpendicular}} = \frac{\text{side adjacent to angle }\theta}{\text{side opposite to angle }\theta} = \frac{AB}{AC}$$

$$\sec\theta = \frac{h}{b} = \frac{\text{hypotenuse}}{\text{base}} = \frac{\text{hypotenuse}}{\text{side adjacent to angle }\theta} = \frac{BC}{AB}$$

$$\operatorname{cosec}\theta = \frac{h}{p} = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{\text{hypotenuse}}{\text{side opposite to angle }\theta} = \frac{BC}{AC}$$

Clearly sin θ and cosec θ are reciprocals to each other. Similarly cos θ and sec θ are reciprocals to each other while tan θ and cot θ are reciprocals to each other.

$\therefore$ $\sin\theta \cdot \operatorname{cosec}\theta = 1$

$\sec\theta \cdot \cos\theta = 1$

$\tan\theta \cdot \cot\theta = 1$

Since $p^2 + b^2 = h^2$ i.e. $(\text{perpendicular})^2 + (\text{base})^2 = (\text{hypotenuse})^2$, therefore we can find the other trigonometric ratio when any one of them is known.

Following relations among natural number will help in solving the problems on trigonometric ratio angle.

$3^2 + 4^2 = 5^2$, $\quad 6^2 + 8^2 = 10^2$

$5^2 + 12^2 = 13^2$, $\quad 10^2 + 24^2 = 26^2$

$8^2 + 15^2 = 17^2$, $\quad 16^2 + 30^2 = 34^2$

$20^2 + 21^2 = 29^2$ $\quad 9^2 + 40^2 = 41^2$ etc.

2. Meaning of $\sin^2\theta$, $\cos^2\theta$:

In trigonometric ratio $(\sin\theta)^2$ is written as $\sin^2\theta$, $(\cos\theta)^2$ is written as $\cos^2\theta$, $(\tan\theta)^3$ is written as $\tan^3\theta$ etc.

3. Value of some specific angle of trigonometrical (t)-ratio function :

We must learn the following table to solve the questions based on trigonometrical (t)-ratio angle 0°, 30°, 45°, 60°, 90°.

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefine
$\cot\theta$	undefine	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefine
$\text{cosec}\,\theta$	undefine	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Tricks to learn the table are as follows :

3.1 Values of $\sin\theta$ are respectively $\sqrt{\frac{0}{4}}, \sqrt{\frac{1}{4}}, \sqrt{\frac{2}{4}}, \sqrt{\frac{3}{4}}, \sqrt{\frac{4}{4}}$

3.2 Value of $\cos\theta$ are in reverse order to that of $\sin\theta$.

Thus values of $\cos\theta$ are respectively $\sqrt{\frac{4}{4}}, \sqrt{\frac{3}{4}}, \sqrt{\frac{2}{4}}, \sqrt{\frac{1}{4}}, \sqrt{\frac{0}{4}}$

3.3 Divide $\sin\theta$ by $\cos\theta$ to get the values of $\tan\theta$.

3.4 Divide $\cos\theta$ by $\sin\theta$ to get the values of $\cot\theta$.

3.5 Values of $\sec\theta$ are reciprocals to the values of $\cos\theta$ i.e., values of $\sec\theta$ are respectively $\sqrt{\frac{4}{0}}, \sqrt{\frac{4}{1}}, \sqrt{\frac{4}{2}}, \sqrt{\frac{4}{3}}, \sqrt{\frac{4}{4}}$ here $\sqrt{\frac{4}{0}}$ is undefined.

3.6 Values of $\text{cosec}\,\theta$ are reciprocals to the values of $\sin\theta$.

We must note that values of $\sin\theta$, $\tan\theta$ and $\sec\theta$ are increasing in 0° to 90°. While values of '*c*' functions i.e., $\cos\theta$, $\cot\theta$ and $\text{cosec}\,\theta$ are decreasing in 0° to 90°.

4. Minimum and Maximum value of t-ratio functions when $0 \le \theta \le 90°$.

From t-ratio table it is clear that for $0 \le \theta \le 90°$.

4.1 Maximum value of each of $\sin\theta$ and $\cos\theta$ is 1.

4.2 Minimum value of each of $\sin\theta$ and $\cos\theta$ is 0.

4.3 Maximum value of $\tan\theta$ and $\cot\theta$ are not defined.

4.4 Minimum value of $\tan\theta$ and $\cot\theta$ are 0.

4.5 Maximum value of $\sec\theta$ and $\operatorname{cosec}\theta$ are undefined.

4.6 Minimum value of each of $\sec\theta$ and $\operatorname{cosec}\theta$ is 1.

5. Some important tricks for Maximum and Minimum values :

5.1 Maximum and Minimum value of $a\cos\theta + b\sin\theta$ are respectively $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$, where θ be any angle.

5.2 If $a > b$ then maximum and minimum value of $a\cos^2\theta + b\sin^2\theta$ are respectively a and b. If $a < b$ then maximum and minimum value of $a\cos^2\theta + b\sin^2\theta$ are respectively b and a.

[i.e., which ever is greater between a and b is the greater value and smaller one is the least value].

5.3 Minimum value of $a\tan^2\theta + b\cot^2\theta$ is $2\sqrt{ab}$ where a and b are positive quantities.

5.4 Minimum value of $a\sec^2\theta + b\cos^2\theta$ is $2\sqrt{ab}$ where a and b are positive quantities.

5.5 Minimum and Maximum value of $a\operatorname{cosec}^2\theta + b\sin^2\theta$ is $2\sqrt{ab}$ where a and b are positive quantities.

5.6 If m and n are positive integers then $(\sin\theta)^n \le \sin\theta \le 1$ and $(\cos\theta)^m \le \cos\theta \le 1$

[Explanation : $\because 0 \le \sin^2\theta \le 1$ i.e., $\sin^2\theta$ is a proper fraction, value of $(\sin^{2n}\theta)$ decreases as its power increases; e.g. when $\theta = 30°$

$\sin\theta = \frac{1}{2}, (\sin\theta)^2 = \frac{1}{4}, (\sin\theta)^3 = \frac{1}{8}$......

Clearly $\sin^3\theta < \sin^2\theta < \sin\theta$]

5.7 $\because$ From above mentioned point we can say that $(\sin^2\theta)^n \le \sin^2\theta$ and $(\cos^2\theta)^m \le \cos^2\theta$,

adding, $\sin^{2n}\theta + \cos^{2m}\theta \le \sin^2\theta + \cos^2\theta$

or, $\boxed{\sin^{2n}\theta + \cos^{2m}\theta \le 1}$

e.g., $\sin^4\theta + \cos^4\theta \le 1$,

$\sin^6\theta + \cos^6\theta \le 1$,

$\sin^8\theta + \cos^{10}\theta \le 1$ etc.

6. Complementary Angle.

For a given angle θ its complementary angle is $(90° - \theta)$.

From definition,

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\text{and } \cos(90° - \theta) = \frac{\text{side along with angle }(90° - \theta)}{\text{hypotenuse}} = \frac{BC}{AC}$$

∴ $\boxed{\sin\theta = \cos(90° - \theta)}$

Similarly, we can prove that

∴ $\boxed{\cos\theta = \sin(90° - \theta)}$, $\tan(90° - \theta) = \cot\theta$, $\cot(90° - \theta) = \tan\theta$,

$\sec(90° - \theta) = \text{cosec}\,\theta$, $\text{cosec}(90° - \theta) = \sec\theta$

7. Trigonometric Inequalities :

If $0 < \theta < 90°$ then

7.1 Value of sin θ increase as θ increase.

Value of tanθ increase as θ increase.

Value of secθ increase as θ increase.

Thus, sinθ, tanθ and secθ follow the same sign of inequality i.e.,

$$\begin{array}{lll} \sin\theta_1 > \sin\theta_2 & \Rightarrow & \theta_1 > \theta_2 \\ \tan\theta_1 > \tan\theta_2 & \Rightarrow & \theta_1 > \theta_2 \\ \sec\theta_1 > \sec\theta_2 & \Rightarrow & \theta_1 > \theta_2 \end{array}$$

7.2 Value of cosθ decreases as θ increases.

Value of cotθ decreases as θ increases.

Value of cosec θ decreases as θ increases.

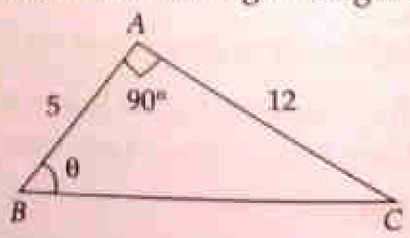
Thus cosθ, cotθ and cosecθ follow the opposite sign of inequality i.e.,

$$\begin{array}{lll} \cos\theta_1 > \cos\theta_2 & \Rightarrow & \theta_1 < \theta_2 \\ \cot\theta_1 > \cot\theta_2 & \Rightarrow & \theta_1 < \theta_2 \\ \text{cosec}\,\theta_1 > \text{cosec}\,\theta_2 & \Rightarrow & \theta_1 < \theta_2 \end{array}$$

Trick : Sign of inequality reverse in 'c' function i.e., in cosθ, cotθ and cosecθ

Solved Example

1. Write all the six t-ratios value in the given figure :



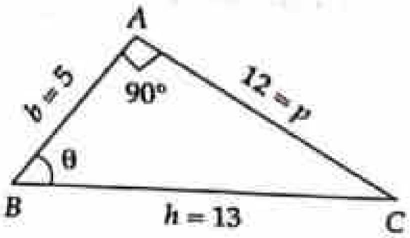
...tion : Given ΔABC is, a right angle triangle with $\angle A = 90°$,

Let $AC = 12 = p$ and $AB = 5 = b$

Then from Pythagoras theorem,

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144} = \sqrt{169} = 13$$



Here side opposite to θ is AC which is p.

Side adjacent to θ is AB, which is b.

Side opposite to right angle is BC, which is hypotenuse h.

$\therefore \sin\theta = \frac{p}{h} = \frac{12}{13}$ $\qquad$ $\operatorname{cosec}\theta = \frac{h}{p} = \frac{13}{12}$

$\cos\theta = \frac{b}{h} = \frac{5}{13}$ $\qquad$ $\sec\theta = \frac{h}{b} = \frac{13}{5}$

$\tan\theta = \frac{p}{b} = \frac{12}{5}$ $\qquad$ $\cot\theta = \frac{b}{p} = \frac{5}{12}$

If $15\cot\theta = 8$ then calculate the remaining trigonometric ratio.

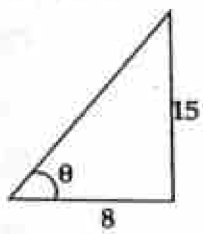
...tion : Given, $\cot\theta = \frac{8}{15} = \frac{b}{p}$

Let $b = 8k$ and $p = 15k$

From Pythagoras Theorem, $h^2 = p^2 + b^2 = (15k)^2 + (8k)^2$

or, $h^2 = 225k^2 + 64k^2 = 289k^2$

or, $h = \sqrt{289k^2} = 17k$



Hence, $\sin\theta = \frac{p}{h} = \frac{15k}{17k} = \frac{15}{17}$ $\qquad$ $\cos\theta = \frac{b}{h} = \frac{8k}{17k} = \frac{8}{17}$

$\tan\theta = \frac{p}{b} = \frac{15k}{8k} = \frac{15}{8}$ $\qquad$ $\sec\theta = \frac{h}{b} = \frac{17k}{8k} = \frac{17}{8}$

$\operatorname{cosec}\theta = \frac{h}{p} = \frac{17k}{15k} = \frac{17}{15}$

If $\tan\theta = \frac{q}{p}$ then find the value of $\frac{p\sin\theta + q\cos\theta}{p\cos\theta + q\sin\theta}$

...lution : $\tan\theta = \frac{q}{p} = \frac{\text{perpendicular}}{\text{base}}$

Let perpendicular $= qk$ and base $= pk$ then

$$\text{hypotenuse} = \sqrt{(\text{perpendicular})^2 + (\text{base})^2} = \sqrt{q^2k^2 + p^2k^2} = \sqrt{q^2 + p^2}\,k$$

$$\therefore \sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{qk}{\sqrt{q^2 + p^2}\,k} = \frac{q}{\sqrt{q^2 + p^2}}$$

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{pk}{k\sqrt{q^2 + p^2}} = \frac{p}{\sqrt{q^2 + p^2}}$$

$$\text{Hence, given expression} = \frac{p\sin\theta + q\cos\theta}{p\cos\theta + q\sin\theta}$$

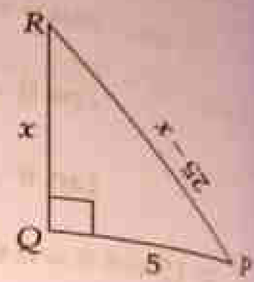
$$= \frac{\dfrac{pq}{\sqrt{q^2+p^2}} + \dfrac{qp}{\sqrt{q^2+p^2}}}{p\cdot\dfrac{p}{\sqrt{q^2+p^2}} + q\cdot\dfrac{q}{\sqrt{q^2+p^2}}}$$

$$= \frac{pq+qp}{p^2+q^2} = \frac{2pq}{p^2+q^2}$$

4. In ΔPQR, with $\angle Q$ at right angle, given that $PR + QR = 25$ cm and $PQ = 5$ cm. Find the value of :

(i) $\sin P$, $\cos P$ and $\tan P$.

(ii) $\sec R - \cot P$.



Solution : Given, $\angle Q = 90^\circ$

$PQ = 5$ cm, $PR + QR = 25$ cm

Let $QR = x$ cm then $PR = (25 - x)$ cm

From Pythagoras theorem, $RP^2 = RQ^2 + QP^2$

or, $(25-x)^2 = x^2 + 5^2$ or, $625 - 50x + x^2 = x^2 + 25$

or, $-50x = 25 - 625 = -600$ or, $x = \dfrac{-600}{-50} = 12$

$\therefore$ $RQ = 12$ cm and $RP = (25 - 12)$ cm $= 13$ cm

$$\therefore \quad \sin P = \frac{\text{side opposite to } \angle P}{\text{hypotenuse}} = \frac{RQ}{RP} = \frac{12}{13}$$

$$\cos P = \frac{\text{side adjacent to } \angle P}{\text{hypotenuse}} = \frac{PQ}{RP} = \frac{5}{13}$$

$$\tan P = \frac{\text{side opposite to } \angle P}{\text{side associated with } \angle P} = \frac{RQ}{PQ} = \frac{12}{5}$$

$$\cot P = \frac{\text{side associated with } \angle P}{\text{side opposite to } \angle P} = \frac{PQ}{RQ} = \frac{5}{12}$$

$$\sec R = \frac{\text{hypotenuse}}{\text{side associated to } \angle R} = \frac{RP}{RQ} = \frac{13}{12}$$

$$\therefore \quad \sec R - \cot P = \frac{13}{12} - \frac{5}{12} = \frac{8}{12} = \frac{2}{3}$$

5. ABC is an isosceles triangle with $AB = AC = 13$ cm and $BC = 20$ cm. If $\angle ABC = \theta$ then find the value of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$ and $\operatorname{cosec}\theta$.

Solution : In ΔABC given that, $AB = AC = 13$, $BC = 20$ and $\angle ABD = \theta$ (see the figure)

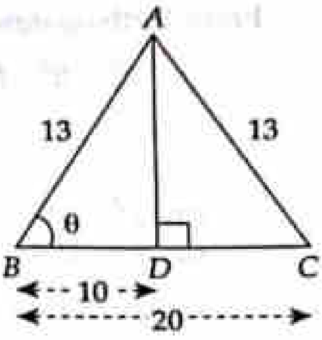
Draw $AD \perp$r BC.

Then $\angle ADB = \angle ADC = 90°$

and $BD = DC = 10$ cm

Clearly $\angle ABC = \angle ABD = \theta$

Using Pythagoras theorem in ΔABD.



$$AD^2 + BD^2 = AB^2$$

or, $AD^2 + 10^2 = 13^2$

or, $AD^2 = 13^2 - 10^2 = 169 - 100 = 69$

or, $AD = \sqrt{69}$

Now, In ΔABD,

$$\sin\theta = \frac{AD}{AB} = \frac{\sqrt{69}}{13} \qquad \cos\theta = \frac{BD}{AB} = \frac{10}{13}$$

$$\tan\theta = \frac{AD}{BD} = \frac{\sqrt{69}}{10} \qquad \cot\theta = \frac{BD}{AD} = \frac{10}{\sqrt{69}}$$

$$\sec\theta = \frac{AB}{BD} = \frac{13}{10} \qquad \text{cosec}\,\theta = \frac{AB}{AD} = \frac{13}{\sqrt{69}}$$

6. $PQRS$ is a rhombus whose diagonals are $PR = 6$ cm and $QS = 8$ cm. If O is the point of intersection of diagonals and $\angle PQO = \theta$ then find the value of $\sin\theta$, $\tan\theta$ and $\sec\theta$.

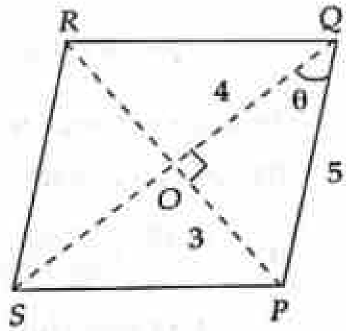
Solution : We know that diagonals of a rhombus intersect at right angle.

Hence in ΔPOQ (see the figure)

$$\angle POQ = 90°,\ OQ = \frac{QS}{2} = 4 \text{ cm and } OP = \frac{PR}{2} = 3 \text{ cm}$$

Using Pythagoras Theorem,

$$PQ = \sqrt{OP^2 + OQ^2} = \sqrt{3^2 + 4^2} = 5$$



In triangle POQ,

$$\sin\theta = \frac{OP}{PQ} = \frac{3}{5}$$

$$\tan\theta = \frac{OP}{OQ} = \frac{3}{4}$$

$$\sec\theta = \frac{PQ}{OQ} = \frac{5}{4}$$

7. If $\sec\theta = x + \frac{1}{4x}$ then prove that

$$\sec\theta + \tan\theta = 2x \quad \text{or} \quad \frac{1}{2x}$$

Solution : Given, $\sec\theta = x + \frac{1}{4x} = \frac{4x^2+1}{4x} = \frac{h}{b}$

Let $h = (4x^2 + 1)k$ and $b = (4x)k$

From Pythagoras theorem,

$$p^2 = h^2 - b^2 = (4x^2+1)^2 k^2 - (16x^2)k^2$$
$$= (16x^4 + 8x^2 + 1)k^2 - (16x^2)k^2$$
$$= (16x^4 - 8x^2 + 1)k^2$$

or, $p^2 = (4x^2-1)^2k^2$

or, $p = \pm(4x^2-1)k$ or, $(1-4x^2)k$

$\therefore$ $p = (4x^2-1)k$

[It must be noted that if $x^2 > \frac{1}{4}$ then $p = (4x^2-1)k$ and if $x^2 < \frac{1}{4}$ then $p = (1-4x^2)k$ as p is always a positive quantity]

If $p = (4x^2-1)k$ then

$$\sec\theta + \tan\theta = \frac{h}{b} + \frac{p}{b} = \frac{h+p}{b}$$
$$= \frac{(4x^2+1)k + (4x^2-1)k}{(4x)k}$$
$$= \frac{(4x^2+1+4x^2-1)k}{(4x)k} = \frac{8x^2}{4x} = 2x$$

If $p = (1-4x^2)k$ then

$$\sec\theta + \tan\theta = \frac{h}{b} + \frac{p}{b} = \frac{h+p}{b}$$
$$= \frac{(4x^2+1)k - (4x^2-1)k}{4xk}$$
$$= \frac{(4x^2+1-4x^2+1)k}{4xk} = \frac{2}{4x} = \frac{1}{2x}$$

$\therefore$ $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$; **Proved.**

8. Find the value of following expressions :

(i) $\sec^2 60^\circ \cos^2 45^\circ - \operatorname{cosec}^2 30^\circ$

(ii) $\dfrac{\tan 45^\circ \cot^2 60^\circ + \tan^2 30^\circ \cot 45^\circ}{\sin 30^\circ + \cos 60^\circ}$

Solution : (i) From the t-ratio table,

$$\sec 60^\circ = 2, \cos 45^\circ = \frac{1}{\sqrt{2}}, \operatorname{cosec} 30^\circ = 2$$

$$\therefore \sec^2 60^\circ \cos^2 45^\circ - \operatorname{cosec}^2 30^\circ = 2^2\left(\frac{1}{\sqrt{2}}\right)^2 - 2^2 = \frac{4}{2} - 4 = -2$$

(ii) We know that

$\tan 45^\circ = 1, \cot 60^\circ = \frac{1}{\sqrt{3}}, \tan 30^\circ = \frac{1}{\sqrt{3}}, \cot 45^\circ = 1, \sin 30^\circ = \frac{1}{2}$

and $\cos 60^\circ = \frac{1}{2}$

$\therefore \dfrac{\tan 45^\circ \cot^2 60^\circ + \tan^2 30^\circ \cot 45^\circ}{\sin 30^\circ + \cos 60^\circ}$

$$= \frac{1\cdot\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2\cdot 1}{\frac{1}{2}+\frac{1}{2}} = \frac{\frac{1}{3}+\frac{1}{3}}{1} = \frac{2}{3}$$

Find the acute angle θ if $4\sin^2\theta = 3$.

Solution : Given that $4\sin^2\theta = 3$

or, $\sin^2\theta = \dfrac{3}{4}$

or, $\sin\theta = \sqrt{\dfrac{3}{4}} = \dfrac{\sqrt{3}}{2}$ $\qquad \therefore \theta = 60^\circ$

Find the maximum value of $\dfrac{1}{\operatorname{cosec}\theta}$

Solution : $\because \dfrac{1}{\operatorname{cosec}\theta} = \sin\theta$ and maximum value of $\sin\theta$ is 1.

$\therefore$ Maximum value of $\dfrac{1}{\operatorname{cosec}\theta}$ is 1.

Prove that minimum value of $a\tan^2\theta + b\cot^2\theta$ is $2\sqrt{ab}$ where a and b are positive quantities. Find the minimum value of $16\tan^2\theta + 9\cot^2\theta$.

Solution : $a\tan^2\theta + b\cot^2\theta = (\sqrt{a}\tan\theta - \sqrt{b}\cot\theta)^2 + 2\sqrt{a}\sqrt{b}\tan\theta\cot\theta$

$= (\sqrt{a}\tan\theta - \sqrt{b}\cot\theta)^2 + 2\sqrt{ab}$ $\qquad (\because \tan\theta\cdot\cot\theta = 1)$

But $(\sqrt{a}\tan\theta - \sqrt{b}\cot\theta)^2$ is either 0 or greater than zero.

$\therefore$ $a\tan^2\theta + b\cot^2\theta \geq 0 + 2\sqrt{ab}$

or, $a\tan^2\theta + b\cot^2\theta \geq 2\sqrt{ab}$

Since value of $a\tan^2\theta + b\cot^2\theta$ is greater than or equal to $2\sqrt{ab}$, its minimum value is $2\sqrt{ab}$.

[A special comment : Do not write the given expression as $(\sqrt{a}\tan\theta + \sqrt{b}\cot\theta)^2 - 2\sqrt{ab}\tan\theta\cot\theta$.

In this situation minimum value of, $\sqrt{a}\tan\theta + \sqrt{b}\cot\theta$ cannot be zero.

Second part : Minimum value of $16\tan^2\theta + 9\cot^2\theta$

$$= 2\sqrt{16\times 9} = 2\sqrt{144} = 2\times 12 = 24$$

12. Find the difference between square of greatest and least value of $15\cos\theta - 8\sin\theta + 5$.

Solution : We know that maximum and minimum value of $a\cos\theta + b\sin\theta$ are respectively $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$.

Here, $a = 15$, $b = 8$

$\therefore \quad \sqrt{a^2+b^2} = \sqrt{15^2+8^2} = 17$

Hence, maximum value = 17 + 5 = 22

and minimum value = – 17 + 5 = – 12

∴ Difference between maximum and minimum value

$= (22)^2 - (-12)^2 = 22^2 - 12^2 = (22+12)(22-12) = 34 \times 10 = 340$

13. If $\tan 3\theta = \cot(75° - 2\theta)$ then find the value of $\sin 4\theta$.

Solution : $\tan 3\theta = \cot(75° - 2\theta) = \tan(90° - (75° - 2\theta))$

or, $3\theta = 15° + 2\theta$ or, $\theta = 15°$

$\therefore \quad \sin 4\theta = \sin 60° = \frac{\sqrt{3}}{2}$

14. Choose the correct statements among following :

(a) $\cos 40° > \cos 70°$
(b) $\sin 35° > \sin 65°$
(c) $\tan 45° < \tan 46°$
(d) $\cot 40° < \cot 39°$
(e) $\sec 20° > \sec 40°$
(f) $\text{cosec}\, 20° < \text{cosec}\, 30°$

Solution : When $0 < \theta < 90°$,

(a) Value of $\cos\theta$ decreases as θ increases, hence, $40° < 70° \Rightarrow \cos 40° > \cos 70°$. The statement is true.

(b) As θ increases, value of $\sin\theta$ also increase, hence $65° > 35°$ $\Rightarrow \sin 65° > \sin 35°$. Hence, the statement is false.

(c) As θ increases value of $\tan\theta$ increases, hence $45° < 46°$ $\Rightarrow \tan 45° < \tan 46°$. Hence, given statement is true.

(d) As θ increases, value of $\cot\theta$ decreases, hence $40° > 39°$ $\Rightarrow \cot 40° < \cot 39°$. Hence, the statement is true.

(e) As θ increases, value of $\sec\theta$ increases hence $20° < 40°$ $\Rightarrow \sec 20° < \sec 40°$. Hence, the statement is false.

(f) As θ increases, value of $\text{cosec}\,\theta$ decreases, Hence $20° < 30°$ $\Rightarrow \text{cosec}\, 20° > \text{cosec}\, 30°$. Hence, given statement is false.

Exercise–10A

1. If $\tan x = \frac{3}{4}, 0 < x < 90°$ then what is the value of $\sin x \cos x$?

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{12}{25}$ (d) $\frac{13}{25}$

2. Which of the following is true ?

(a) $\tan x > 1;\ 45° < x < 90°$

(b) $\sin x > \frac{1}{2};\ 0° < x < 30°$

(c) $\cos x > \frac{1}{2};\ 60° < x < 90°$

(d) For some value of x in $30° < x < 45°$, $\sin x = \cos x$

3. **Statement (A)** : $\sin 1° < \cos 1°$
Reason (R) : $\sin\theta < \cos\theta$ When $0° < \theta < 90°$ then
(a) Both A and R are correct and R is the correct explanation of A.
(b) Both A and R are correct but R is not a correct explanation of A.
(c) A is correct, R is incorrect.
(d) A is incorrect, R is correct.

4. If $\cos A = \frac{5}{13}$ then what is the value of $\frac{\sin A - \cot A}{2\tan A}$?
(a) $\frac{395}{3644}$ (b) $\frac{395}{3844}$ (c) $\frac{395}{3744}$ (d) $\frac{385}{3744}$

5. If $\sin x = \cos y$ where x and y are acute angle then what is the relation between x and y ?
(a) $x - y = \frac{\pi}{2}$ (b) $x + y = \frac{3\pi}{2}$ (c) $x + y = \frac{\pi}{2}$ (d) $x + y = \frac{\pi}{4}$

6. In a right angled triangle base $BC = 15$ cm and $\sin B = \frac{4}{5}$, then what is the length of hypotenuse AB ?
(a) 25 cm (b) 20 cm (c) 5 cm (d) 4 cm

7. If $\sin\theta = \frac{m^2 - n^2}{m^2 + n^2}$, then value of $\tan\theta$ is
(a) $\frac{m^2 + n^2}{m^2 - n^2}$ (b) $\frac{2mn}{m^2 + n^2}$ (c) $\frac{m^2 - n^2}{2mn}$ (d) $\frac{m^2 + n^2}{2mn}$

8. If $\sin(x - y) = \frac{1}{2}$ and $\cos(x + y) = \frac{1}{2}$, then value of x is
(a) 15° (b) 30° (c) 45° (d) 60°

9. If $1 + \tan\theta = \sqrt{2}$, then value of $\cot\theta - 1$ is
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $\frac{1}{2}$

10. If $\sin(x + 54°) = \cos x$, where $0 < x,\ x + 54° < 90°$ then what is the value of x?
(a) 54° (b) 36° (c) 27° (d) 18°

11. If $x\cos 60° + y\cos 0° = 3$ and $4x\sin 30° - y\cot 45° = 2$, then what is the value of x ?
(a) –1 (b) 0 (c) 1 (d) 2

12. If $\cos x + \cos^2 x = 1$ then what is the value of $\sin^2 x + \sin^4 x$?
(a) 0 (b) 1 (c) 2 (d) 4

13. If $x + y = 90°$ and $\sin x : \sin y = \sqrt{3} : 1$ then $x : y$ equals,
(a) $1 : 1$ (b) $1 : 2$ (c) $2 : 1$ (d) $3 : 2$

14. If $0 \le x \le \frac{\pi}{2}$ then which one of the following is always true ?
(a) $\sin^2 x < \frac{1}{2}$ and $\cos^2 x > \frac{1}{2}$ (b) $\sin^2 x > \frac{1}{2}$ and $\cos^2 x < \frac{1}{2}$

(c) $\sin^2 x < \frac{1}{2}$ and $\cos^2 x < \frac{1}{2}$

(d) At least one of $\sin^2 x$, $\cos^2 x$ is less than 1.

15. If $p = \tan^2 x + \cot^2 x$ then which one is true ?

(a) $p \le 2$ (b) $p \ge 2$ (c) $p < 2$ (d) $p > 2$

16. Value of $\frac{5\sin 75^\circ \sin 77^\circ + 2\cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin 77^\circ} - \frac{7\sin 81^\circ}{\cos 9^\circ}$ is

(a) -1 (b) 0 (c) 1 (d) 2

17. If $0 < x < 45^\circ$ and $45^\circ < y < 90^\circ$, then which of the following is true.

(a) $\sin x = \sin y$ (b) $\sin x < \sin y$ (c) $\sin x > \sin y$ (d) $\sin x \le \sin y$

18. What is the value of $\sin^3 60^\circ \cot 30^\circ - 2\sec^2 45^\circ + 3\cos 60^\circ \tan 45^\circ - \tan^2 60^\circ$?

(a) $\frac{35}{8}$ (b) $\frac{-35}{8}$ (c) $\frac{-11}{8}$ (d) $\frac{11}{8}$

19. If $\tan\theta = \frac{p}{q}$, then what is the value of $\frac{p\sec\theta - q\,\text{cosec}\,\theta}{p\sec\theta + q\,\text{cosec}\,\theta}$?

(a) $\frac{p-q}{p+q}$ (b) $\frac{q^2-p^2}{q^2+p^2}$ (c) $\frac{p^2-q^2}{q^2+p^2}$ (d) 1

20. Value of $\text{cosec}^2\theta - 2 + \sin^2\theta$ is always

(a) less than zero (b) non negative

(c) zero (d) 1

21. If $\cot\theta = \frac{2xy}{x^2-y^2}$, then which one is equal to $\cos\theta$?

(a) $\frac{x^2-y^2}{x^2+y^2}$ (b) $\frac{x^2+y^2}{x^2-y^2}$ (c) $\frac{2xy}{x^2+y^2}$ (d) $\frac{2xy}{\sqrt{x^2+y^2}}$

22. For what value of θ $(\sin\theta + \text{cosec}\,\theta) = 2.5$, where $0 < \theta \le 90^\circ$?

(a) 30° (b) 45° (c) 60° (d) 90°

23. If $0 < \theta < \phi < 90^\circ$, then which one of the following is true ?

(a) $(\sin\theta + \cos\theta)^2 > 2$ (b) $(\sin^2\theta + \cos^2\phi) \le 2$

(c) $(\sin^2\theta + \cos^2\phi) < 2$ (d) $(\sin^2\theta + \cos^2\phi) > 2$

24. If $\tan\theta = \frac{2t}{1-t^2}$ then $\sin\theta + \cos\theta$ equals

(a) $\frac{t^2-1}{t^2+1}$ (b) $\frac{t^2-2t-1}{t^2+1}$ (c) $\frac{(t-1)^2}{t^2+1}$ (d) $\frac{1+2t-t^2}{1+t^2}$

25. If $0 \le \theta < \frac{\pi}{2}$ and $p = \sec^2\theta$ then which one of the following is true ?

(a) $p < 1$ (b) $p = 1$ (c) $p > 1$ (d) $p \ge 1$

26. In a ΔABC, $\angle ABC = 90^\circ$, $\angle ACB = 30^\circ$, $AB = 5$ cm. What is the length of AC?

(a) 10 cm (b) 5 cm (c) $5\sqrt{2}$ cm (d) $5\sqrt{3}$ cm

If $0 < \theta < \frac{\pi}{2}$ and $\cos\theta + \sqrt{3}\sin\theta = 2$, then what is the value of θ ?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

Suppose ABC is a right angled triangle with right angled at C. If length of sides opposite to angle A, B, C are respectively u, v and w then $\tan A + \tan B$ equals

(a) $\frac{u^2}{vw}$ (b) 1 (c) $u + v$ (d) $\frac{w^2}{uv}$

ABC is a right angled triangle with right angle at A. If $\tan B = \frac{1}{\sqrt{3}}$, and which one is the form of a hypotenuse for real k ?

(a) $3k$ (b) $2k$ (c) $5k$ (d) $9k$

If α is an acute angle and $\sin\alpha = \sqrt{\frac{x-1}{2x}}$ then $\tan\alpha$ is equal to which of the following ?

(a) $\sqrt{\frac{x-1}{x+1}}$ (b) $\sqrt{\frac{x+1}{x-1}}$

(c) $\sqrt{x^2-1}$ (d) $\sqrt{x^2+1}$

If θ is in first quadrant and $\cos\theta \geq \frac{1}{2}$ then which one of the following is true ?

(a) $\theta \leq \frac{\pi}{3}$ (b) $\theta \geq \frac{\pi}{3}$ (c) $\theta \leq \frac{\pi}{6}$ (d) $\theta \geq \frac{\pi}{6}$

What is the value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 90^\circ$?

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) 2

If $A = \frac{\pi}{6}$ and $B = \frac{\pi}{3}$, then among the following which is/are true ?

I. $\sin A + \sin B = \cos A + \cos B$

II. $\tan A + \tan B = \cot A + \cot B$

Use the alternative given below to get the correct answer

(a) only I (b) only II

(c) both I and II (d) Neither I nor II

If $\sin 17^\circ = \frac{a}{b}$ then value of $\sec 17^\circ - \sin 73^\circ$ is

(a) $\frac{a}{b\sqrt{a^2+b^2}}$ (b) $\frac{b^2}{a\sqrt{b^2-a^2}}$

(c) $\frac{a^2}{b\sqrt{b^2-a^2}}$ (d) 0

Consider the Earth as a sphere of radius R, radius of circle at latitude 40°S is

(a) $R\cos 40^\circ$ (b) $R\sin 80^\circ$ (c) $R\sin 40^\circ$ (d) $R\tan 40^\circ$

36. If $\frac{\operatorname{cosec}\theta + \sin\theta}{\operatorname{cosec}\theta - \sin\theta} = \frac{5}{3}$ and $0° < \theta < 90°$ then what is the value of $\tan\theta$?

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

37. Choose the correct statement :

(a) $\sin 40° > \sin 20°$ (b) $\sin 40° > \sin 50°$

(c) $\cos 20° > \cos 10°$ (d) Both (a) and (c) are correct

38. Given below are respectively base and hypotenuse of four right angle triangles :

1 and $\sqrt{5}$, 2 and $\sqrt{13}$, 3 and 5, 4 and $\sqrt{41}$

$\theta_1, \theta_2, \theta_3, \theta_4$ are respectively angle included between them. What are the increasing order of these values.

1. $\sin\theta_1$ 2. $\tan\theta_2$ 3. $\cos\theta_3$ 4. $\sec\theta_4$

Choose the correct code among following :

Code :

(a) 4 – 1 – 2 – 3 (b) 1 – 4 – 3 – 2 (c) 3 – 1 – 2 – 4 (d) 3 – 1 – 4 – 2

39. Consider the following statements about the expression

$\sin^3\theta + 2\sin^2\theta + 3\sin\theta$

1. For any $\theta \in R$, maximum value of this expression is 6.
2. For any $\theta \in R$, value of this expression cannot be zero.

Among above statement which is/are true ?

(a) only 1 (b) only 2

(c) Both 1 and 2 (d) Neither 1 nor 2

40. Consider right angled ΔABC with $\angle B = 90°$. If $\angle ACB = 60°$, then $AB : BC : CA$ equals

(a) $\sqrt{3}:1:2$ (b) $1:\sqrt{3}:2$ (c) $1:1:\sqrt{2}$ (d) $\sqrt{2}:1:\sqrt{3}$

41. In ΔABC, $\angle ABC = 60°$ and AD is perpendicular from A to BC. If $AB = x$ and $AC = \frac{3x}{2}$ the CD is equal to which of the following ?

(a) $\frac{x}{2}$ (b) $\sqrt{\frac{3}{2}}\,x$ (c) $\frac{3x}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2x}$

42. If $0° \le \theta \le 90°$ then for any value of θ which one is correct ?

(a) $\sin\theta = \sqrt{2}$ (b) $\sin\theta + \cos\theta = 2$

(c) $\sin\theta + \cos\theta = 0$ (d) $\sin\theta - \cos\theta = 1$

43. If θ is acute then value of $\sin\theta + \cos\theta$ will be

(a) less than 1 (b) equal to 2

(c) more than 1 (d) more than one and less than 2

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44. If $\sin\theta + \operatorname{cosec}\theta = 2$ then value of $\sin^4\theta + \cos^4\theta$?

(a) 2 (b) 2^2 (c) 2^3 (d) 1

If $\sec\theta = \frac{13}{5}$, then what is the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$?

(a) 1 (b) 2 (c) 3 (d) 4

If $0° \le x \le 90°$ and $\sin x + \sqrt{3}\cos x = 1$, then what is the value of x ?

(a) 30° (b) 45° (c) 60° (d) 90°

In a right angled ΔABC if $\angle B = 90°$, $AC = 2\sqrt{5}$ and $AB - BC = 2$ then what is the value of $\cos^2 A - \cos^2 C$?

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{\sqrt{7}}{4}$ (d) $\frac{1}{\sqrt{5}}$

Statement (A) : If $\tan\theta + \cot\theta = 2$, then for all $n \in N$, $\tan^n\theta + \cot^n\theta = 2$

Reason (R) : For all $n \in N$, $\tan\theta + \cot\theta = \tan^n\theta + \cot^n\theta$

(a) Both A and R are correct and R is the correct explanation of A.
(b) Both A and R are correct but R is not a correct explanation of A.
(c) A is correct, R is incorrect.
(d) A is incorrect, R is correct.

What is the value of expression $\cos^2\frac{\pi}{8} + 4\cos^2\frac{\pi}{4} - \sec\frac{\pi}{3} + 5\tan^2\frac{\pi}{3} + \sin^2\frac{\pi}{8}$?

(a) 8 (b) 10 (c) 16 (d) 18

50. If $q\operatorname{cosec}\theta = p$ and θ is acute then value of $\left(\sqrt{p^2 - q^2}\right)\tan\theta$ is

(a) p (b) q (c) pq (d) $\sqrt{p^2 + q^2}$

51. If $2x^2\cos 60° - 4\cot^2 45° - 2\tan 60° = 0$, then what is the value of x ?

(a) 2 (b) 3 (c) $\sqrt{3} - 1$ (d) $\sqrt{3} + 1$

52. If $13\cos\theta = 12k - 5$ where $0 \le \theta \le 90°$ and k is an integer then number of possible value of k is

(a) 0 (b) 1 (c) 2 (d) more than 2

53. **Statement (A) :** $\tan 50° > 1$.

Reason (R) : For $0° < \theta < 90°$, $\tan\theta > 1$

(a) Both A and R are correct and R is the correct explanation of A.
(b) Both A and R are correct but R is not a correct explanation of A.
(c) A is correct, R is incorrect.
(d) A is incorrect, R is correct.

54. Which one of the following is true ?

(a) $\sin 35° > \cos 55°$ (b) $\cos 61° > \frac{1}{2}$

(c) $\sin 32° > \frac{1}{2}$ (d) $\tan 44° > 1$

55. What is the value of x in the expression

$$\frac{x\operatorname{cosec}^2 30° \sec^2 45°}{8\cos^2 45° \sin^2 60°} = \tan^2 60° - \tan^2 30° \text{ ?}$$

(a) $x = 1$ (b) $x = 2$ (c) $x = \frac{1}{2}$ (d) $x = \frac{3}{2}$

56. If $\sin\alpha + \cos\beta = 2(0^\circ \le \beta < \alpha \le 90^\circ)$, then $\sin\left(\frac{2\alpha+\beta}{3}\right) =$

(a) $\sin\frac{\alpha}{2}$ (b) $\cos\frac{\alpha}{3}$ (c) $\sin\frac{\alpha}{3}$ (d) $\cos\frac{2\alpha}{3}$

57. Value of $\cot 10^\circ \cdot \cot 20^\circ \cdot \cot 60^\circ \cdot \cot 70^\circ \cdot \cot 80^\circ$ is

(a) 1 (b) -1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

58. If $\tan\theta = 1$ then what is the value of $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$?

(a) 2 (b) $2\frac{1}{2}$ (c) 3 (d) $\frac{4}{5}$

59. What of the value of $\tan\frac{\pi}{8} \cdot \tan\frac{\pi}{12} \cdot \tan\frac{\pi}{4} \cdot \tan\frac{3\pi}{8} \tan\frac{5\pi}{12}$?

(a) 0 (b) 1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

60. If $\tan 15^\circ = 2 - \sqrt{3}$ then value of $\tan 15^\circ \cot 75^\circ + \tan 75^\circ \cot 15^\circ$ is

(a) 14 (b) 12 (c) 10 (d) 8

Answer–10A

1. (c)	2. (a)	3. (c)	4. (c)	5. (c)	6. (a)	7. (c)	8. (c)
9. (b)	10. (d)	11. (d)	12. (b)	13. (c)	14. (d)	15. (b)	16. (b)
17. (b)	18. (b)	19. (c)	20. (b)	21. (c)	22. (a)	23. (c)	24. (d)
25. (d)	26. (a)	27. (a)	28. (d)	29. (b)	30. (a)	31. (a)	32. (b)
33. (c)	34. (c)	35. (a)	36. (b)	37. (a)	38. (c)	39. (a)	40. (a)
41. (b)	42. (d)	43. (d)	44. (d)	45. (c)	46. (d)	47. (a)	48. (a)
49. (c)	50. (b)	51. (d)	52. (c)	53. (c)	54. (c)	55. (a)	56. (b)
57. (d)	58. (a)	59. (b)	60. (a)				

Explanation

1. (c) $\because \tan x = \frac{3}{4} = \frac{p}{b}$

$$\therefore h = \sqrt{p^2 + b^2} = \sqrt{3^2 + 4^2} = 5$$

$$\therefore \sin x \cdot \cos x = \frac{p}{h} \cdot \frac{b}{h} = \frac{3}{5} \cdot \frac{4}{5} = \frac{12}{25}$$

2. (a) $\tan 45^\circ = 1$ and $\tan\theta > 1$. When $45^\circ < \theta < 90^\circ$.

3. (c) A is true R is false.

4. (c) Given,

$$\cos A = \frac{5}{13} = \frac{b}{h}$$

$$\therefore p = \sqrt{h^2 - b^2} = \sqrt{13^2 - 5^2} = 12$$

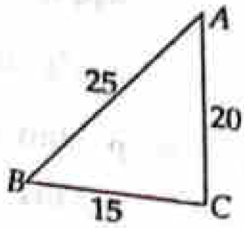
$$\therefore \frac{\sin A - \cot A}{2\tan A} = \frac{\frac{12}{13} - \frac{5}{12}}{2 \times \frac{12}{5}} = \frac{144 - 65}{13 \times 12 \times 2 \times \frac{12}{5}} = \frac{395}{3744}$$

(a) $\sin x$ ...

$x = \frac{\pi}{2} - y \quad \Rightarrow x + y = \frac{\pi}{2}$

(b) Given, $\sin B = \frac{4}{5} \Rightarrow \cos B = \frac{3}{5} = \frac{BC}{AB}$

$BC = 15$ cm



$\therefore AB = BC \times \frac{5}{3} = 15 \times \frac{5}{3} = 25$ cm.

(c) $\sin\theta = \frac{m^2 - n^2}{m^2 + n^2} = \frac{p}{h}$

$\therefore \quad b = \sqrt{h^2 - p^2}$

$= \sqrt{m^4 + n^4 + 2m^2n^2 - (m^4 + n^4 - 2m^2n^2)} = \sqrt{4m^2n^2} = 2mn$

$\therefore \tan\theta = \frac{p}{b} = \frac{m^2 - n^2}{2mn}$

(c) Given, $\sin(x - y) = \frac{1}{2}$ and $\cos(x + y) = \frac{1}{2}$

$\Rightarrow x - y = 30°$ and $x + y = 60°$ solving

$x = 45°$ and $y = 15°$

(b) Given, $1 + \tan\theta = \sqrt{2} \Rightarrow \tan\theta = \sqrt{2} - 1$

$\therefore \cot\theta - 1 = \frac{1}{\sqrt{2} - 1} - 1 = \frac{\sqrt{2} + 1}{1} - 1 = \sqrt{2}$

10. (d) Given, $\sin(x + 54°) = \cos x = \sin(90° - x)$

$\Rightarrow x + 54° = 90° - x$

$\Rightarrow 2x = 36° \quad \Rightarrow \quad x = 18°$

11. (d) Given, $x\cos 60° + y\cos 0° = 3$

$\Rightarrow \frac{x}{2} + y = 3$

$\Rightarrow x + 2y = 6 \qquad \ldots \text{(i)}$

and $4x \sin 30° - y\cot 45° = 2$

$\Rightarrow 4x \times \frac{1}{2} - y \cdot 1 = 2$

$\Rightarrow 2x - y = 2 \qquad \ldots \text{(ii)}$

Solving equation (i) & (ii), $\quad x = y = 2$

12. (b) Given, $\cos x + \cos^2 x = 1$

$\Rightarrow \cos x = 1 - \cos^2 x = \sin^2 x$

Squaring both sides,

$\cos^2 x = \sin^4 x \Rightarrow 1 - \sin^2 x = \sin^4 x$

$\Rightarrow 1 = \sin^2 x + \sin^4 x$

13. (c) $\sin x : \sin y = \sqrt{3} : 1 = \frac{\sqrt{3}}{2} : \frac{1}{2} = \sin 60° : \sin 30°$

$\therefore \quad x : y = 60 : 30 \quad \Rightarrow x : y = 2 : 1$

14. (d) For $0 \le x \le \frac{\pi}{2}$,

$\cos^2 x$ and $\sin^2 x$, lie between 0 and 1. Hence option (d) is correct

15. (b) $p = \tan^2 x + \cot^2 x = (\tan x - \cot x)^2 + 2\tan x \cot x$

$= (\tan x - \cot x)^2 + 2 \ge 0 + 2 = 2$

16. (b) $\frac{5\sin 75° \sin 77° + 2\cos 13° \cos 15°}{\cos 15° \sin 77°} - \frac{7\sin 81°}{\cos 9°}$

$= \frac{5\cos 15° \sin 77° + 2\sin 77° \cos 15°}{\cos 15° \sin 77°} - \frac{7\cos 9°}{\cos 9°} = 7 - 7 = 0$

17. (b) We know that value of $\sin x$, θ increases as θ increases from 0° to 90°

$\therefore$ $\sin y > \sin x$

18. (b) $\sin^3 60° \cot 30° - 2\sec^2 45° + 3\cos 60° \tan 45° - \tan^2 60°$

$= \left(\frac{\sqrt{3}}{2}\right)^3 (\sqrt{3}) - 2(\sqrt{2})^2 + 3\left(\frac{1}{2}\right)(1) - (\sqrt{3})^2$

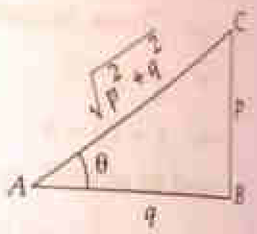
$= \frac{9}{8} - 4 + \frac{3}{2} - 3 = -\frac{35}{8}$

19. (c) Given, $\tan\theta = \frac{p}{q}$

$\Rightarrow \sec\theta = \frac{\sqrt{p^2+q^2}}{q}$ and $\operatorname{cosec}\theta = \frac{\sqrt{p^2+q^2}}{p}$

$\therefore \quad \frac{p\sec\theta - q\operatorname{cosec}\theta}{p\sec\theta + q\operatorname{cosec}\theta}$

$$= \frac{p\left(\frac{\sqrt{p^2+q^2}}{q}\right) - q\left(\frac{\sqrt{p^2+q^2}}{p}\right)}{p\left(\frac{\sqrt{p^2+q^2}}{q}\right) + q\left(\frac{\sqrt{p^2+q^2}}{p}\right)}$$



$$= \frac{\frac{p}{q} - \frac{q}{p}}{\frac{p}{q} + \frac{q}{p}} = \frac{p^2 - q^2}{p^2 + q^2}$$

20. (b) $\operatorname{cosec}^2\theta - 2 + \sin^2\theta = (\sin\theta - \operatorname{cosec}\theta)^2$

Hence it is always non negative.

21. (c) Given, $\cot\theta = \frac{2xy}{x^2 - y^2} = \frac{b}{p}$

In ΔABC,

$\therefore \quad h^2 = (x^2 - y^2)^2 + (2xy)^2$

$\Rightarrow$ $h^2 = (x^2 + y^2)^2$

$\Rightarrow$ $h = x^2 + y^2$

$\therefore$ $\cos\theta = \frac{b}{h} = \frac{2xy}{x^2 + y^2}$

22. (a) Given, $(\sin\theta + \operatorname{cosec}\theta) = 2.5$

$\Rightarrow$ $\left(\sin\theta + \frac{1}{\sin\theta}\right) = \frac{5}{2}$

$\Rightarrow$ $2\sin^2\theta - 5\sin\theta + 2 = 0$

$\Rightarrow$ $2\sin^2\theta - 4\sin\theta - \sin\theta + 2 = 0$

$\Rightarrow$ $2\sin\theta\,(\sin\theta - 2) - 1\,(\sin\theta - 2) = 0$

$\Rightarrow$ $(2\sin\theta - 1)(\sin\theta - 2) = 0$

$\Rightarrow$ $\sin\theta = \frac{1}{2}$ $\qquad (\because \sin\theta \neq 2)$

$\Rightarrow$ $\theta = 30^\circ$

23. (c) When $0 < \theta < \phi < 90^\circ$

$0 < \sin^2\theta, \cos^2\theta < 1$

$\therefore$ $\sin^2\theta + \cos^2\phi < 2$

24. (d) $\tan\theta = \frac{2t}{1-t^2} = \frac{p}{b}$

$\therefore$ $h = \sqrt{p^2 + b^2} = \sqrt{(2t)^2 + (1-t^2)^2} = \sqrt{4t^2 + 1 + t^4 - 2t^2} = 1 + t^2$

Hence, $\sin\theta + \cos\theta = \frac{p}{h} + \frac{b}{h} = \frac{2t + 1 - t^2}{1 + t^2}$

25. (d) We know that when $\theta \in \left[0, \frac{\pi}{2}\right]$, value of $\sec^2\theta$, increases from 1 to ∞.

$\therefore$ $p \geq 1$

26. (a) In ΔBAC, $\cos 60^\circ = \frac{AB}{AC}$

$\Rightarrow$ $\frac{1}{2} = \frac{5}{AC}$ $\quad\therefore\quad AC = 10$ cm

27. (a) Given, $\cos\theta + \sqrt{3}\,\sin\theta = 2$

$\Rightarrow$ $\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = 1$

$\Rightarrow$ $\sin 30^\circ \cos\theta + \cos 30^\circ \sin\theta = 1$

$\Rightarrow$ $\sin(30^\circ + \theta) = \sin 90^\circ$

$\Rightarrow$ $30^\circ + \theta = 90^\circ$

$\Rightarrow$ $\theta = 60^\circ = \frac{\pi}{3}$

28. (d) In ΔABC, $\tan A = \frac{BC}{AC} = \frac{u}{v}$, $\tan B = \frac{v}{u}$

Also, $u^2 + v^2 = w^2$ (Pythagoras theorem)

$\therefore \quad \tan A + \tan B = \frac{u}{v} + \frac{v}{u} = \frac{u^2 + v^2}{vu} = \frac{w^2}{uv}$

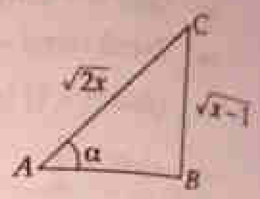
29. (b) Given, $\tan B = \frac{k}{\sqrt{3}k}$

From Pythagoras theorem, $AB^2 + AC^2 = BC^2$

$\Rightarrow \quad (\sqrt{3}k)^2 + (1k)^2 = BC^2 \quad \Rightarrow \quad BC^2 = 4K^2 \quad \Rightarrow \quad BC = 2k$

30. (a) Given, $\sin\alpha = \sqrt{\frac{x-1}{2x}}$

Applying Pythagoras theorem in ΔABC,



$$AC^2 = AB^2 + BC^2$$

$\Rightarrow \quad 2x = AB^2 + (x-1)$

$\Rightarrow \quad AB^2 = x + 1$

$\Rightarrow \quad AB = \sqrt{x+1} \qquad \therefore \quad \tan\alpha = \frac{BC}{AB} = \frac{\sqrt{x-1}}{\sqrt{x+1}}$

31. (a) We know that as θ increases value of $\cos\theta$ decreases,

$\therefore \quad \cos\theta \geq \frac{1}{2}$

$\Rightarrow \quad \cos\theta \geq \cos\frac{\pi}{3} \quad \Rightarrow \quad \theta \leq \frac{\pi}{3}$

32. (b) $\because \quad \cos 90^\circ = 0$

$\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots\ldots\ldots \cos 90^\circ = 0$

33. (c) If $A = \frac{\pi}{6}$ and $B = \frac{\pi}{3}$

I. $\sin A + \sin B = \sin\frac{\pi}{6} + \sin\frac{\pi}{3}$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

and $\cos A + \cos B = \cos\frac{\pi}{6} + \cos\frac{\pi}{3} = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1+\sqrt{3}}{2}$

$\Rightarrow \quad \sin A + \sin B = \cos A + \cos B$

II. $\tan A + \tan B = \tan\frac{\pi}{6} + \tan\frac{\pi}{3}$

$$= \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

and $\cot A + \cot B = \cot\frac{\pi}{6} + \cot\frac{\pi}{3} = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

$\Rightarrow \quad \tan A + \tan B = \cot A + \cot B$

Alternate method, $A + B = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$ ... (i)

I. $\sin A + \sin B = \sin\left(\frac{\pi}{2} - B\right) + \sin\left(\frac{\pi}{2} - A\right) = \cos B + \cos A = \cos A + \cos B$

33. $\tan A + \tan B = \tan\left(\frac{\pi}{2} - B\right) + \tan\left(\frac{\pi}{2} - A\right)$

$$= \cot B + \cot A = \cot A + \cot B$$

Hence both statements are true.

34. (c) $\sin 17^\circ = \frac{a}{b} = \frac{\text{perpendicular}}{\text{hypotenuse}} \Rightarrow \text{base} = \sqrt{b^2 - a^2}$

$\therefore \quad \sec 17^\circ = \frac{\text{hypotenuse}}{\text{base}} = \frac{b}{\sqrt{b^2 - a^2}}$

and $\sin 73^\circ = \sin(90^\circ - 17^\circ) = \cos 17^\circ = \frac{\text{base}}{\text{hypotenuse}} = \frac{\sqrt{b^2 - a^2}}{b}$

Hence, $\sec 17^\circ - \sin 73^\circ = \frac{b}{\sqrt{b^2 - a^2}} - \frac{b^2 - a^2}{b} = \frac{b^2 - b^2 + a^2}{b\sqrt{b^2 - a^2}} = \frac{a^2}{b\sqrt{b^2 - a^2}}$

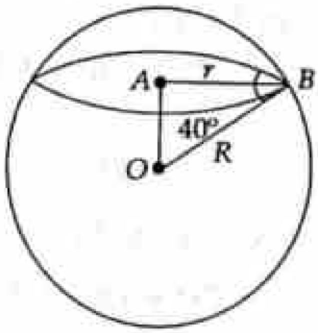
35. (a) See the figure,

In ΔOAB,

$\cos 40^\circ = \frac{AB}{OB}$

$\Rightarrow \cos 40^\circ = \frac{r}{R}$

$\therefore \ r = R\cos 40^\circ$



36. (b) $\because \ \frac{\operatorname{cosec}\theta + \sin\theta}{\operatorname{cosec}\theta - \sin\theta} = \frac{5}{3} \Rightarrow \frac{1 + \sin^2\theta}{1 - \sin^2\theta} = \frac{5}{3}$

$\Rightarrow 3 + 3\sin^2\theta = 5 - 5\sin^2\theta$

$\Rightarrow 8\sin^2\theta = 2 \quad \Rightarrow \quad \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 30^\circ$

$\therefore \ \tan\theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$

37. (a) In first quadrant, value of $\sin\theta$ increases as θ increases and value of $\cos\theta$ decreases as θ increases. Thus $40^\circ > 20^\circ \Rightarrow \sin 40^\circ > \sin 20^\circ$.

38. (c) $\because \ \sin\theta_1 = \frac{2}{\sqrt{5}} = 0.894$

$\tan\theta_2 = \frac{3}{2} = 1.5$

$\cos\theta_3 = \frac{3}{5} = 0.6$

and $\sec\theta_4 = \frac{\sqrt{41}}{4} = 1.6$

$\therefore$ Correct order is $3 - 1 - 2 - 4$.

39. (a) We know the maximum value of $\sin\theta = 1$

$\therefore$ Maximum value of $\sin^3\theta + 2\sin^2\theta + 3\sin\theta = 1 + 2 + 3 = 6$

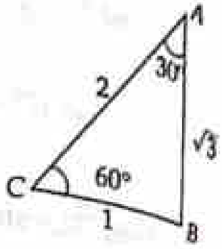
at $\theta = 0°$, $\sin\theta = 0$ $\therefore \sin^3\theta + 2\sin^2\theta + 3\sin\theta = 0$

Hence only statement (1) is correct.

40. (a) $\because \angle C = 60°$ and $\angle B = 90°$

$\therefore \angle A = 180° - (90° + 60°)$

$= 180° - 150° = 30°$



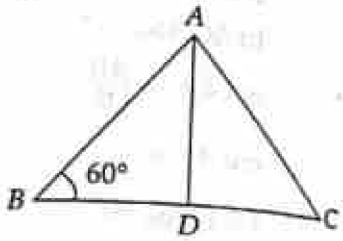
$\therefore \sin C = \sin 60° = \frac{\sqrt{3}}{2}$

and $\sin A = \sin 30° = \frac{1}{2}$

From figure it is clear that $AB : BC : CA = \sqrt{3} : 1 : 2$

41. (b) In ΔABD,

$\cos 60° = \frac{BD}{AB} \Rightarrow \frac{1}{2} = \frac{BD}{x} \Rightarrow BD = \frac{x}{2}$



In ΔABD,

$AD = \sqrt{AB^2 - BD^2}$

$= \sqrt{x^2 - \frac{x^2}{4}} = \frac{\sqrt{3}\,x}{2}$

Now, In ΔADC,

$CD = \sqrt{AC^2 - AD^2}$

$= \sqrt{\frac{9x^2}{4} - \frac{3x^2}{4}} = \sqrt{\frac{6x^2}{4}} = \sqrt{\frac{3}{2}}\,x$

42. (d) (a) $\sin\theta = \sqrt{2}$ is not possible as $\sin\theta \le 1$

(b) Maximum value of $\sin\theta$ is 1 and Maximum value of $\cos\theta$ is also 1 but both cannot be attained simultaneously. Hence $\sin\theta + \cos\theta = 2$ is not possible.

(c) $\sin\theta + \cos\theta = 0$

$\Rightarrow \sin\theta - \cos\theta \Rightarrow \tan\theta = -1$, but when $0° < \theta < 90°$ $\tan\theta$ is positive. So, option (c) is incorrect.

(d) $\sin\theta - \cos\theta = 1$ is true when $\theta = 90°$.

Hence option (d) is correct.

43. (d) Since maximum value of each of $\sin\theta$ and $\cos\theta$ is 1, but they cannot be 1 simultaneously.

Hence $\sin\theta + \cos\theta \ne 2$. Again when $\theta = 45°$.

$\sin\theta + \cos\theta = \cos 45° + \cos 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$, which indicates that (d) is correct.

14. (d) $\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$

$\Rightarrow \sin\theta = \sin 90° \Rightarrow \theta = 90°$

$\therefore \sin^4\theta + \cos^4\theta = \sin^4 90° + \cos^4 90° = 1 + 0 = 1$

15. (c) $\because \sec\theta = \frac{13}{5} = \frac{h}{b}$

$\therefore p = \sqrt{h^2 - p^2} = \sqrt{13^2 - 5^2} = 12$

Required expression $= \dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \dfrac{2\cdot\frac{p}{h} - 3\cdot\frac{b}{h}}{4\cdot\frac{p}{h} - 9\cdot\frac{b}{h}}$

$$= \frac{2p - 3b}{4p - 9b} = \frac{2\times 12 - 3\times 5}{4\times 12 - 9\times 5} = 3$$

16. (d) $\because \sin x + \sqrt{3}\cos x = 1$

$\Rightarrow \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{1}{2}$

$\Rightarrow \sin x \sin 30° + \cos x \cos 30° = \frac{1}{2}$

$\Rightarrow \cos(x - 30°) = \cos 60°$

$\Rightarrow x - 30° = 60° \quad \therefore x = 90°$

17. (a) $\because AC^2 = AB^2 + BC^2$

$\therefore (2\sqrt{5})^2 = (BC + 2)^2 + (BC)^2$

$\Rightarrow 20 = 2BC^2 + 4BC + 4$

$\Rightarrow 2BC^2 + 4BC - 16 = 0$

$\Rightarrow 2(BC^2 + 2BC - 8) = 0$

$\Rightarrow (BC + 4)(BC - 2) = 0$

$\Rightarrow BC = 2$

($\because$ BC cannot be negative)

Hence, $AB - BC = 2 \Rightarrow AB = 2 + BC = 2 + 2 = 4$

In ΔABC, $\cos^2 A - \cos^2 C = \left(\frac{4}{2\sqrt{5}}\right)^2 - \left(\frac{2}{2\sqrt{5}}\right)^2 = \frac{12}{20} = \frac{3}{5}$

18. (a) $\tan\theta + \cot\theta = 2$

$\Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta - 1)^2 = 0 \Rightarrow \tan\theta = 1$

$\therefore \tan^n\theta + \cot^n\theta = 1 + 1 = 2$

Hence (A) and (R) is correct and R is correct statement of A.

49. (c) $\left(\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}\right)+4\cos^2 45^\circ-\sec 60^\circ+5\tan^2 60^\circ$

$$=1+4\left(\frac{1}{\sqrt{2}}\right)^2-2+5(\sqrt{3})^2=1+2-2+15=16$$

50. (b) Given, $\text{cosec}\theta=\frac{p}{q}=\frac{\text{hypotenuse}}{\text{perpendicular}}$

Now, $\tan\theta=\frac{\text{perpendicular}}{\text{base}}$ $\therefore$ base $=\sqrt{(\text{hypotenuse})^2-(\text{perpendicular})^2}$

$$=\sqrt{p^2-q^2}$$

$\therefore\quad \tan\theta=\frac{q}{\sqrt{p^2-q^2}}$

Now, $\sqrt{p^2-q^2}.\tan\theta=\left(\sqrt{p^2-q^2}\right)\frac{\text{perpendicular}}{\text{base}}$

$$=\sqrt{p^2-q^2}\times\frac{q}{\sqrt{p^2-q^2}}=q$$

51. (d) $2x^2\times\frac{1}{2}-4(1)^2-2\sqrt{3}=0$

$x^2-4-2\sqrt{3}=0$

$x^2=4+2\sqrt{3}=(\sqrt{3}+1)^2\quad\therefore\quad x=\sqrt{3}+1$

52. (c) $\cos\theta=\frac{12k-5}{13}$

$\because\quad 0\le\cos\theta\le 1$

$\therefore\quad 0\le\frac{12k-5}{13}\le 1$

or, $0\le 12k-5\le 13$ or, $5\le 12k\le 18$

or, $\frac{5}{12}\le k\le\frac{18}{12}$ or, $\frac{5}{12}\le k\le\frac{3}{2}$

Clearly it is true for integral value of $k=0,1$. Hence k has two value.

53. (c) We know that as θ increases from 0° to 90°, value of $\tan\theta$ increases.

$\therefore\quad \tan 50^\circ>\tan 45^\circ=1$

54. (c) $\sin 32^\circ>\sin 30^\circ\Rightarrow\sin 32^\circ>\frac{1}{2}$

55. (a) $\frac{x\times(2)^2\times(\sqrt{2})^2}{8\times\left(\frac{1}{\sqrt{2}}\right)^2\times\left(\frac{\sqrt{3}}{2}\right)^2}=(\sqrt{3})^2-\left(\frac{1}{\sqrt{3}}\right)^2$

or, $\frac{x\times 4\times 2}{8\times\frac{1}{2}\times\frac{3}{4}}=3-\frac{1}{3}$ or, $\frac{8x}{3}=\frac{8}{3}$ $\therefore$ $x=1$

56. (b) $\because$ $\sin\alpha \le 1; \cos\beta \le 1$

$\therefore$ $\sin\alpha + \cos\beta = 2$ is possible only when $\sin\alpha = 1$ and $\cos\beta = 1$

$\Rightarrow$ $\alpha = 90°; \beta = 0°$

$\therefore$ $\sin\left(\frac{2\alpha+\beta}{3}\right) = \sin\left(\frac{180°}{3}\right) = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos\frac{\alpha}{3} = \cos 30° = \frac{\sqrt{3}}{2}$

57. (d) $\because$ $\cot 10°\cdot\cot 80°\cdot\cot 20°\cdot\cot 70°\cdot\cot 60°$

$= \cot 10°\cdot\tan 10°\cdot\cot 20°\cdot\tan 20°\cdot\cot 60°$ $\left[\begin{array}{c}\because \tan(90°-\theta) = \cot\theta \\ \tan\theta\cdot\cot\theta = 1\end{array}\right]$

$= 1\cdot 1\cdot\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

58. (a) $\because$ $\tan\theta = 1 = \tan 45°$

$\therefore$ $\theta = 45°$

$\therefore$ $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta} = \frac{8\sin 45° + 5\cos 45°}{\sin^3 45° - 2\cos^3 45° + 7\cos 45°}$

$= \frac{8\times\frac{1}{\sqrt{2}} + 5\times\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^3 - 2\times\left(\frac{1}{\sqrt{2}}\right)^3 + 7\times\frac{1}{\sqrt{2}}} = \frac{\frac{1}{\sqrt{2}}(8+5)}{\frac{1}{\sqrt{2}}\left(\frac{1}{2} - 1 + 7\right)} = \frac{13}{\left(\frac{13}{2}\right)} = 2$

59. (b) $\frac{\pi}{8} = \frac{180°}{8} = 22\frac{1°}{2}$, $\frac{\pi}{12} = \frac{180°}{12} = 15°$, $\frac{\pi}{4} = 45°$

$\tan\frac{3\pi}{8} = \tan\frac{3\times 180°}{8} = \tan 67\frac{1°}{2} = \cot 22\frac{1°}{2}$

$\tan\frac{5\pi}{12} = \tan(5\times 15°) = \tan 75° = \cot 15°$

$\therefore$ Required value $= \tan 22\frac{1°}{2}\ \tan 15°\tan 45°\cot 22\frac{1°}{2}\cot 15 = 1$

$(\because \tan\theta\cdot\cot\theta = 1)$

60. (a) $\tan 15°\cdot\cot 75° + \tan 75°\cdot\cot 15°$

$= \tan 15°\cdot\cot(90° - 15°) + \tan(90° - 15°)\cdot\cot 15°$

$= \tan^2 15° + \cot^2 15°$ ... (i) $\left[\begin{array}{c}\because \tan(90°-\theta) = \cot\theta \\ \cot(90°-\theta) = \tan\theta\end{array}\right]$

$\because$ $\tan 15° = 2 - \sqrt{3}$

$\therefore$ $\cot 15° = \frac{1}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})} = 2 + \sqrt{3}$

$\therefore$ $\tan^2 15° + \cot^2 15°$

$= (2-\sqrt{3})^2 + (2+\sqrt{3})^2 = 2(4+3) = 14$

Exercise-10B

1. If $\sin(x+y) = \cos[3(x+y)]$ then what is the value of $\tan[2(x+y)]$?
(a) $\sqrt{3}$ (b) 1 (c) 0 (d) $\frac{1}{\sqrt{3}}$
[SSC Tier-I 2012]

2. If $\sin 2\theta = \frac{1}{2}$ then what is the value of $\cos(75^\circ - \theta)$?
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$
[SSC Tier-I 2012]

3. In a right angled triangle ABC, $AB = 2.5$ cm, $\cos B = 0.5$, $\angle ACB = 90^\circ$. Length of side AC in cm is
(a) $5\sqrt{3}$ (b) $\frac{5}{2}\sqrt{3}$ (c) $\frac{5}{4}\sqrt{3}$ (d) $\frac{5}{16}\sqrt{3}$
[SSC Tier-I 2012]

4. If $\cos\theta = \frac{4}{5}$, then value of $\frac{\text{cosec}\,\theta}{1+\cot\theta}$ is
(a) $\frac{7}{5}$ (b) $\frac{2}{7}$ (c) $\frac{5}{7}$ (d) $\frac{4}{7}$
[SSC Tier-I 2012]

5. Maximum value of $24\sin\theta + 7\cos\theta$ is
(a) 7 (b) 17 (c) 24 (d) 25
[SSC Tier-I 2012]

6. In ΔABC, $\angle A$ is right angle and AD is perpendicular to BC. If $AD = 4$ cm, $BC = 12$ cm, then value of $(\cot B + \cot C)$ is
(a) 6 (b) 3 (c) 4 (d) $\frac{3}{2}$
[SSC Tier-I 2012]

7. If $5\tan\theta = 4$ then value of $\left(\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}\right)$ is
(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{5}{7}$ (d) $\frac{2}{5}$
[SSC Tier-I 2012]

8. What is the minimum value of $4\text{cosec}^2\alpha + 9\sin^2\alpha$?
(a) 10 (b) 11 (c) 12 (d) 14
[SSC Tier-I 2012]

9. In a right angled triangle $\angle B$ is right angle and $AC = 2\sqrt{5}$ cm. If $AB - BC = 2$ cm then what is the value of $(\cos^2 A - \cos^2 C)$?
(a) $\frac{3}{5}$ (b) $\frac{6}{5}$ (c) $\frac{3}{10}$ (d) $\frac{2}{5}$
[SSC Tier-I 2012]

10. If $\sin(A+B) = 1$ and $\cos(A-B) = \frac{\sqrt{3}}{2}$, where A and B are positive acute angle and $A \geq B$ then A and B are
(a) $A = 60^\circ, B = 30^\circ$ (b) $A = 45^\circ, B = 45^\circ$
(c) $A = 75^\circ, B = 15^\circ$ (d) None of these
[SSC Tier-I 2012]

Answer-10B

1. (b) 2. (b) 3. (c) 4. (c) 5. (a) 6. (b) 7. (a) 8. (c)
9. (a) 10. (a)

Explanation

1. (b) $\sin(x+y) = \cos(3(x+y)) = \sin\left(\frac{\pi}{2} - 3(x+y)\right)$

$\therefore$ $(x+y) = 90° - 3(x+y)$

or $4(x+y) = 90°$

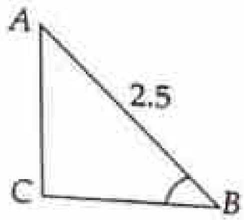
or $2(x+y) = 45°$

$\therefore$ $\tan(2(x+y)) = \tan 45° = 1$

2. (b) $\sin 2\theta = \frac{1}{2} \Rightarrow 2\theta = 30° \Rightarrow \theta = 15°$

$\therefore$ $\cos(75° - \theta) = \cos(75° - 15°) = \cos 60° = \frac{1}{2}$

3. (c) $\cos B = 0.5 = \frac{1}{2} \Rightarrow B = 60°$



Now, $\sin B = \frac{AC}{AB}$

$\Rightarrow$ $AC = AB \sin B$

$= (2.5)\sin 60° = \frac{5}{2}\,\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{4}$

4. (c) $\cos\theta = \frac{4}{5} = \frac{b}{h}$

$\therefore$ $p = \sqrt{h^2 - b^2} = \sqrt{25 - 16} = 3$

Now, $\frac{\operatorname{cosec}\theta}{1 + \cot\theta} = \frac{h/p}{1 + \frac{b}{p}} = \frac{h}{p+b} = \frac{5}{3+4} = \frac{5}{7}$

5. (d) Recall that maximum and minimum value of $a\cos\theta + b\sin\theta$ are respectively $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$.

Hence maximum value of $24\sin\theta + 7\cos\theta = \sqrt{24^2 + 7^2}$

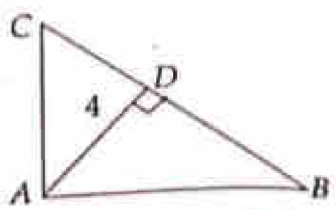
$= \sqrt{576 + 49} = \sqrt{625} = 25$

6. (b) In right angle ΔABD, $\cot B = \frac{BD}{4}$

In right angle ΔACD, $\cot C = \frac{CD}{4}$

$\therefore$ $\cot B + \cot C = \frac{BD + CD}{4}$

$= \frac{BC}{4} = \frac{12}{4} = 3$ cm



7. (a) (tricky solution), $5\tan\theta = 4$

$\Rightarrow \dfrac{5\sin\theta}{\cos\theta} = 4 \Rightarrow \dfrac{5\sin\theta}{3\cos\theta} = \dfrac{4}{3}$

by componendo-dividendo

$\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta} = \dfrac{4-3}{4+3} = \dfrac{1}{7}$

8. (c) $4\,\text{cosec}^2\alpha + 9\sin^2\alpha$

$= (2\text{cosec}\alpha - 3\sin\alpha)^2 + 2\cdot 2\text{cosec}\alpha\cdot 3\sin\alpha$

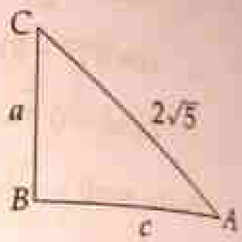
$= (2\text{cosec}\alpha - 3\sin\alpha)^2 + 12$

$\because\ 2\text{cosec}\alpha - 3\sin\alpha \geq 0$

$\therefore$ Required value ≥ 12

9. (a) Given $AB - BC = 2$ i.e., $c - a = 2$

and $c^2 + a^2 = 20$ *(Pythagoras Theorem)*



putting $c = a + 2$ in $c^2 + a^2 = 20$

$(a+2)^2 + a^2 = 20$

or, $2a^2 + 4a + 4 = 20$

or, $a^2 + 2a + 2 = 10$

or, $a^2 + 2a - 8 = 0$

or, $a = 2, -4$

$\therefore c = 4$

Now, $\cos^2 A - \cos^2 C = \left(\dfrac{c}{2\sqrt{5}}\right)^2 - \left(\dfrac{a}{2\sqrt{5}}\right)^2 = \dfrac{c^2 - a^2}{20} = \dfrac{16-4}{20} = \dfrac{3}{5}$

10. (a) $\sin(A - B) = 1 \quad \Rightarrow A + B = 90°$

$\cos(A - B) = \dfrac{\sqrt{3}}{2} \quad \Rightarrow A - B = 30°$

Adding, $2A = 120° \Rightarrow A = 60°$

$\therefore B = 30°$

★★★

11

Elementary Trigonometric Identities

1. Trigonometric Identities

Three basic identities in trigonometry are

1.1 $\sin^2\theta + \cos^2\theta = 1$

1.2 $\sec^2\theta - \tan^2\theta = 1$

1.3 $\text{cosec}^2\theta - \cot^2\theta = 1$

All these identities can be proved with the Pythagoras theorem $p^2 + b^2 = h^2$

e.g.,

$$\sin^2\theta + \cos^2\theta = \left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = \frac{p^2+b^2}{h^2} = \frac{h^2}{h^2} = 1 \text{ etc.}$$

Exchange the sides of these identities to obtain more identities, which are as follows,

1.4 $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$ or, $\cos^2\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta}$ or, $\cos\theta = \sqrt{1-\sin^2\theta}$

1.5 $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$ or, $\tan^2\theta = \sec^2\theta - 1$

$\Rightarrow \sec\theta = \sqrt{1+\tan^2\theta}$ or, $\tan\theta = \sqrt{\sec^2\theta - 1}$

1.6 $\text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \text{cosec}^2\theta = 1 + \cot^2\theta$ or, $\cot^2\theta = \text{cosec}^2\theta - 1$

$\Rightarrow \text{cosec}\,\theta = \sqrt{1+\cot^2\theta}$ or, $\cot\theta = \sqrt{\text{cosec}^2\theta - 1}$

2. Other Identities : Following identities given in the previous chapter are also helpful in this chapter. Learn it properly.

2.1 $\sin\theta \cdot \text{cosec}\,\theta = 1 \Rightarrow \sin\theta = \frac{1}{\text{cosec}\,\theta} \Rightarrow \text{cosec}\,\theta = \frac{1}{\sin\theta}$

2.2 $\cos\theta \cdot \sec\theta = 1 \Rightarrow \cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta}$

2.3 $\tan\theta \cdot \cot\theta = 1 \Rightarrow \tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{1}{\tan\theta}$

2.4 $\tan\theta = \frac{\sin\theta}{\cos\theta}$ 2.5 $\cot\theta = \frac{\cos\theta}{\sin\theta}$

2.6 $\sin(90^\circ - \theta) = \cos\theta$, $\cos(90^\circ - \theta) = \sin\theta$

2.7 $\tan(90^\circ - \theta) = \cot\theta$, $\cot(90^\circ - \theta) = \tan\theta$

2.8 $\sec(90^\circ - \theta) = \text{cosec}\,\theta$, $\text{cosec}(90^\circ - \theta) = \sec\theta$

Points to remember

1. $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta \Rightarrow \cos^2\theta = 1 - \sin^2\theta$
2. $\sec^2\theta - \tan^2\theta = 1 \Rightarrow 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - 1 = \tan^2\theta$
3. $\text{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow 1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \text{cosec}^2\theta - 1 = \cot^2\theta$
4. $\sin\theta \cdot \text{cosec}\,\theta = 1 \Rightarrow \sin\theta = \frac{1}{\text{cosec}\,\theta} \Rightarrow \text{cosec}\,\theta = \frac{1}{\sin\theta}$
5. $\cos\theta \cdot \sec\theta = 1 \Rightarrow \cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta}$
6. $\tan\theta \cdot \cot\theta = 1 \Rightarrow \tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{1}{\tan\theta}$
7. $\cot\theta = \frac{\cos\theta}{\sin\theta}$
8. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
9. (i) $\sin(90° - \theta) = \cos\theta$ (ii) $\cos(90° - \theta) = \sin\theta$
10. (i) $\tan(90° - \theta) = \cot\theta$ (ii) $\cot(90° - \theta) = \tan\theta$
11. (i) $\sec(90° - \theta) = \text{cosec}\,\theta$ (ii) $\text{cosec}(90° - \theta) = \sec\theta$

Solved example

1. Prove that $\frac{1+\cos A}{\sin A} + \frac{\sin A}{1+\cos A} = 2\,\text{cosec}\,A$

Solution : LHS $= \frac{1+\cos A}{\sin A} + \frac{\sin A}{1+\cos A}$

$$= \frac{(1+\cos A)^2 + (\sin A)^2}{\sin A(1+\cos A)} = \frac{1+2\cos A+\cos^2 A+\sin^2 A}{\sin A(1+\cos A)}$$

$$= \frac{1+2\cos A+1}{\sin A(1+\cos A)} = \frac{2+2\cos A}{\sin A(1+\cos A)} \qquad (\because \sin^2 A + \cos^2 A = 1)$$

$$= \frac{2(1+\cos A)}{\sin A(1+\cos A)} = \frac{2}{\sin A} = 2\,\text{cosec}\,A = \text{RHS}.$$

2. Prove that $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\,\text{cosec}\,\theta$

Solution : LHS $= \sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$

$$= \frac{(\sqrt{\sec\theta-1})^2 + (\sqrt{\sec\theta+1})^2}{\sqrt{\sec\theta+1}\cdot\sqrt{\sec\theta-1}}$$

$$= \frac{\sec\theta-1+\sec\theta+1}{\sqrt{\sec^2\theta-1}}$$

$$= \frac{2\sec\theta}{\sqrt{\tan^2\theta}} \qquad (\because \sec^2\theta - 1 = \tan^2\theta)$$

$$= \frac{2\sec\theta}{\tan\theta} = 2\cdot\frac{1}{\cos\theta\cdot\frac{\sin\theta}{\cos\theta}} = \frac{2}{\sin\theta} = 2\,\text{cosec}\,\theta$$

If $\cot\theta + \cos\theta = m$ and $\cot\theta - \cos\theta = n$ then prove that $m^2 - n^2 = 4\sqrt{mn}$

Solution : $\text{LHS} = m^2 - n^2$

$$= (\cot\theta + \cos\theta)^2 - (\cot\theta - \cos\theta)^2$$

$$= (\cot^2\theta + \cos^2\theta + 2\cot\theta\cos\theta) - (\cot^2\theta + \cos^2\theta - 2\cot\theta\cos\theta)$$

$$= 4\cot\theta\cos\theta \qquad \text{... (i)}$$

$$\text{RHS} = 4\sqrt{mn} = 4\sqrt{(\cot\theta + \cos\theta)(\cot\theta - \cos\theta)}$$

$$= 4\sqrt{\cot^2\theta - \cos^2\theta}$$

$$= 4\sqrt{\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta} = 4\sqrt{\frac{\cos^2\theta - \cos^2\theta\sin^2\theta}{\sin^2\theta}}$$

$$= 4\sqrt{\frac{\cos^2\theta(1-\sin^2\theta)}{\sin^2\theta}} = 4\sqrt{\frac{\cos^2\theta\cdot\cos^2\theta}{\sin^2\theta}}$$

$$= 4\sqrt{\cot^2\theta\cos^2\theta} \qquad \left(\because \frac{\cos\theta}{\sin\theta} = \cot\theta\right)$$

$$= 4\cot\theta\cos\theta \qquad \text{... (ii)}$$

From (i) and (ii), LHS = RHS, **Proved.**

Prove that $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$ *[SSC Tier-I 2014]*

Solution : $\text{LHS} = \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$

$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$

$$= \frac{\{(\sin\theta + \cos\theta) - 1\}\{(\cos\theta + \sin\theta) + 1\}}{\sin\theta\cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta\cos\theta}$$

[Let $\sin\theta + \cos\theta = a$, $1 = b$ and use $(a-b)(a+b) = a^2 - b^2$]

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 = \text{RHS; } \textbf{Proved.}$$

5. If $\sec\theta + \tan\theta = p$ then find the value of $\sin\theta$

Solution : since $\sec^2\theta - \tan^2\theta = 1$

$\therefore \quad (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1 \qquad \text{... (i)}$

or, $(\sec\theta - \tan\theta)\,p = 1 \qquad$ or, $\sec\theta - \tan\theta = \frac{1}{p}$

But, $\sec\theta + \tan\theta = p$

Adding, $2\sec\theta = \frac{1}{p} + p = \frac{1+p^2}{p}$

or, $\sec\theta = \frac{1+p^2}{2p} = \frac{h}{b}$

$\therefore \quad p^2 = h^2 - b^2 = (1+p^2)^2 - (2p)^2 = (1-p^2)^2$

Hence $\sin\theta = \frac{p}{h} = \frac{1-p^2}{1+p^2}$

Trick Learn that : If $\sec\theta + \tan\theta = p$ then $\sec\theta = \frac{1}{2}\left(p+\frac{1}{p}\right)$

If $\sec\theta - \tan\theta = p$ then also $\sec\theta = \frac{1}{2}\left(p+\frac{1}{p}\right)$

If $\text{cosec}\,\theta + \cot\theta = p$ then $\text{cosec}\,\theta = \frac{1}{2}\left(p+\frac{1}{p}\right)$

If $\text{cosec}\,\theta - \cot\theta = p$ then also $\text{cosec}\,\theta = \frac{1}{2}\left(p+\frac{1}{p}\right)$

6. (i) Express $\cos\theta$ in terms of $\tan\theta$
(ii) Express $\cos\theta$ in terms of $\text{cosec}\,\theta$

Solution : (i) $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{1+\tan^2\theta}}$

(ii) $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{1}{\text{cosec}^2\theta}}$

7. Prove that $\sec^2\theta + \text{cosec}^2\theta = \sec^2\theta\,\text{cosec}^2\theta$

Solution : LHS $= \sec^2\theta + \text{cosec}^2\theta$

$$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}$$

$$= \frac{1}{\cos^2\theta\sin^2\theta} = \frac{1}{\cos^2\theta}\cdot\frac{1}{\sin^2\theta} = \sec^2\theta\cdot\text{cosec}^2\theta$$

Second method : RHS

$\sec^2\theta\cdot\text{cosec}^2\theta$

$= (1+\tan^2\theta)(1+\cot^2\theta)$

$= 1 + \cot^2\theta + \tan^2\theta + \tan^2\theta\cot^2\theta$

$= 1 + \cot^2\theta + \tan^2\theta + 1$

$= (1+\cot^2\theta) + (1+\tan^2\theta)$

$= \text{cosec}^2\theta + \sec^2\theta$

8. Prove that

$$\frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\cos\theta}{1-\sin\theta}$$

Solution : $\text{LHS} = \frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1}$

$$= \frac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1} \quad (\because \sec^2\theta - \tan^2\theta = 1)$$

$$= \frac{(\sec\theta + \tan\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{(\tan\theta - \sec\theta + 1)} = \sec\theta + \tan\theta$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)} = \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)} = \frac{\cos\theta}{1-\sin\theta}; \text{ Proved}$$

[**Note** : To write $1 = \sec^2\theta - \tan^2\theta$ is very important. In some question we may write $1 = \operatorname{cosec}^2\theta - \cot^2\theta$]

. Prove that $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ *[SSC Tier-I 2014]*

Solution : We know that $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$\therefore \sin^6\theta + \cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$

$$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$$

$$= 1^3 - 3\sin^2\theta\cos^2\theta \cdot 1 = 1 - 3\sin^2\theta\cos^2\theta$$

. Prove that $\sin\theta(1 + \tan\theta) + \cos\theta(1 + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$

Solution : $\text{LHS} = \sin\theta(1 + \tan\theta) + \cos\theta(1 + \cot\theta)$

$$= \sin\theta\left(1 + \frac{\sin\theta}{\cos\theta}\right) + \cos\theta\left(1 + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \sin\theta\left(\frac{\cos\theta + \sin\theta}{\cos\theta}\right) + \cos\theta\left(\frac{\sin\theta + \cos\theta}{\sin\theta}\right)$$

$$= (\cos\theta + \sin\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) = \cos\theta + \sin\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= (\cos\theta + \sin\theta)\frac{1}{\cos\theta\sin\theta} = \frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{1}{\cos\theta} = \operatorname{cosec}\theta + \sec\theta$$

$$= \sec\theta + \operatorname{cosec}\theta = \text{RHS; Proved}$$

11. Prove that $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

Solution : $\text{LHS} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

dividing numerator and denominator by $\sin A$

$$\text{LHS} = \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} = \frac{(\cot A + \operatorname{cosec} A) - 1}{(\cot A - \operatorname{cosec} A) + 1} \times \frac{\cot A - \operatorname{cosec} A}{\cot A - \operatorname{cosec} A}$$

$$= \frac{(\cot^2 A - \text{cosec} \ldots}{\{(\cot A - \text{cosec}\, A)+1\}(\cot A - \text{cosec}\, A)}$$

$$= \frac{-1 - \cot A + \text{cosec}\, A}{(\cot A - \text{cosec}\, A + 1)(\cot A - \text{cosec}\, A)}$$

$$= \frac{-1}{\cot A - \text{cosec}\, A} = \frac{1}{\text{cosec}\, A - \cot A} = \text{RHS} \text{ ; Proved}$$

12. If $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$ then find the value of $\cos\theta$. Find θ if $0° < \theta < 90°$

Solution : Given $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

or, $\frac{\cos\theta(1+\sin\theta) + \cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$

or, $\frac{\cos\theta + \cos\theta\sin\theta + \cos\theta - \cos\theta\sin\theta}{1-\sin^2\theta} = 4$

or, $\frac{2\cos\theta}{\cos^2\theta} = 4$ or, $\frac{2}{\cos\theta} = 4$

or, $\cos\theta = \frac{2}{4} = \frac{1}{2}$ or, $\cos\theta = \cos 60°$

or, $\theta = 60°$

13. If $5\cos\theta + 12\sin\theta = 13$ then find the value of $\tan\theta$ and $\text{cosec}\,\theta$.

Solution : given, $12\sin\theta = 13 - 5\cos\theta$

or, $144\sin^2\theta = 169 + 25\cos^2\theta - 130\cos\theta$ (squaring)

or, $144(1-\cos^2\theta) = 169 + 25\cos^2\theta - 130\cos\theta$

or, $144 - 144\cos^2\theta = 169 + 25\cos^2\theta - 130\cos\theta$

or, $169\cos^2\theta - 130\cos\theta + 25 = 0$

or, $(13\cos\theta)^2 - 2\cdot(13\cos\theta)\cdot 5 + 5^2 = 0$

or, $(13\cos\theta - 5)^2 = 0$

or, $13\cos\theta - 5 = 0$

or, $\cos\theta = \frac{5}{13} = \frac{b}{h}$ (say)

From Pythagoras theoram

$$p = \sqrt{h^2 - b^2} = \sqrt{13^2 - 5^2} = 12$$

$\therefore$ $\tan\theta = \frac{p}{b} = \frac{12}{5}$ and $\text{cosec}\,\theta = \frac{h}{p} = \frac{13}{12}$

14. If $\text{cosec}^2\theta + 2\cot^2\theta = 10$ then find the value of $\sin\theta + \cos\theta$ when $0° < \theta < 90°$

Solution : Given, $\text{cosec}^2\theta + 2\cot^2\theta = 10$

or, $1 + \cot^2\theta + 2\cot^2\theta = 10$

or, $3\cot^2\theta = 9$

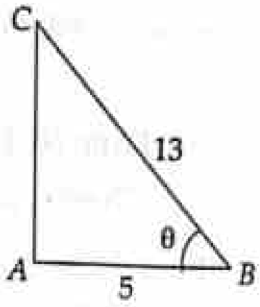
or $\cot^2\theta = 3$

or $\cot\theta = \sqrt{3} = \cot 30°$

or $\theta = 30°$

$\therefore$ $\sin\theta + \cos\theta = \sin 30° + \cos 30°$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}+1}{2}$$



5. Prove that $\tan 1° \tan 2° \tan 3° \ldots\ldots\ldots \tan 88° \tan 89° = 1$

Solution : $\because \tan(90° - \theta) = \cot\theta$

$\therefore$ $\tan 89° = \tan(90° - 1°) = \cot 1°$

$\tan 88° = \tan(90° - 2°) = \cot 2°$

....

$\tan 46° = \tan(90° - 44°) = \cot 44°$

$\therefore$ LHS $= \tan 1° \tan 2° \tan 3° \ldots\ldots\ldots \tan 44° \tan 45° \tan 46° \ldots\ldots\ldots \tan 88° \cot 44°$

$= \tan 1° \tan 2° \tan 3° \ldots \tan 44° \tan 45° \cot 44° \ldots \cot 2° \cot 1°$

$= (\tan 1° \cot 1°)(\tan 2° \cot 2°) \ldots\ldots\ldots (\tan 44° \cot 44°) \tan 45°$

$= 1 \cdot 1 \cdot 1 \ldots\ldots\ldots 1$ $\quad (\because \tan\theta \cot\theta = 1)$

$= 1$

This question is based on complementary angle. Two angles are called complementary when their sum is 90° For such question we must not that $\sin\theta \sin(90 - \theta) = 1$, $\cos\theta \cos(90 - \theta) = 1$ etc.

e.g. $\sin 40 \cdot \sin 50 = 1$, $\cos 35 \cdot \cos 65 = 1$ etc]

6. Evaluate

$$\frac{\sec 29°}{\operatorname{cosec} 61°} + 2\cot 8° \cot 17° \cot 45° \cot 73° \cot 82° - 3(\sin^2 38° + \sin^2 52°) + 4\{\cos\theta \sin(90° - \theta) + \sin\theta \cos(90° - \theta)\}$$

Solution : $\dfrac{\sec 29°}{\operatorname{cosec} 61°} = \dfrac{\sec 29°}{\operatorname{cosec}(90° - 29°)} = \dfrac{\sec 29°}{\sec 29°} = 1$ ... (i)

$2\cot 8° \cot 17° \cot 45° \cot 73° \cot 82°$

$= 2\cot 8° \cot 17° \cdot 1 \cot(90° - 17°) \cot(90° - 8°)$ $\quad (\because \cot 45° = 1)$

$= 2\cot 8° \cot 17° \tan 17° \tan 8°$ $\quad (\because \cot(90° - \theta) = \tan\theta)$

$= 2(\cot 8° \tan 8°)(\cot 17° \tan 17°)$ $\quad (\because \cot\theta \cdot \tan\theta = 1)$

$= 2 \cdot 1 \cdot 1$ ... (ii)

$= 2$

$3(\sin^2 38° + \sin^2 52°) = 3\{\sin^2 38° + \sin^2(90° - 38°)\}$

$= 3\{\sin^2 38° + \cos^2 38°\} = 3 \cdot 1 = 3$ ... (iii)

$4\{\cos\theta \cdot \sin(90° - \theta) + \sin\theta \cos(90° - \theta)\}$

$= 4(\cos\theta\cos\theta + \sin\theta\sin\theta)$

$= 4(\cos^2\theta + \sin^2\theta) = 4$ ...(iv)

From (i), (ii), (iii) and (iv)

Given expression $= 1 + 2 - 3 + 4 = 4$. Ans.

17. Find $\sin 2x + \cos 4x$ if $\tan 2x \cdot \tan 4x = 1$ [SSC Tier-I 2014]

Solution : (This question is also based on complementary angle. See the solution carefully)

$\tan 2x . \tan 4x = 1$

$\Rightarrow \tan 2x = \frac{1}{\tan 4x} = \cot 4x$

$\Rightarrow \tan 2x = \tan(90° - 4x))$ or, $2x = 90° - 4x$

or, $6x = 90°$ or, $x = \frac{90°}{6} = 15°$

$\therefore \sin 2x + \cos 4x = \sin 30° + \cos 60°$

$= \frac{1}{2} + \frac{1}{2} = 1$

18. Prove that $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} - \sec\theta = \sec\theta - \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

Solution : L.H.S. $= \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}} - \sec\theta$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} - \sec\theta = \frac{1+\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}$

$= \frac{1+\sin\theta-1}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$ ...(i)

R.H.S. $= \sec\theta - \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$

$= \sec\theta - \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \frac{1}{\cos\theta} - \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1-(1-\sin\theta)}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$ ...(ii)

From equation (i) & (ii), L.H.S. = R.H.S.

19. If $a\cos\theta - b\sin\theta = c$,

then prove than $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

Solution : Given that $a\cos\theta - b\sin\theta = c$

squaring both sides,

$a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta = c^2$

or, $a^2(1-\sin^2\theta) + b^2(1-\cos^2\theta) - 2ab\cos\theta\sin\theta = c^2$

or, $a^2 + b^2 - (a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta) = c^2$

or, $a^2 + b^2 - (a\sin\theta + b\cos\theta)^2 = c^2$

or, $(a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

or, $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

If $\operatorname{cosec}\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = n$ then find a relation between m and n, independent of θ

Solution : $\operatorname{cosec}\theta - \sin\theta = m \Rightarrow \frac{1}{\sin\theta} - \sin\theta = m$

$$\Rightarrow \frac{1-\sin^2\theta}{\sin\theta} = m \Rightarrow \frac{\cos^2\theta}{\sin\theta} = m \quad \text{... (i)}$$

and $\sec\theta - \cos\theta = n \Rightarrow \frac{1}{\cos\theta} - \cos\theta = n$

$$\Rightarrow \frac{1-\cos^2\theta}{\cos\theta} = n \Rightarrow \frac{\sin^2\theta}{\cos\theta} = n \quad \text{... (ii)}$$

Eliminating $\cos\theta$ from (i) and (ii)

$$\frac{\cos^2\theta}{\sin\theta}\cdot\left(\frac{\sin^2\theta}{\cos\theta}\right)^2 = mn^2$$

$$\Rightarrow \sin^3\theta = mn^2 \Rightarrow \sin\theta = \left(mn^2\right)^{\frac{1}{3}} \quad \text{... (iii)}$$

Again, eliminating $\sin\theta$ from (i) and (ii)

$$\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 \frac{\sin^2\theta}{\cos\theta} = m^2 n$$

$$\Rightarrow \cos^3\theta = m^2 n \Rightarrow \cos\theta = \left(m^2 n\right)^{\frac{1}{3}} \quad \text{... (iv)}$$

From (iii) and (iv)

$$\sin^2\theta + \cos^2\theta = \left\{\left(mn^2\right)^{\frac{1}{3}}\right\}^2 + \left\{\left(m^2 n\right)^{\frac{1}{3}}\right\}^2$$

$\therefore \quad \left(mn^2\right)^{\frac{2}{3}} + \left(m^2 n\right)^{\frac{2}{3}} = 1$. which is required relation.

If $x > 0$ and $2\cos^2\left(x - \frac{1}{x}\right) = x + \frac{1}{x}$ then prove that $x^2 + \frac{1}{x^2} = 2$

Solution : $\because \quad \cos^2\theta \le 1$

$$\therefore \quad 2\cos^2\left(x - \frac{1}{x}\right) \le 2 \quad \text{... (i)}$$

again, from $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \ge 0$ से

$$x + \frac{1}{x} - 2 \ge 0$$

or $x + \frac{1}{x} \ge 2$ ... (ii)

From (i)and (ii)

$2\cos^2\left(x - \frac{1}{x}\right) = x + \frac{1}{x}$ is possible only when

$2\cos^2\left(x - \frac{1}{x}\right) = 2$ and $x + \frac{1}{x} = 2$ simultaneously.

Clearly at $x = 1$ each of them is 2.

$$\therefore \quad x^2 + \frac{1}{x^2} = 1^2 + 1^2 = 2$$

Exercise

1. Expression $\frac{\tan x}{1+\sec x}-\frac{\tan x}{1-\sec x}$ is equal to

(a) cosecx (b) 2 cosecx (c) 2 sinx (d) 2 cosx

2. Expression $(\sin^4 x - \cos^4 x + 1)\,\mathrm{cosec}^2 x$ is eaqual to

(a) 1 (b) 2 (c) 0 (d) –1

3. If $l\cos^2\theta + m\sin^2\theta = \frac{\cos^2\theta\left(\mathrm{cosec}^2\theta+1\right)}{\mathrm{cosec}^2\theta-1}$ then what is the value of $\tan^2\theta$?

(a) $\frac{l-2}{m-1}$ (b) $\frac{l-1}{2-m}$ (c) $\frac{l-2}{l-m}$ (d) $\frac{2-l}{1-m}$

4. Assertion (A) : $\sec^2 23° - \tan^2 23° = 1$

Reason (R) : For every real value of θ, $\sec^2\theta - \tan^2\theta = 1$

(a) both A and R are true and R is a correct explanation of A.

(b) both A and R are true but R is not a correct explanation of A

(c) A is true, R is false

(d) A is false, R is true.

5. If $\sin x \cos x = \frac{1}{2}$ then the value of $\sin x - \cos x$ is

(a) 2 (b) 1 (c) 0 (d) – 1

6. If $\tan^2 y\,\mathrm{cosec}^2 x - 1 = \tan^2 y$ then which one of the following is true.

(a) $x - y = 0$ (b) $x = 2y$ (c) $y = 2x$ (d) $x - y = 1°$

7. If $\frac{\cos x}{1+\mathrm{cosec}\,x}+\frac{\cos x}{\mathrm{cosec}\,x-1}=2$ then which one is a value of x ?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

8. If $\sin x + \sin y = a$ and $\cos x + \cos y = b$

then value of $\sin x \cdot \sin y + \cos x \cdot \cos y$ is

(a) $a + b - ab$ (b) $a + b + ab$ (c) $a^2 + b^2 - 2$ (d) $\left(\frac{a^2+b^2-2}{2}\right)$

9. If α is an angle in first quadrant such that $\mathrm{cosec}^4\alpha = 17 + \cot^4\alpha$, then what is the value of sinα ?

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{9}$ (d) $\frac{1}{16}$

10. If $x + \left(\frac{1}{x}\right) = 2\cos\alpha$ then the value of $x^2 + \left(\frac{1}{x^2}\right)$ is

(a) $4\cos^2\alpha$ (b) $4\cos^2\alpha - 1$

(c) $2\cos^2\alpha - 2\sin^2\alpha$ (d) $\cos^2\alpha - \sin^2\alpha$

11. If $\sin\theta + \cos\theta = a$ and $\sec\theta + \mathrm{cosec}\,\theta = b$ then which of the following relation is true ?

(a) $a = b\,(a^2 - 1)$ (b) $b = a(b^2 - 1)$

(c) $2a = b(a^2 - 1)$ (d) $2b = a(a^2 - 1)$

Among given values of θ which one satisfies the equation $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

If $7\cos^2\theta + 3\sin^2\theta = 4$ and $0 < \theta < \frac{\pi}{2}$, then value of $\tan\theta$ is.

(a) $\sqrt{7}$ (b) $\frac{7}{3}$ (c) 3 (d) $\sqrt{3}$

[SSC Tier-I 2014]

14. When $0 < \theta < 90°$ then value of $[(1-\sin^2\theta)\sec^2\theta + \tan^2\theta](\cos^2\theta + 1)$ is

(a) 2 (b) > 2 (c) ≥ 2 (d) < 2

15. What is the value of $\sin^2 15° + \sin^2 20° + \sin^2 25° + \ldots\ldots + \sin^2 75°$?

(a) $\tan^2 15° + \tan^2 20° + \tan^2 25° + \ldots\ldots + \tan^2 75°$

(b) $\cos^2 15° + \cos^2 20° + \cos^2 25° + \ldots\ldots + \cos^2 75°$

(c) $\cot^2 15° - \cot^2 20° + \cot^2 25° + \ldots\ldots + \cot^2 75°$

(d) $\sec^2 15° + \sec^2 20° + \sec^2 25° + \ldots\ldots + \sec^2 75°$

16. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is equal to

(a) $\sec\theta - \tan\theta$ (b) $\sec\theta + \tan\theta$ (c) $\text{cosec}\theta + \cot\theta$ (d) $\text{cosec}\theta - \cot\theta$

17. If $0° < \theta < 90°$ and $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2$, then which of the following is equal to θ ?

(a) 30° (b) 45° (c) 60° (d) 75°

18. If $\sin 3\theta = \cos(\theta - 2°)$ where 3θ and $(\theta - 2°)$ are acute angle then θ is ?

(a) 22° (b) 23° (c) 24° (d) 25°

19. Expression $\frac{\sin^6\theta - \cos^6\theta}{\sin^2\theta - \cos^2\theta}$ is equal to

(a) $\sin^4\theta - \cos^4\theta$ (b) $1 - \sin^2\theta\cos^2\theta$

(c) $1 + \sin^2\theta\cos^2\theta$ (d) $1 - 3\sin^2\theta\cos^2\theta$

20. If $\sin^4 x + \sin^2 x = 1$, then the value of $\cot^4 x + \cot^2 x$ is

(a) $\cos^2 x$ (b) $\sin^2 x$ (c) $\tan^2 x$ (d) 1

21. If $x\cos\theta + y\sin\theta = 2$ and $x\cos\theta - y\sin\theta = 0$, then which one of the following is true ?

(a) $x^2 + y^2 = 1$ (b) $\frac{1}{x^2} + \frac{1}{y^2} = 1$ (c) $xy = 1$ (d) $x^2 - y^2 = 1$

22. Expression $\sin A(1 + \tan A) + \cos A(1 + \cot A)$ is equal to

(a) $\sec A + \text{cosec} A$ (b) $2\,\text{cosec} A(\sin A + \cos A)$

(c) $\tan A + \cot A$ (d) $\sec A\,\text{cosec} A$

23. If $0° < \theta < 90°$ and $\cos^2\theta - \sin^2\theta = \frac{1}{2}$, then value of θ is ?

(a) 30° (b) 45° (c) 60° (d) 90°

24. If $3\sin\theta + 4\cos\theta = 5$, then $3\cos\theta - 4\sin\theta$ is equal to ?

(a) 0 (b) 3 (c) 4 (d) 5

25. On simplification $\frac{(1-\sin A\cos A)(\sin^2 A-\cos^2 A)}{\cos A(\sec A-\text{cosec} A)(\sin^3 A+\cos^3 A)}$ equals

(a) $\sin A$ (b) $\cos A$ (c) $\sec A$ (d) $\text{cosec} A$

26. For $0° < \theta < 90°$ which of the following expression is/are independent of θ ?

(i) $\cos\theta\,(1-\sin\theta)^{-1}+\cos\theta\,(1+\sin\theta)^{-1}$

(ii) $\cos\theta\,(1+\text{cosec}\theta)^{-1}+\cos\theta\,(\text{cosec}\theta-1)^{-1}$

Choose the correct code among following .

(a) Only (i) (b) Only(ii)

(c) Both (i) and (ii) (d) Neither (i) Nor (ii)

27. If $a\cos\theta - b\sin\theta = c$, then the value of $a\sin\theta + b\cos\theta$ is

(a) $\pm\sqrt{a^2+b^2+c^2}$ (b) $\pm\sqrt{a^2-b^2+c^2}$

(c) $\pm\sqrt{a^2+b^2-c^2}$ (d) $\pm\sqrt{b^2-c^2-a^2}$

28. Expression $\tan^2\alpha+\cot^2\alpha$ is

(a) ≥ 2 (b) ≤ 2

(c) ≥ -2 (d) None of these

29. Maximum value of $\sin^8\theta+\cos^{14}\theta$ is

(a) $\sqrt{2}$ (b) 2 (c) 1 (d) $\frac{1}{\sqrt{2}}$

30. If $P=\frac{1}{2}\sin^2\theta+\frac{1}{3}\cos^2\theta$, then

(a) $\frac{1}{3}\leq P\leq\frac{1}{2}$ (b) $P\geq\frac{1}{2}$

(c) $2\leq P\leq 3$ (d) $-\frac{\sqrt{13}}{6}\leq P\leq\frac{\sqrt{13}}{6}$

31. Minimum value of $5\cos\theta+12$ is

(a) 5 (b) 12 (c) 7 (d) 17

32. If $a\sin^3\theta+b\cos^3\theta=\sin\theta\cos\theta$, $0<\theta<90°$ and $a\sin\theta=b\cos\theta$ then the value of a^2+b^2 is

(a) ab (b) $2ab$ (c) 1 (d) 2

33. $\sin^2 17.5°+\sin^2 72.5°$ is equal to

(a) $\cos^2 90°$ (b) $\tan^2 45°$ (c) $\cos^2 30°$ (d) $\sin^2 45°$

34. A cow is tied in a pole with a rope. The cow moves in a circular path keeping rope straight. When it covers a distance of 44 meter an angle of 72° is subtended at the centre. The length of the rope is

(a) 45 m (b) 35 m (c) 22 m (d) 56m

35. $(\sin\theta+\cos\theta)(\tan\theta+\cot\theta)=$

(a) 1 (b) $\sin\theta\cdot\cos\theta$ (c) $\sec\theta\cdot\text{cosec}\theta$ (d) $\sec\theta+\text{cosec}\theta$

37. If $\sec\alpha$, $\text{cosec}\,\alpha$ are roots of equation $x^2+px+q=0$ then

(a) $p^2=p+2q$ (b) $q^2=p+2q$ (c) $p^2=q\,(q+2)$

(d) $q^2=p\,(p+2)$ (e) $p^2=q\,(q-2)$

If $\sec\theta$ and $\tan\theta$ are roots of equation $ax^2 + bx + c = 0$ $(a, b \neq 0)$ then the value of $\sec\theta - \tan\theta$ is

(a) $-\frac{a}{b}$ (b) $\frac{\sqrt{b^2-4ac}}{a}$ (c) $1-\frac{a}{b}$

(d) $1+\frac{a^2}{b^2}$ (e) $\frac{a}{b}$

If $x = h + a\sec\theta$ and $y = k + b\,\mathrm{cosec}\theta$ then

(a) $\frac{a^2}{(x+h)^2} - \frac{b^2}{(y+k)^2} = 01$ (b) $\frac{a^2}{(x-h)^2} + \frac{b^2}{(y-k)^2} = 1$

(c) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (d) $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

If $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$, then the value of $\cos A + \sqrt{6}\sin A$ is

(a) $\sqrt{6}\sin A$ (b) $-\sqrt{7}\sin A$

(c) $\sqrt{6}\cos A$ (d) $\sqrt{7}\cos A$

11. If $\sin\theta$ and $\cos\theta$ are roots of equation $ax^2 + bx + c = 0$ then

(a) $(a-c)^2 = b^2 - c^2$ (b) $(a-c)^2 = b^2 + c^2$

(c) $(a+c)^2 = b^2 - c^2$ (d) $(a+c)^2 = b^2 + c^2$

12. Maximum value of $\sin(\cos x)$ is—

(a) $\sin 1$ (b) 1 (c) $\sin\left(\frac{1}{\sqrt{2}}\right)$ (d) $\sin\left(\frac{\sqrt{3}}{2}\right)$

13. If $\cos x + \cos^2 x = 1$ then the value of $\sin^{12} x + 3\sin^{10} x + 3\sin^8 x + \sin^6 x - 1$ is

(a) 2 (b) 1 (c) -1 (d) 0

14. If $3\sin\theta + 5\cos\theta = 5$ then the value of $5\sin\theta - 3\cos\theta$ is

(a) 5 (b) 3 (c) 4 (d) None of these

15. If $\tan\theta + \sec\theta = p$ then the value of $\sec\theta$ is

(a) $\frac{p^2+1}{p^2}$ (b) $\frac{p^2+1}{\sqrt{p}}$ (c) $\frac{p^2+1}{2p}$ (d) $\frac{p+1}{2p}$

16. If $\sin\theta - \cos\theta = \sqrt{2}\cos\theta$ then the value of $\sin\theta + \cos\theta$ is

(a) $2\cos\theta$ (b) $2\sin\theta$ (c) $\sqrt{2}\sin\theta$ (d) $\sqrt{2}\cos\theta$

17. If $\tan(\theta + 30)\tan(2\theta + 3\theta) = 1$ then the value of $\sin(5\theta - 2\theta)$ is

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{1}{\sqrt{2}}$

18. If $\sec x = \mathrm{cosec}\, y$ then the value of $\mathrm{cosec}(x+y)$ is

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) undefined

19. If $\tan 2\theta = \cot(\theta - 18^\circ)$ then then value of $\sin\left(\frac{5\theta}{4}\right) + \cos\left(\frac{5\theta}{4}\right)$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $2\sqrt{2}$ (c) 1 (d) $\sqrt{2}$

50. If $\sin\theta + \cos\theta = 1$ then the value of $\sin\theta - \cos\theta$ is

(a) 0 (b) $\pm\sqrt{2}$ (c) ± 1 (d) $\pm\frac{1}{\sqrt{2}}$

51. If $k = (1-\sin\alpha)(1-\sin\beta)(1-\sin\gamma) = (1+\sin\alpha)(1+\sin\beta)(1+\sin\gamma)$ then the value of k is

(a) $\pm\sin\alpha\sin\beta\sin\gamma$ (b) $\pm\cos\alpha\cos\beta\cos\gamma$

(c) $\pm\sec\alpha\sec\beta\sec\gamma$ (d) $\pm\operatorname{cosec}\alpha\operatorname{cosec}\beta\operatorname{cosec}\gamma$

52. If $p = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$
$= (\sec A + \tan A)(\sec B \times \tan B)(\sec C + \tan C)$ then value of p is

(a) $\pm\tan A\tan B\tan C$ (b) $\pm\sec A\sec B\sec C$

(c) ± 1 (d) None of these

53. **The value of** $(1+\cot\theta+\operatorname{cosec}\theta)(1+\cot\theta-\operatorname{cosec}\theta)$ is

(a) $2\tan\theta$ (b) $2\cot\theta$ (c) $2\sec\theta$ (d) $2\operatorname{cosec}\theta$

54. **Which of the following is not equal to** $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$

(a) $\sec\theta+\tan\theta$ (b) $\frac{1}{\sec\theta-\tan\theta}$

(c) $\frac{1+\sin\theta}{\cos\theta}$ (d) $\frac{1-\sin\theta}{\cos\theta}$

55. If $m = \tan\theta+\sin\theta$ and $n = \tan\theta-\sin\theta$ then the value of m^2-n^2 is

(a) $2\sqrt{mn}$ (b) $4\sqrt{mn}$ (c) $\sqrt{mn}$ (d) $\sqrt{2mn}$

56. The value of $\frac{\cos\theta}{\tan\theta+\sec\theta}-\frac{\cos\theta}{\tan\theta-\sec\theta}$ is

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $2\cos\theta$

57. $\sec^2\theta+\operatorname{cosec}^2\theta$ is equal to which of the following ?

(a) $\sec\theta\tan\theta$ (b) $\sec\theta\operatorname{cosec}\theta$

(c) $\sec^2\theta\operatorname{cosec}^2\theta$ (d) $\sin^4\theta+\cos^4\theta$

58. The identity $(1+\tan\theta-\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$ equals

(a) 2 (b) 1 (c) $2\tan\theta$ (d) $2\cot\theta$

59. Which is equal to $\sec\theta.\operatorname{cosec}\theta$?

(a) $\sin\theta+\cos\theta$ (b) $\tan\theta+\cot\theta$

(c) $2(\tan\theta+\cot\theta)$ (d) $2(\sin\theta+\cos\theta)$

60. The value of $\tan^4 A+\tan^2 A$ in terms of $\sec A$ is

(a) $\sec^4 A+\sec^2 A$ (b) $\sec^2 A+\sec^4 A$

(c) $\sec^4 A+\sec^2 A-1$ (d) $\sec^4 A-\sec^2 A$

61. If $0<\theta<90°$ then what is the minimum value of
$\sin^2\theta+\cos^2\theta+\tan^2\theta+\cot^2\theta+\sec^2\theta+\operatorname{cosec}^2\theta$?

(a) 10 (b) 5 (c) 6 (d) 7

62. If $\cos^2\alpha+\cos^2\beta = 2$ then what is the value of $\tan^3\alpha+\sin^5\beta$?

(a) -1 (b) 0 (c) 1 (d) $\frac{1}{\sqrt{3}}$

If $A = \tan 11^\circ \tan 29^\circ$, $B = 2\cot 61^\circ \cot 79^\circ$ then which among the following is true ?

(a) $A = 2B$ (b) $A = -2B$ (c) $2A = B$ (d) $2A = -B$

On simplification $(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$ yields

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

The value of $\sin^2 1^\circ + \sin^2 5^\circ + \sin^2 9^\circ + \ldots\ldots + \sin^2 89^\circ$ is

(a) $11\frac{1}{2}$ (b) $11\sqrt{2}$ (c) 11 (d) $\frac{11}{\sqrt{2}}$

The numeric value of $\cot 18^\circ \left(\cot 72^\circ \cos^2 22^\circ + \frac{1}{\tan 72^\circ \sec^2 68^\circ} \right)$ is

(a) 1 (b) $\sqrt{2}$ (c) 3 (d) $\frac{1}{\sqrt{3}}$

67. If $\sin\alpha \sec(30^\circ + \alpha) = 1$ $(0 < \alpha < 60^\circ)$ then the value of $\sin\alpha + \cos 2\alpha$ is

(a) 1 (b) $\frac{2+\sqrt{3}}{2\sqrt{3}}$ (c) 0 (d) $\sqrt{2}$

68. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $2\cos^2\theta$ is

(a) $\frac{5}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

69. If θ is a positive acute angle and $\cos^2\theta + \cos^4\theta = 1$ then the value of $\tan^2\theta + \tan^4\theta$ is

(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) 0

70. If θ is an acute angle and $\tan\theta + \cot\theta = 2$ then the value of $\tan^5\theta + \cot^{10}\theta$ is

(a) 1 (b) 2 (c) 3 (d) 4

71. $(\sin^2 1^\circ + \tan^2 3^\circ + \sin^2 5^\circ + \tan^2 7^\circ + \ldots\ldots + \tan^2 87^\circ + \sin^2 89^\circ)$ equals

(a) 23 (b) 22 (c) $22\frac{1}{2}$ (d) $23\frac{1}{2}$

72. If $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$, $(0^\circ < \theta < 90^\circ)$ then the value of $2\sin\theta + \cos\theta$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{3}{\sqrt{2}}$ (d) $\frac{\sqrt{2}}{3}$

73. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$ then the value of $\sin^4\theta - \cos^4\theta$ is

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

74. If $\sec^2\theta + \tan^2\theta = 7$, then the value of θ is

(a) 60° (b) 30° (c) 0° (d) 90°

75. $(\sec x \cdot \sec y + \tan x \cdot \tan y)^2 - (\sec x \cdot \tan y\ \tan x \cdot \sec y)^2$ in its simplest form, is

(a) -1 (b) 0 (c) $\sec^2 x$ (d) 1

76. If $\frac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$ and $0^\circ < \theta < 90^\circ$ then the value of θ is

(a) 30° (b) 45° (c) ... (d) None of these

77. If $\sin\theta - \cos\theta = \frac{7}{13}$ and $0 < \theta < 90°$ then the value of $\sin\theta + \cos\theta$ is

(a) $\frac{17}{13}$ (b) $\frac{13}{17}$ (c) $\frac{1}{13}$ (d) $\frac{1}{17}$

Answer–11A

1. (b)	2. (b)	3. (b)	4. (a)	5. (c)	6. (a)	7. (c)	8. (d)
9. (a)	10. (c)	11. (c)	12. (c)	13. (d)	14. (b)	15. (b)	16. (b)
17. (b)	18. (b)	19. (b)	20. (d)	21. (b)	22. (a)	23. (a)	24. (a)
25. (b)	26. (d)	27. (c)	28. (a)	29. (c)	30. (a)	31. (c)	32. (a)
33. (b)	34. (b)	35. (d)	37. (c)	38. (b)	39. (b)	40. (b)	41. (d)
42. (a)	43. (d)	44. (b)	45. (c)	46. (c)	47. (a)	48. (a)	49. (d)
50. (c)	51. (b)	52. (c)	53. (b)	54. (d)	55. (b)	56. (c)	57. (c)
58. (a)	59. (b)	60. (d)	61. (d)	62. (b)	63. (c)	64. (c)	65. (a)
66. (a)	67. (a)	68. (a)	69. (b)	70. (b)	71. (c)	72. (c)	73. (c)
74. (a)	75. (d)	76. (c)	77. (a)				

Explanation

1. **(b)** $\frac{\tan x}{1+\sec x} - \frac{\tan x}{1-\sec x}$

$$= \frac{\tan x(1-\sec x - 1 - \sec x)}{1-\sec^2 x}$$

$$= \frac{\tan x(-2\sec x)}{-\tan^2 x} \qquad (\because \ -\sec^2 x + 1 = -\tan^2 x)$$

$$= \frac{-2\tan x \sec x}{-\tan^2 x} = \frac{\frac{2}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{2}{\sin x} = 2\operatorname{cosec} x$$

2. **(b)** $(\sin^4 x - \cos^4 x + 1)\operatorname{cosec}^2 x$

$$= \{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) + 1\}\operatorname{cosec}^2 x$$

$$[\because \ a^2 - b^2 = (a+b)(a-b)]$$

$$= \{\sin^2 x - \cos^2 x + 1\}\operatorname{cosec}^2 x$$

$$= (\sin^2 x + \sin^2 x)\operatorname{cosec}^2 x \qquad (\because 1 - \cos^2 x = \sin^2 x)$$

$$= 2\sin^2 x \frac{1}{\sin^2 x} = 2$$

3. **(b)** Given, $l\cos^2\theta + m\sin^2\theta = \frac{\cos^2\theta(\operatorname{cosec}^2\theta + 1)}{\cot^2\theta}$ $(\because \operatorname{cosec}^2\theta - 1 = \cot^2\theta)$

or, $l\cos^2\theta + m\sin^2\theta = \frac{\cos^2\theta(\operatorname{cosec}^2\theta + 1)\cdot\sin^2\theta}{\cos^2\theta}$

or, $l\cos^2\theta + m\sin^2\theta = \sin^2\theta(\operatorname{cosec}^2\theta + 1) = 1 + \sin^2\theta$

or $l\cos^2\theta + m\sin^2\theta = \sin^2\theta + \cos^2\theta + \sin^2\theta = 2\sin^2\theta + \cos^2\theta$

$(\because 1 = \sin^2\theta + \cos^2\theta)$

or $(l-1)\cos^2\theta = (2-m)\sin^2\theta$

or $\frac{l-1}{2-m} = \tan^2\theta$

(a) $\sec^2\theta - \tan^2\theta = 1$ is not true when $\theta = 90°$, because $\tan\theta$ and $\sec\theta$ are not defined at $= 90°$

(c) Now, $(\sin x - \cos x)^2 = (\sin^2 x + \cos^2 x) - 2\sin x \cos x = 1 - 2\left(\frac{1}{2}\right) = 0$

$\Rightarrow \sin x - \cos x = 0$

(a) Given, $\tan^2 y \operatorname{cosec}^2 x - 1 = \tan^2 y$

$\Rightarrow \tan^2 y(\operatorname{cosec}^2 x - 1) = 1$

$\Rightarrow \tan^2 y \cot^2 x = 1 \Rightarrow \tan^2 y = \frac{1}{\cot^2 x} = \tan^2 x \qquad \therefore \quad x = y$

(c) Given, $\frac{\cos x}{1+\operatorname{cosec} x} + \frac{\cos x}{\operatorname{cosec} x - 1} = 2$

$\Rightarrow \frac{2\cos x \operatorname{cosec} x}{\operatorname{cosec}^2 x - 1} = 2 \qquad \Rightarrow \frac{\cos x \operatorname{cosec} x}{\cot^2 x} = 1$

$\Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

(d) Given, $\sin x + \sin y = a$ and $\cos x + \cos y = b$, squaring

$\Rightarrow \sin^2 x + \sin^2 y + 2\sin x \sin y = a^2$ ...(i)

and $\cos^2 x + \cos^2 y + 2\cos x \cos y = b^2$ ...(ii)

adding (i) and (ii)

$(\sin^2 x + \cos^2 x) + (\sin^2 y + \cos^2 y) + 2(\sin x \sin y + \cos x \cos y) = a^2 + b^2$

$\Rightarrow (\sin x \sin y + \cos x \cos y) = \frac{a^2 + b^2 - 2}{2}$

(a) Given, $\operatorname{cosec}^4\alpha - \cot^4\alpha = 17$

$\Rightarrow (\operatorname{cosec}^2\alpha - \cot^2\alpha)(\operatorname{cosec}^2\alpha + \cot^2\alpha) = 17$

$\Rightarrow 1.\left(\frac{1}{\sin^2\alpha} + \frac{\cos^2\alpha}{\sin^2\alpha}\right) = 17$

$\Rightarrow \left(\frac{1+\cos^2\alpha}{\sin^2\alpha}\right) = 17$

$\Rightarrow 2 - \sin^2\alpha = 17\sin^2\alpha \quad \Rightarrow \sin^2\alpha = \frac{1}{9} \Rightarrow \sin\alpha = \frac{1}{3}$

10. (c) Given, $x + \frac{1}{x} = 2\cos\alpha$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4\cos^2\alpha$

$\Rightarrow x^2 + \frac{1}{x^2} = 2(2\cos^2\alpha - 1) = 2(\cos^2\alpha - (1 - \cos^2\alpha)) = 2\cos^2\alpha - 2\sin^2\alpha$

11. (c) $b = \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} = \frac{a}{\sin\theta\cos\theta}$

$\therefore \quad \frac{a}{b} = \sin\theta\cos\theta$ ...(i)

$\sin\theta + \cos\theta = a$ squaring

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = a^2$

or, $1 + 2\sin\theta\cos\theta = a^2$

or, $\sin\theta\cos\theta = \frac{a^2-1}{2}$ ...(ii)

From (i) and (ii)

$$\frac{a}{b} = \frac{a^2-1}{2} \quad \Rightarrow 2a = b(a^2-1)$$

12. (c) Given, $\frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta} = 2$

$\Rightarrow \quad \frac{\cos\theta + \sin\theta\cos\theta - \cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta} = 2$

$\Rightarrow \quad 2\sin\theta\cdot\cos\theta = 2\cos^2\theta \Rightarrow 2\sin\theta = 2\cos\theta$

$\Rightarrow \quad \tan\theta = 1 \qquad \Rightarrow \tan\theta = \frac{\pi}{4}$

13. (d) Given, $7\cos^2\theta + 3\sin^2\theta = 4$

$\Rightarrow \quad 7(1-\sin^2\theta) + 3(\sin^2\theta) = 4$

$\Rightarrow \quad 7 - 4\sin^2\theta = 4$

$\Rightarrow \quad 4\sin^2\theta = 3$

$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \theta = 60^\circ \qquad \therefore \quad \tan\theta = \tan 60^\circ = \sqrt{3}$

14. (b) $[(1-\sin^2\theta)\sec^2\theta + \tan^2\theta](\cos^2\theta + 1)$

$= (\sec^2\theta - \tan^2\theta + \tan^2\theta)(\cos^2\theta + 1)$

$= 1 + \sec^2\theta > 1 + 1 > 2 \qquad [\because \sec^2\theta > 1, 0 < \theta < 90^\circ]$

15. (b) $\sin^2 15^\circ + \sin^2 20^\circ + \sin^2 25^\circ + \ldots + \sin^2 75^\circ$

$= \sin^2(90^\circ - 75^\circ) + \sin^2(90^\circ - 70^\circ) + \ldots + \sin^2(90^\circ - 15^\circ)$

$= \cos^2 75^\circ + \cos^2 70^\circ + \ldots + \cos^2 15^\circ$

16. (b) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1+\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$

(a) If $0° < \theta < 90°$

and $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta + \cos^2\theta = 2\sin\theta\cdot\cos\theta$

$\Rightarrow (\sin^2\theta + \cos^2\theta - 2\sin\theta\cdot\cos\theta) = 0$

$\Rightarrow (\sin\theta - \cos\theta)^2 = 0$

$\Rightarrow \sin\theta - \cos\theta = 0 \qquad \Rightarrow \tan\theta = 1 = \tan\frac{\pi}{4}$

$\Rightarrow \theta = \frac{\pi}{4}$ or, $45°$

(b) $\sin 3\theta = \cos(\theta - 2°)$

$\Rightarrow \sin 3\theta = \sin[90° - (\theta - 2°)]$

$\Rightarrow 3\theta = 90° - \theta + 2°$

$\Rightarrow 4\theta = 92° \qquad \Rightarrow \theta = \frac{92}{4} = 23°$

(b) $\frac{\sin^6\theta - \cos^6\theta}{\sin^2\theta - \cos^2\theta} = \frac{(\sin^2\theta)^3 - (\cos^2\theta)^3}{\sin^2\theta - \cos^2\theta}$

$= \frac{(\sin^2\theta - \cos^2\theta)(\sin^4\theta + \cos^4\theta + \sin^2\theta\cos^2\theta)}{\sin^2\theta - \cos^2\theta}$

$= \sin^4\theta + \cos^4\theta + \sin^2\theta\cos^2\theta$

$= \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta$

$= 1 - \sin^2\theta\cos^2\theta$

1 (d) $\sin^4 x + \sin^2 x = 1$

$\Rightarrow \sin^4 x = 1 - \sin^2 x = \cos^2 x$ ... (i)

$\therefore \cot^4 x + \cot^2 x = \cot^2 x(1 + \cot^2 x) = \cot^2 x\cdot\text{cosec}^2 x$

$\frac{\cos^2 x}{\sin^2 x}\cdot\frac{1}{\sin^2 x} = \frac{\cos^2 x}{\sin^4 x} = 1 \qquad (\because \sin^4 x = \cos^2 x)$

1 (b) Given,

$x\cos\theta + y\sin\theta = 2$ ... (i)

and $x\cos\theta - y\sin\theta = 0$ ... (ii)

Solving equation (i) and (ii),

$\Rightarrow x\cos\theta = 1$ and $y\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{x}$ and $\sin\theta = \frac{1}{y}$

$\because \cos^2\theta + \sin^2\theta = 1$

$\therefore \frac{1}{x^2} + \frac{1}{y^2} = 1$

22. (a) $\sin A(1+\tan A)+\cos A(1+\cot A)$

$$\sin A\left(\frac{\sin A+\cos A}{\cos A}\right)+\cos A\left(\frac{\cos A+\sin A}{\sin A}\right)$$

$$=(\sin A+\cos A)\left(\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)$$

$$=(\sin A+\cos A)\left(\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}\right)$$

$$=\frac{\sin A}{\sin A\cos A}+\frac{\cos A}{\sin A\cos A}=\sec A+\operatorname{cosec} A$$

23. (a) $\because \quad \cos^2\theta-\sin^2\theta=\frac{1}{2}$

or, $(1-\sin^2\theta)-\sin^2\theta=\frac{1}{2}$

or, $1-\frac{1}{2}=2\sin^2\theta$

or, $\sin^2\theta=\frac{1}{4}$ or $\theta=30^\circ$

24. (a) $\because \quad (3\sin\theta+4\cos\theta)=5$

Squaring both sides $(3\sin\theta+4\cos\theta)^2=25$

$\Rightarrow 9\sin^2\theta+16\cos^2\theta+24\sin\theta\cos\theta=25$

$\Rightarrow 9(1-\cos^2\theta)+16(1-\sin^2\theta)+24\sin\theta\cos\theta=25$

$\Rightarrow 9-9\cos^2\theta+16-16\cos^2\theta+24\sin\theta\cos\theta=25$

$\Rightarrow 9\cos^2\theta+16\sin^2\theta-24\sin\theta\cos\theta=0$

$\Rightarrow (3\cos\theta-4\sin\theta)^2=0 \quad \Rightarrow 3\cos\theta-4\sin\theta=0$

25. (b) $$\frac{(1-\sin A\cos A)(\sin^2 A-\cos^2 A)}{\cos A(\sec A-\operatorname{cosec} A)(\sin^3 A+\cos^3 A)}$$

$$=\frac{(1-\sin A\cos A)(\sin^2 A-\cos^2 A)}{\cos A\left(\frac{1}{\cos A}-\frac{1}{\sin A}\right)(\sin A+\cos A)\left(\sin^2 A+\cos^2 A-\sin A\cos A\right)}$$

$$=\frac{(1-\sin A\cos A)(\sin A-\cos A)}{\cos A(\sin A-\cos A)(\sin A+\cos A)(1-\sin A\cos A)}=\sin A$$

26. (d) (i) $$\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}$$

$$=\frac{\cos\theta(1+\sin\theta+1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}=\frac{2\cos\theta}{1-\sin^2\theta}=\frac{2\cos\theta}{\cos^2\theta}=\frac{2}{\cos\theta}$$

(ii) $$\frac{\cos\theta}{1+\operatorname{cosec}\theta}+\frac{\cos\theta}{\operatorname{cosec}\theta-1}$$

$$=\frac{\cos\theta[\operatorname{cosec}\theta-1+\operatorname{cosec}\theta+1]}{(\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-1)}$$

$$=\frac{2\cos\theta\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-1}=\frac{2\cot\theta}{\cot^2\theta}=\frac{2}{\cot\theta}$$

Neither 1 nor 2 is independent θ

$a\cos\theta - b\sin\theta = c,$

Squaring

$a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta.\sin\theta = c^2$

$\Rightarrow a^2(1-\sin^2\theta) + b^2(1-\cos^2\theta) - 2ab\cos\theta.\sin\theta = c^2$

$\Rightarrow a^2\sin^2\theta - b^2\cos^2\theta - 2ab\cos\theta.\sin\theta = c^2 - a^2 - b^2$

$\Rightarrow a^2\sin^2\theta + b^2\cos^2\theta + 2ab\cos\theta.\sin\theta = a^2 + b^2 - c^2$

$\Rightarrow (a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$

$\Rightarrow a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

(a) $(\tan\alpha - \cot\alpha)^2 \geq 0$

$\Rightarrow \tan^2\alpha + \cot^2\alpha - 2\tan\alpha\cot\alpha \geq 0$

$\Rightarrow \tan^2\alpha + \cot^2\alpha - 2 \geq 0 \qquad (\because \tan\alpha\cot\alpha = 1)$

(d) Since values of $\sin^2\theta$ and $\cos^2\theta$ lie between and 0 and 1, therefore its value decreases as power of $\sin\theta$ and $\cos\alpha$ increases.

Hence $\sin^8\theta \leq \sin^2\theta$ and $\cos^{14}\theta \leq \cos^2\theta$; adding

$\sin^8\theta + \cos^{14}\theta \leq \sin^2\theta + \cos^2\theta$ or, $\sin^8\theta + \cos^{14}\theta \leq 1$

(a) $P = \frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta = \frac{3\sin^2\theta + 2\cos^2\theta}{6} = \frac{\sin^2\theta + 2}{6}$

$\because 0 \leq \sin^2\theta \leq 1$

$\therefore$ When $\sin^2\theta = 0, P = \frac{2}{6} = \frac{1}{3}, \qquad \sin^2\theta = 1, P = \frac{3}{6} = \frac{1}{2}$

(d) $\because$ Minimum value of $\cos\theta$ is -1

$\therefore$ Minimum value of $5\cos\theta + 12 = -5 + 12 = 7$

(a) $a\sin\theta = b\cos\theta \quad \Rightarrow \quad \tan\theta = \frac{b}{a}$

$\therefore \sin\theta = \frac{b}{\sqrt{a^2+b^2}}$ and $\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$

Given relation $a\sin^3\theta + b\cos^3\theta = \sin\theta\cdot\cos\theta$

$$\Rightarrow \frac{ab^3}{\left(\sqrt{a^2+b^2}\right)^3} + \frac{ba^3}{\left(\sqrt{a^2+b^2}\right)^3} = \frac{ba}{\left(\sqrt{a^2+b^2}\right)^2}$$

$$\Rightarrow \frac{ab\left(b^2+a^2\right)}{\left(\sqrt{a^2+b^2}\right)^3} = \frac{ab}{\left(\sqrt{a^2+b^2}\right)^2}$$

or, $\frac{b^2+a^2}{\sqrt{a^2+b^2}} = 1$

$\Rightarrow \sqrt{a^2+b^2} = 1 \Rightarrow a^2 + b^2 = 1$

(b) $\sin^2 17.5^\circ + \sin^2 72.5^\circ$

$= \sin^2 17.5 + \cos^2 17.5^\circ = 1 = \tan^2 45^\circ$

$\Rightarrow 360° = \frac{44}{72} \times 360 = 220$ ...

Perimeter $2\pi r = 220$ meters

$\therefore \quad r = \frac{220}{2\pi} = 110 \times \frac{7}{22} = 35$ meters

35. (d) $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$$= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} = \sec\theta + \operatorname{cosec}\theta$$

37. (c) Sum of roots $= \sec\alpha + \operatorname{cosec}\alpha = p$ ...(i)

Product of roots $= \sec\alpha \operatorname{cosec}\alpha = q$ ...(ii)

from (i) $\frac{1}{\cos\alpha} + \frac{1}{\sin\alpha} = p \Rightarrow \frac{\sin\alpha + \cos\alpha}{\cos\alpha\sin\alpha} = p$ ...(iii)

from (ii) $\frac{1}{\cos\alpha\sin\alpha} = q \Rightarrow \sin\alpha\cos\alpha = \frac{1}{q}$ ...(iv)

from (iii) and (iv) $\sin\alpha + \cos\alpha = \frac{p}{q}$ ...(v)

but $(\sin\alpha + \cos\alpha)^2 = \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha$

$\Rightarrow \left(\frac{p}{q}\right)^2 = 1 + \frac{2}{q} \quad \Rightarrow \frac{p^2}{q^2} = \frac{q+2}{q} \quad \Rightarrow p^2 = q^2 + 2q = q(q+2)$

38. (b) $\sec\theta - \tan\theta = \sqrt{(\sec\theta + \tan\theta)^2 - 4\sec\theta\tan\theta}$

$$= \sqrt{\left(\frac{-b}{a}\right)^2 - \frac{4c}{a}} = \sqrt{\frac{b^2 - 4ac}{a^2}} = \frac{\sqrt{b^2 - 4ac}}{a}$$

39. (b) $x - h = a\sec\theta \Rightarrow \cos\theta = \frac{a}{x-h}$

$y - k = b\operatorname{cosec}\theta \Rightarrow \sin\theta = \frac{b}{y-k}$

$\because \quad \cos^2\theta + \sin^2\theta = 1$

$\therefore \quad \left(\frac{a}{x-h}\right)^2 + \left(\frac{b}{y-k}\right)^2 = 1$

40. (b) $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$ squaring both sides

$\sin^2 A + 6\cos^2 A - 2\sqrt{6}\sin A\cos A = 7\cos^2 A$

$\Rightarrow \sin^2 A = \cos^2 A + 2\sqrt{6}\sin A\cos A$

Adding $6\sin^2 A$ both sids

$7\sin^2 A = \cos^2 A + 2\sqrt{6}\sin A\cos A + 6\sin^2 A$

or, $7\sin^2 A = (\cos A + \sqrt{6}\sin A)^2$

$\therefore \quad \cos A + \sqrt{6}\sin A = \pm\sqrt{7}\sin A$

sum of roots $\sin\theta + \cos\theta = -\frac{b}{a}$

product of roots $\sin\theta \cdot \cos\theta = \frac{c}{a}$

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta \cdot \cos\theta$

$\therefore \left(\frac{-b}{a}\right)^2 = 1 + 2\frac{c}{a}$

$\therefore a^2 - b^2 + 2ac = 0$

or $a^2 + 2ac = b^2$

or Adding c^2, $(a + c)^2 = b^2 + c^2$

2. (a) $\because$ Maximum value of $\cos x$ is 1

$\therefore$ Maximum value of $\sin(\cos x)$ is $\sin 1$

3. (d) Given $\cos x + \cos^2 x = 1$

$\Rightarrow \cos x = 1 - \cos^2 x = \sin^2 x$ ... (i)

Now, $\sin^{12}x + 3\sin^{10}x + 3\sin^8 x + \sin^6 x - 1$

$= \cos^6 x + 3\cos^5 x + 3\cos^4 x + \cos^3 x - 1$ $(\because \sin^2 x = \cos x)$

$= (\cos^2 x + \cos x)^3 - 1 = 1^3 - 1 = 0$ $(\because \cos^2 x + \cos x = 1)$

4. (b) Given $3\sin\theta + 5\cos\theta = 5$, Squaring

$9\sin^2\theta + 25\cos^2\theta + 30\sin\theta\cos\theta = 25$

or, $9(1 - \cos^2\theta) + 25(1 - \sin^2\theta) + 30\sin\theta\cos\theta = 25$

or, $9 + 25 - (9\cos^2\theta + 25\sin^2\theta - 30\sin\theta\cos\theta) = 25$

or, $9 = (5\sin\theta - 3\cos\theta)^2$

$\therefore$ $5\sin\theta - 3\cos\theta = 3$

5. (c) $\because \sec^2\theta - \tan^2\theta = 1$

$\therefore (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\therefore (\sec\theta - \tan\theta)\, p = 1$

$\therefore \sec\theta - \tan\theta = \frac{1}{p}$ ... (i)

and $\sec\theta + \tan\theta = p$ ... (ii)

Adding $2\sec\theta = \frac{1}{p} + p = \frac{1+p^2}{p}$ or, $\sec\theta = \frac{1+p^2}{2p}$

6. (c) $(\sin\theta - \cos\theta) = \sqrt{2}\cos\theta$

Squaring $\sin^2\theta + \cos^2\theta - 2\sin\theta.\cos\theta = 2\cos^2\theta$

or, $\sin^2\theta = \cos^2\theta + 2\sin\theta\cos\theta$

Adding $\sin^2\theta$ both sids

$2\sin^2\theta = \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta$

or, $2\sin^2\theta = (\sin\theta + \cos\theta)^2$

$\therefore$ $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$

47. (a) Given $\tan 4\theta \tan 5\theta = 1$

or, $\tan 4\theta = \frac{1}{\tan 5\theta} = \cot 5\theta$ or, $\tan 4\theta = \tan(90^\circ - 5\theta)$

or, $4\theta = 90^\circ - 5\theta$ or, $9\theta = 90^\circ$

or, $\theta = 10^\circ$

$\therefore \sin(5\theta - 2\theta) = \sin 3\theta = \sin 30^\circ = \frac{1}{2}$

48. (a) $\sec x = \operatorname{cosec} y = \sec(90^\circ - y)$

$\therefore x = 90^\circ - y \Rightarrow x + y = 90^\circ$

$\therefore \operatorname{cosec}(x + y) = \operatorname{cosec} 90^\circ = 1$

49. (d) $\tan 2\theta = \cot(\theta - 18^\circ) = \tan(90^\circ - (\theta - 18^\circ))$

$\therefore 2\theta = 90^\circ - \theta + 18^\circ$

or, $3\theta = 108^\circ$

or, $\theta = 36^\circ$ or, $\frac{5\theta}{4} = 45^\circ$

Hence $\sin\left(\frac{5\theta}{4}\right) + \cos\left(\frac{5\theta}{4}\right) = \sin 45^\circ + \cos 45^\circ = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

50. (c) Solve as Question number 46

51. (b) $k^2 = (1 - \sin\alpha)(1 - \sin\beta)(1 - \sin\gamma)(1 + \sin\alpha)(1 + \sin\beta)(1 + \sin\gamma)$

$= (1 - \sin^2\alpha)(1 - \sin^2\beta)(1 - \sin^2\gamma)$

$= \cos^2\alpha \cos^2\beta \cos^2\gamma$

52. (c) Solve as Question number 51 and use $\sec^2\theta - \tan^2\theta = 1$

53. (b) Required value $= (1 + \cot\theta)^2 - (\operatorname{cosec}^2\theta)$

$= 1 + \cot^2\theta + 2\cot\theta - \operatorname{cosec}^2\theta$

$= \operatorname{cosec}^2\theta + 2\cot\theta - \operatorname{cosec}^2\theta = 2\cot\theta$

54. (d) See solved example 8.

55. (b) Do as in solved example 3

56. (c) Take L C M and apply $\sec^2\theta - \tan^2\theta = 1$

57. (c) $\sec^2\theta + \operatorname{cosec}^2\theta = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}$

$= \frac{1}{\cos^2\theta \sin^2\theta} = \sec^2\theta \operatorname{cosec}^2\theta$

58. (a) See solved example -4

59. (b) $\sec\theta \operatorname{cosec}\theta = \frac{1}{\cos\theta \sin\theta} = \frac{\cos^2\theta + \sin^2\theta}{\cos\theta \sin\theta} = \cot\theta + \tan\theta$

60. (d) $\tan^4 A + \tan^2 A = (\sec^2 A - 1)^2 + (\sec^2 A - 1)$

$= \sec^4 A - 2\sec^2 A + 1 + \sec^2 A - 1$

$= \sec^4 A - \sec^2 A$

61. (d) Givan expression $= \sin^2\theta + \cos^2\theta + \tan^2\theta + \cot^2\theta + \sec^2\theta + \text{cosec}^2\theta$

$= (\sin^2\theta + \cos^2\theta) + \tan^2\theta + \cot^2\theta + (1 + \tan^2\theta) + (1 + \cot^2\theta)$

$= 1 + 1 + 1 + 2\,(\tan^2\theta + \cot^2\theta)$

$= 3 + 2\,\{(\tan\theta - \cot\theta)^2 + 2\}$

But $(\tan\theta - \cot\theta)^2 \geq 0$

$\therefore$ Given expression $\geq 3 + 2(0 + 2) \Rightarrow$ Given expression ≥ 7

62. (b) $\cos^2\alpha + \cos^2\beta = 2$

It is possible only when each of $\cos^2\alpha$ and $\cos^2\beta$ is equal to 1 as their individual value cannot exeed 1.

$\therefore$ $\cos^2\alpha = 1$ and $\cos^2\beta = 1$ $\Rightarrow$ $\alpha = \beta = 0^\circ$

Hence $\tan^3\alpha + \sin^5\beta$

$= (\tan 0^\circ)^3 + (\sin 0^\circ)^5 = 0$

63. (c) $A = \tan 11^\circ \cdot \tan 29^\circ$ ... (i)

$B = 2\cot 61^\circ \cdot \cot 79^\circ$

$= 2\cot(90^\circ - 29^\circ)\cdot\cot(90^\circ - 11^\circ)$

$= 2\tan 29^\circ \cdot \tan 11^\circ = 2\tan 11^\circ \cdot \tan 29 = 2A$

64. (c) $(\sec A - \cos A)^2 + (\text{cosec}A - \sin A)^2 - (\cot A - \tan A)^2$

$= \sec^2 A + \cos^2 A - 2\sec A\cdot\cos A + \text{cosec}^2 A + \sin^2 A - 2\sin A\cdot\text{cosec}A -$
$\cot^2 A - \tan^2 A + 2\cot A\cdot\tan A$

$= \sin^2 A + \cos^2 A + \sec^2 A - \tan^2 A + \text{cosec}^2 A - \cot^2 A - 2 - 2 + 2$

$= 1 + 1 + 1 - 2 = 1$

65. (a) $\sin^2 1^\circ + \sin^2 5^\circ + \sin^2 9^\circ + \ldots + \sin^2 89^\circ$

$= \sin^2 1^\circ + \sin^2 89^\circ + \sin^2 5^\circ + \sin^2 85^\circ + \ldots + \sin^2 41^\circ + \sin^2 49^\circ + \sin^2 45^\circ$

$= \sin^2 1^\circ + \sin^2(90 - 1)^\circ + \sin^2 5^\circ + \sin^2(90 - 5)^\circ + \ldots\ldots + \sin^2 41^\circ$
$+ \sin^2(90^\circ - 41^\circ) + \sin^2 45^\circ$

$= (\sin^2 1^\circ + \cos^2 1^\circ) + (\sin^2 5^\circ + \cos^2 5^\circ) + \ldots\ldots + \sin^2 45^\circ$

$= 1 + 1 + \ldots\ldots 11 \text{ to term} + \left(\frac{1}{\sqrt{2}}\right)^2 \quad = 11 + \frac{1}{2} = 11\frac{1}{2}$

66. (a) $\cot 18^\circ\left(\cot 72^\circ \cos^2 22^\circ + \dfrac{1}{\tan 72^\circ \sec^2 68^\circ}\right)$

$= \cot 18^\circ(\cot 72^\circ \cos^2 22^\circ + \cot 72^\circ \cos^2 68^\circ)$

$= \cot 18^\circ \cot 72^\circ[\cos^2 22^\circ + \cos^2(90 - 22)^\circ]$

$= \cot 18^\circ \cot 72^\circ[\cos^2 22^\circ + \sin^2 22^\circ]$

$= \cot 18^\circ \cot(90 - 18)^\circ \times 1 = \cot 18^\circ \tan 18^\circ = 1$

67. (a) $\because \quad \cos(30^\circ + \alpha)$

$\Rightarrow \quad \dfrac{\sin\alpha}{\sin(90^\circ - 30 - \alpha)} = 1$

$\Rightarrow \quad \dfrac{\sin\alpha}{\sin(60^\circ - \alpha)} = 1$

$\Rightarrow \quad \sin\alpha = \sin(60^\circ - \alpha)$

$\Rightarrow \quad 2\alpha = 60^\circ \qquad \therefore \quad \alpha \Rightarrow 30^\circ$

$\therefore \quad \sin\alpha + \cos2\alpha = \sin30^\circ + \cos60^\circ = \frac{1}{2} + \frac{1}{2} = 1$

68. (a) $\because \quad \cos^4\theta - \sin^4\theta = \frac{2}{3}$

$\Rightarrow \quad (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}$

$\Rightarrow \quad \cos^2\theta - \sin^2\theta = \frac{2}{3}$

$\Rightarrow \quad \cos^2\theta - (1 - \cos^2\theta) = \frac{2}{3}$

$\Rightarrow \quad 2\cos^2\theta - 1 = \frac{2}{3} \qquad \Rightarrow 2\cos^2\theta = \frac{2}{3} + 1 = \frac{5}{3}$

69. (b) $\because \quad \cos^2\theta + \cos^4\theta = 1$

$\Rightarrow \quad \cos^2\theta + \cos^4\theta = \sin^2\theta + \cos^2\theta$

$\therefore \quad \cos^4\theta = \sin^2\theta \Rightarrow \cos^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} \quad \Rightarrow \cos^2\theta = \tan^2\theta$

Hence $\tan^2\theta + \tan^4\theta = \cos^2\theta + \cos^4\theta = 1$

70. (b) $\because \quad \tan\theta + \cot\theta = 2$

$\Rightarrow \quad \tan\theta + \dfrac{1}{\tan\theta} = 2$

$\Rightarrow \quad \tan^2\theta + 1 = 2\tan\theta \qquad \Rightarrow \quad (\tan\theta - 1)^2 = 0$

$\Rightarrow \quad \tan\theta = 1 \qquad \Rightarrow \quad \cot\theta = 1$

$\therefore \quad \tan^5\theta = \cot^{10}\theta = 1 + 1 = 2$

71. (c) $(\sin^2 1^\circ + \sin^2 89^\circ) + (\tan^2 3^\circ + \tan^2 87^\circ) + (\sin^2 5^\circ + \sin^2 85^\circ) + \ldots\ldots$
$(\tan^2 43^\circ + \tan^2 47^\circ) + \sin^2 45^\circ$

$= 1 + 1 \ldots\ldots$ to 22 terms $+ \frac{1}{2}$

$(\because \quad \sin^2 1^\circ + \sin^2 89^\circ = \sin^2 1^\circ + \cos^2 1^\circ = 1$ etc)

$= 22 + \frac{1}{2} = 22\frac{1}{2}$

72. (c) $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$, Squaring $\quad 4\cos^2\theta + \sin^2\theta - 4\cos\theta\sin\theta = \frac{1}{2}$

$\Rightarrow \quad 4(1 - \sin^2\theta) + (1 - \cos^2\theta) - 4\cos\theta\cdot\sin\theta = \frac{1}{2}$

$\Rightarrow \quad 5 - \frac{1}{2} = 4\sin^2\theta + \cos^2\theta + 4\cos\theta\cdot\sin\theta$

$\Rightarrow \quad \frac{9}{2} = (2\sin\theta + \cos\theta)^2 \qquad \therefore \quad 2\sin\theta + \cos\theta = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}$

73. (c) $\because \dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$

$\Rightarrow \sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$

$\Rightarrow 4\cos\theta = 2\sin\theta$

$\Rightarrow \tan\theta = 2 = \dfrac{p}{b}$

$\therefore \sin^4\theta - \cos^4\theta$

$$= \left(\frac{p}{h}\right)^4 - \left(\frac{b}{h}\right)^4 = \frac{p^4 - b^4}{h^4} = \frac{p^4 - b^4}{\left(p^2 + b^2\right)^2} = \frac{16-1}{5^2} = \frac{15}{25} = \frac{3}{5}$$

74. (a) $\because \sec^2\theta + \tan^2\theta = 7$

$\Rightarrow 1 + \tan^2\theta + \tan^2\theta = 7$

$\Rightarrow 2\tan^2\theta = 7 - 1 = 6$

$\Rightarrow \tan^2\theta = 3$

$\Rightarrow \tan\theta = \sqrt{3}$ $\qquad \therefore \theta = 60^\circ$

75. (d) $(\sec x \cdot \sec y + \tan x \cdot \tan y)^2 - (\sec x \cdot \tan y + \tan x \cdot \sec y)^2$

$$= \left(\frac{1}{\cos x \cdot \cos y} + \frac{\sin x \cdot \sin y}{\cos x \cdot \cos y}\right)^2 - \left(\frac{1 \cdot \sin y}{\cos x \cdot \cos y} + \frac{\sin x}{\cos x \cdot \cos y}\right)^2$$

$$= \left(\frac{1 + \sin x \cdot \sin y}{\cos x \cdot \cos y}\right)^2 - \left(\frac{\sin x + \sin y}{\cos x \cdot \cos y}\right)^2$$

$$= \frac{1 + \sin^2 x \sin^2 y + 2\sin x \cdot \sin y - \sin^2 x - \sin^2 y - 2\sin x \cdot \sin y}{\cos^2 x \cdot \cos^2 y}$$

$$= \frac{1 + \sin^2 x \cdot \sin^2 y - \sin^2 x - \sin^2 y}{\cos^2 x \cdot \cos^2 y} = \frac{\left(1 - \sin^2 x\right)\left(1 - \sin^2 y\right)}{\cos^2 x \cos^2 y}$$

$$= \frac{\cos^2 x \cdot \cos^2 y}{\cos^2 x \cdot \cos^2 y} = 1$$

76. (c) $\because \dfrac{\cos^2\theta}{\cot^2\theta - \cos^2\theta} = 3$ $\qquad \Rightarrow \cos^2\theta = 3\cot^2\theta - 3\cos^2\theta$

$\Rightarrow 4\cos^2\theta = \dfrac{3\cos^2\theta}{\sin^2\theta}$ $\qquad \Rightarrow \sin\theta = \dfrac{\sqrt{3}}{2}$ $\quad \therefore \theta = 60^\circ$

77. (a) $(\sin\theta - \cos\theta)^2 = \dfrac{49}{169}$

$\Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta \cdot \cos\theta = \dfrac{49}{169} \Rightarrow 2\sin\theta\cos\theta = 1 - \dfrac{49}{169} = \dfrac{120}{169}$

Now $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta \cdot \cos\theta = 1 + \dfrac{120}{169} = \dfrac{289}{169}$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{\dfrac{287}{169}} = \dfrac{17}{13}$

1. If $4x = \sec\theta$ and $\frac{4}{x} = \tan\theta$ then the value of $8\left(x^2 - \frac{1}{x^2}\right)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

[SSC Tier-I 2012]

2. $2 - \cos^2\theta = 3\sin\theta\cos\theta$, $\sin\theta \neq \cos\theta$ then the value of $\tan\theta$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 0

[SSC Tier-I 2012]

3. If $\sin\theta + \cos\theta = \sqrt{2}\cos(90° - \theta)$ then the value of $\cot\theta$ is

(a) $\sqrt{2}$ (b) $\sqrt{2} - 1$ (c) $\sqrt{2} + 1$ (d) 0

[SSC Tier-I 2012]

4. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$; $\sin\theta \neq 0$, $\cos\theta \neq 0$ then the value of $x^2 + y^2$ is

(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

[SSC Tier-I 2012]

5. If A and B are complementary angle then the value of $\sin A\cos B + \cos A\sin B - \tan A\tan B + \sec^2 A - \cot^2 B$ is

(a) 1 (b) –1 (c) 2 (d) 0

[SSC Tier-I 2012]

6. Minimum value of $2\sin^2\theta + 3\cos^2\theta$ is

(a) 1 (b) 2 (c) 3 (d) 5

[SSC Tier-I 2012]

7. $\sec^4\theta - \sec^2\theta$ equals

(a) $\cos^4\theta - \cos^2\theta$ (b) $\cos^2\theta - \cos^4\theta$ (c) $\tan^2\theta - \tan^4\theta$ (d) $\tan^2\theta + \tan^4\theta$

[SSC Tier-I 2012]

8. If $\cos A + \cos^2 A = 1$ then the value $\sin^2 A + \sin^4 A$ is

(a) 0 (b) –1 (c) 1 (d) $\frac{1}{2}$

[SSC Tier-I 2012]

9. If $\sec\theta - \operatorname{cosec}\theta = 0$ then the value of $(\sec\theta + \operatorname{cosec}\theta)$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) 0 (d) $2\sqrt{2}$

[SSC Tier-I 2012]

10. If $P\sin\theta = \sqrt{3}$ and $P\cos\theta = 1$ then the value of P is

(a) $\frac{1}{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{-1}{\sqrt{3}}$ (d) 2

[SSC Tier-I 2012]

11. If $u_n = \cos^n\alpha + \sin^n\alpha$ then the value of $2u_6 - 3u_4 + 1$ is

(a) 1 (b) 4 (c) 6 (d) 0

12. If $\sin(x + y) = \cos[3(x + y)]$ then the value of $\tan[2(x + y)]$ is

(a) $\sqrt{3}$ (b) 1 (c) 0 (d) $\frac{1}{\sqrt{3}}$

[SSC Tier-I 2012]

13. The value of $(1 + \sec 20° + \cot 70°)(1 - \operatorname{cosec} 20° + \tan 70°)$ is
(a) 0 (b) -1 (c) 2 (d) 1
[SSC Tier-I 2012]

14. If $0 \le \alpha \le \frac{\pi}{2}$ and $2\sin\alpha + 15\cos^2\alpha = 7$ then then value of $\cot\alpha$ is
(a) $\frac{1}{2}$ (b) $\frac{5}{4}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$
[SSC Tier-I 2012]

15. The value of θ $[0° < \theta < 90°]$, for which $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$, is
(a) 30° (b) 45° (c) 60° (d) None of these
[SSC Tier-I 2012]

16. If $\sec\theta + \tan\theta = 2$, then the value of $\sec\theta$ is
(a) $\frac{7}{4}$ (b) $\frac{7}{2}$ (c) $\frac{5}{2}$ (d) $\frac{5}{4}$
[SSC Tier-I 2012]

17. If $\tan 2\theta \cdot \tan 4\theta = 1$ then the value of $\tan 3\theta$ is
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2
[SSC Tier-I 2012]

18. If $\cos\theta + \sec\theta = \sqrt{3}$, then the value of $\cos^3\theta + \sec^3\theta$ is
(a) 0 (b) 1 (c) -1 (d) $\sqrt{3}$
[SSC Tier-I 2012]

19. If $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\operatorname{cosec}\theta = 3$, then the relation between x and y is
(a) $2x^2 + y^2 = 2$ (b) $x^2 + 4y^2 = 4$ (c) $x^2 + 4y^2 = 1$ (d) $4x^2 + y^2 = 4$
[SSC Tier-I 2012]

20. If $\sec\theta + \tan\theta = \sqrt{3}$, then the positive value of $\sin\theta$ is
(a) 0 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1
[SSC Tier-I 2012]

21. If $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$, then the value of $\frac{\cos^4\beta}{\cos^2\alpha} + \frac{\sin^4\beta}{\sin^2\alpha}$ is
(a) 4 (b) 0 (c) $\frac{1}{8}$ (d) 1
[SSC Tier-I 2012]

22. Value of $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$ $\left(\text{where } \theta \ne \frac{\pi}{2}\right)$ is
(a) $\frac{1+\sin\theta}{\cos\theta}$ (b) $\frac{1-\sin\theta}{\cos\theta}$ (c) $\frac{1-\cos\theta}{\sin\theta}$ (d) $\frac{1+\cos\theta}{\sin\theta}$
[SSC Tier-I 2012]

23. If x, y are acute positive angle $x + y < 90°$ and $\sin(2x - 20°) = \cos(2y + 20°)$ then the value of $\sec(x + y)$ is
(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) 0
[SSC Tier-I 2012]

24. Minimum value of $(4\sec^2\theta + 9\,\text{cosec}^2\theta)$ is

(a) 1 (b) 19 (c) 25 (d) 7

[SSC Tier-I 2012]

25. If $\tan(x+y)\tan(x-y) = 1$ then the value of $\tan\left(\frac{2x}{3}\right)$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 1

[SSC Tier-I 2012]

26. If $x = \text{cosec}\theta - \sin\theta$ and $y = \sec\theta - \cos\theta$ then the value of $x^2y^2(x^2 + y^2 + 3)$ is

(a) 0 (b) 1 (c) 2 (d) 3

[SSC Tier-I 2012]

27. If $\sin\theta + \sin^2\theta = 1$ then the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$ is

(a) 0 (b) 1 (c) −1 (d) 2

[SSC Tier-I 2012]

28. If $\tan(x+y)\tan(x-y) = 1$ then the value of $\tan x$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

[SSC Tier-I 2012]

29. If $\cot A + \text{cosec} A = 3$ and A is an acute angle then the value of $\cos A$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{5}$

[SSC Tier-I 2012]

30. The simplified value of $1 - \frac{\sin^2 A}{1+\cos A} + \frac{1+\cos A}{\sin A} - \frac{\sin A}{1-\cos A}$ is

(a) 0 (b) 1 (c) $\sin A$ (d) $\cos A$

[SSC Tier-I 2012]

31. If α is an acute angle and $2\sin\alpha + 15\cos^2\alpha = 7$ then value of $\cot\alpha$ is

(a) $\frac{4}{3}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{3}{4}$

[SSC Tier-I 2012]

32. If $\tan\theta - \cot\theta = a$ and $\cos\theta - \sin\theta = b$ then value of $(a^2+4)(b^2-1)^2$ is

(a) 1 (b) 2 (c) 3 (d) 4

[SSC Tier-I 2012]

33. If $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$ then the value of $\tan\theta$ is

(a) $\frac{1}{2}(a^2 - b^2)$ (b) $\frac{1}{2ab}(a^2 - b^2)$ (c) $\frac{1}{2}(a^2 + b^2)$ (d) $\frac{1}{2ab}(a^2 + b^2)$

[SSC Tier-I 2012]

34. $\sin^2 21^\circ + \sin^2 69^\circ$ is equal to

(a) $2\sin^2 21^\circ$ (b) $2\sin^2 69^\circ$ (c) 1 (d) 0

35. $\sin^2 5^\circ + \sin^2 25^\circ + \sin^2 45^\circ + \sin^2 65^\circ + \sin^2 85^\circ$ is equal to

(a) 2.5 (b) 3 (c) 1.5 (d) 2

[SSC Tier-I 2012]

$3\sin^2\alpha + 7$ ... value of $\tan\alpha$ is

(a) $\sqrt{3}$ (b) $\sqrt{6}$ (c) $\sqrt{2}$ (d) $\sqrt{5}$

[SSC Tier-I 2012]

For all real values of α, $x = \cos^4\alpha + \sin^2\alpha$, then range of x is

(a) $\frac{3}{4} \le x \le 1$ (b) $\frac{3}{4} \le x \le \frac{13}{15}$ (c) $\frac{13}{16} \le x \le 1$ (d) $\frac{1}{2} \le x \le 2$

$\sin^2\alpha = \cos^3\alpha$ then the value of $(\cot^6\alpha - \cot^2\alpha)$ is

(a) 1 (b) 0 (c) -1 (d) 2

[SSC Tier-I 2012]

Answers–11B

(a)	2. (c)	3. (b)	4. (a)	5. (a)	6. (b)	7. (d)	8. (c)
(d)	10. (d)	11. (d)	12. (b)	13. (c)	14. (c)	15. (c)	16. (d)
(b)	18. (a)	19. (b)	20. (b)	21. (d)	22. (a)	23. (a)	24. (c)
(a)	26. (b)	27. (a)	28. (a)	29. (d)	30. (d)	31. (d)	32. (d)
(b)	34. (c)	35. (a)	36. (a)	37. (a)	38. (a)		

Explanation

(a) $\because$ $\sec^2\theta - \tan^2\theta = 1$ $\quad\therefore$ $(4x)^2 - \left(\frac{4}{x}\right)^2 = 1$

$\Rightarrow 16\left(x^2 - \frac{1}{x^2}\right) = 1 \quad \Rightarrow 8\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{2}$

(c) $2 - \cos^2\theta = 3\sin\theta\cos\theta$

Dividing both sides by $\cos^2\theta$

$2\sec^2\theta - 1 = 3\tan\theta$

or, $2(1 + \tan^2\theta) - 1 = 3\tan\theta$

or, $2\tan^2\theta - 3\tan\theta + 1 = 0$

$\Rightarrow$ $(2\tan\theta - 1)(\tan\theta - 1) = 0$

$\Rightarrow$ $\tan\theta = \frac{1}{2}, 1$

but $\sin\theta \ne \cos\theta$ $\tan\theta \ne 1$

$\therefore$ $\tan\theta = \frac{1}{2}$

(b) $\sin\theta + \cos\theta = \sqrt{2}\cos(90° - \theta)$

or, $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$

or, $\cos\theta = \left(\sqrt{2} - 1\right)\sin\theta$

Dividing both sides by $\sin\theta$

$\cot\theta = \sqrt{2} - 1$

(a) $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

or, $x\sin\theta\sin^2\theta + y\cos\theta\cos^2\theta = \sin\theta\cos\theta$

or, $y\cos\theta\sin^2\theta + x\sin\theta\cos^2\theta = \sin\theta\cos\theta$ $\quad(\because x\sin\theta = y\cos\theta)$

or, $y\sin\theta + x\cos\theta = 1$

From second relation $x\sin\theta - y\cos\theta = 0$

Squaring and adding (1) and (2)

$(y\sin\theta + x\cos\theta)^2 + (x\sin\theta - y\cos\theta)^2 = 1^2 + 0^2$

$\Rightarrow$ $y^2(\sin^2\theta + \cos^2\theta) + x^2(\cos^2\theta + \sin^2\theta) = 1$

[$2xy\sin\theta\cos\theta$ will be cancelled out]

Second method (Trial Method) :

We can guess from $x\sin\theta = y\cos\theta$ and that $x = \cos\theta$ and $y = \sin\theta$

$x = \cos\theta$ and $y = \sin\theta$

If also satisfies $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\therefore$ $x^2 + y^2 = 1$

(Third Method) :

Let $x\sin\theta = y\cos\theta = k$ then $\sin\theta = \frac{k}{x}$ and $\cos\theta = \frac{k}{y}$

Now from $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{k}{x}\right)^2 + \left(\frac{k}{y}\right)^2 = 1$$

$\Rightarrow$ $k^2\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = 1$ $\qquad\Rightarrow$ $k^2\left(\frac{y^2 + x^2}{x^2y^2}\right) = 1$

$\Rightarrow$ $k^2(x^2 + y^2) = x^2y^2$ ...(i)

Again from $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$$x\left(\frac{k}{x}\right)^3 + y\left(\frac{k}{y}\right)^3 = \frac{k}{x}\cdot\frac{k}{y}$$

$k^3\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = \frac{k^2}{xy}$ $\qquad\Rightarrow$ $k^3\left(\frac{y^2 + x^2}{x^2y^2}\right) = \frac{k^2}{xy}$

$k(x^2 + y^2) = xy$ ...(ii)

Putting $xy = k(x^2 + y^2)$ in equation (i)

$k^2(x^2 + y^2) = k^2(x^2 + y^2)^2$

$\Rightarrow$ $1 = x^2 + y^2$

5. (a) Given $A + B = 90°$ or, $B = 90° - A$

$\therefore$ $\cos B = \sin A$, $\sin B = \cos A$, $\tan B = \cot A$

and $\cot B = \tan A$

Hence given expression

$= \sin A \sin A + \cos A \cos A - \tan A \cot A + \sec^2 A - \tan^2 A$

$= \sin^2 A + \cos^2 A - 1 + 1 = 1 - 1 + 1 = 1$

6. (b) $2\sin^2\theta + 3\cos^2\theta = 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta = 2 + \cos^2\theta$

Since minimum value of $\cos^2\theta$ is zero

$\therefore$ Minimum value of given expression $= 2 + 0 = 2$

7. (d) $\sec^4\theta - \sec^2\theta = \sec^2\theta\,(\sec^2\theta - 1)$

$= \sec^2\theta\tan^2\theta = (1 + \tan^2\theta)\tan^2\theta = \tan^2\theta + \tan^4\theta$

8. (c) $\cos A + \cos^2 A = 1 \Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A$

Now, $\sin^2 A + \sin^4 A = \cos A + \cos^2 A = 1$ $\quad (\because \sin^2 A = \cos A)$

9. (d) $\sec\theta = \operatorname{cosec}\theta \Rightarrow \theta = 45°$ $\quad (\because \sec 45° = \sqrt{2}, \operatorname{cosec} 45° = \sqrt{2})$

$\therefore$ $\sec\theta + \operatorname{cosec}\theta = \sec 45° + \operatorname{cosec} 45° = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

10. (d) Squaring and adding $(P\sin\theta)^2 + (P\cos\theta)^2 = \left(\sqrt{3}\right)^2 + 1^2$

$\Rightarrow$ $P^2(\sin^2\theta + \cos^2\theta) = 3 + 1$ $\quad \Rightarrow P^2 = 4$ $\quad \therefore P = 2$

11. (d) $2u_6 - 3u_4 + 1 = 2(\cos^6\alpha + \sin^6\alpha) - 3(\cos^4\alpha + \sin^4\alpha) + 1$

$= 2\{(\cos^2\alpha + \sin^2\alpha)^3 - 3\sin^2\alpha\cos^2\alpha(\sin^2\alpha + \cos^2\alpha)\}$

$- 3\{(\cos^2\alpha + \sin^2\alpha)^2 - 2\cos^2\alpha\sin^2\alpha\} + 1$

$(\because a^3 + b^3 = (a+b)^3 - 3ab(a+b),\ a^2 + b^2 = (a+b)^2 - 2ab,$

here $a = \cos^2\alpha$ and $b = \sin^2\alpha)$

$= 2\{1 - 3\sin^2\alpha\cos^2\alpha\cdot 1\} - 3\{1 - 2\sin^2\alpha\cos^2\alpha\} + 1$

$= 2 - 6\sin^2\alpha\cos^2\alpha - 3 + 6\sin^2\alpha\cos^2\alpha + 1$

$2u_6 - 3u_4 + 1 = 2(\cos^6\alpha + \sin^6\alpha) - 3(\cos^4\alpha + \sin^4\alpha) + 1$

Trick, since all the options are idependent of α, putting $\alpha = 0$

$2u_6 - 3u_4 + 1 = 2(\cos^6 0 + \sin^6 0) - 3(\cos^4 0 + \sin^4 0) + 1$

$= 2(1 + 0) - 3(1 + 0) + 1 = 0$

We can put any value of α

12. (b) $\sin(x + y) = \cos(3(x + y) = \sin\left(\frac{\pi}{2} - 3\left(x + y\right)\right)$

$\therefore$ $(x + y) = 90° - 3(x + y)$

or, $4(x + y) = 90°$

or, $2(x + y) = 45°$

$\therefore$ $\tan(2(x + y) = \tan 45° = 1$

13. (c) $(1 + \sec 20° + \cot 70°)(1 - \operatorname{cosec} 20° + \tan 70°)$

$= (1 + \sec 20° + \tan 20°)(1 - \operatorname{cosec} 20° + \cot 20°)$

$= \left(1 + \frac{1 + \sin 20°}{\cos 20°}\right)\left(1 - \frac{1 - \cos 20°}{\sin 20°}\right)$

$= \left(\frac{\cos 20° + 1 + \sin 20°}{\cos 20°}\right)\left(\frac{\sin 20° - 1 + \cos 20°}{\sin 20°}\right)$

3...

$$= \frac{(\cos 20^\circ + \sin 20^\circ)^2 - 1}{\cos 20^\circ \sin 20^\circ}$$

$$= \frac{1 + 2\sin 20^\circ \cos 20^\circ - 1}{\cos 20^\circ \sin 20^\circ}$$

$$= \frac{2\sin 20^\circ \cos 20^\circ}{\cos 20^\circ \sin 20^\circ} = 2 \qquad (\because \sin^2 20^\circ + \cos^2 20^\circ = 1)$$

14. (c) $2\sin\alpha + 15\cos^2\alpha = 7$

$\Rightarrow \; 2\sin\alpha + 15(1 - \sin^2\alpha) = 7$

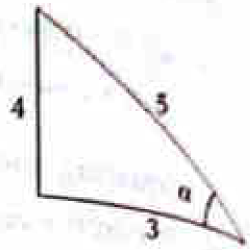
$\Rightarrow \; 15\sin^2\alpha - 2\sin\alpha - 8 = 0$

$\Rightarrow \; 15\sin^2\alpha - 12\sin\alpha + 10\sin\alpha - 8 = 0$

$\Rightarrow \; 3\sin\alpha(5\sin\alpha - 4) + 2(5\sin\alpha - 4) = 0$

$\Rightarrow \; (3\sin\alpha + 2)(5\sin\alpha - 4) = 0$

$\therefore \; \sin\alpha = \frac{-2}{3}, \frac{4}{5}$



But $0 \le \alpha \le \frac{\pi}{2}$

$\therefore \; \sin\alpha = \frac{4}{5} \Rightarrow \cot\alpha = \frac{3}{4}$

15. (c) $\frac{\cos\theta}{1 - \sin\theta} + \frac{\cos\theta}{1 + \sin\theta} = 4$

or, $\frac{\cos\theta(1 + \sin\theta) + \cos\theta(1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)} = 4$

or, $\frac{2\cos\theta}{1 - \sin^2\theta} = 4$

or, $\frac{\cos\theta}{\cos^2\theta} = 2$

$\Rightarrow \; \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

16. (d) Given $\sec\theta + \tan\theta = 2$

$\because \; \sec^2\theta - \tan^2\theta = 1$

$\therefore \; (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$2(\sec\theta - \tan\theta) = 1$

or, $\sec\theta - \tan\theta = \frac{1}{2}$

Adding (1) and (2),

$2\sec\theta = 2 + \frac{1}{2} = \frac{5}{2}$

$\therefore \; \sec\theta = \frac{5}{4}$

17. (b) $\tan 2\theta \tan 4\theta = 1$

or, $\tan 2\theta = \dfrac{1}{\tan 4\theta} = \cot 4\theta$

or, $\tan 2\theta = \tan(90° - 4\theta)$

or, $2\theta = 90° - 4\theta$

or, $6\theta = 90°$

or, $\theta = 15°$

$\therefore$ $\tan 3\theta = \tan 45° = 1$

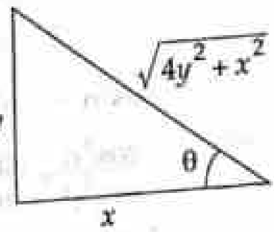
18. (a) $\cos\theta + \sec\theta = \sqrt{3}$

$\cos^3\theta + \sec^3\theta + 3(\cos\theta + \sec\theta)(\cos\theta \cdot \sec\theta) = 3\sqrt{3}$

or, $\cos^3\theta + \sec^3\theta + 3\sqrt{3} \cdot 1 = 3\sqrt{3}$

or, $\cos^3\theta + \sec^3\theta = 0$

19. (b) From first relation, $\tan\theta = \dfrac{2y}{x}$



Here $p = 2y$, $b = x$

$\therefore$ $h = \sqrt{4y^2 + x^2}$

From second relation, $2x \sec\theta - y \operatorname{cosec}\theta = 3$

or, $2x \dfrac{\sqrt{4y^2 + x^2}}{x} - \dfrac{y\sqrt{4y^2 + x^2}}{2y} = 3$

or, $\left(2 - \dfrac{1}{2}\right)\sqrt{4y^2 + x^2} = 3$ or, $\dfrac{3}{2}\sqrt{4y^2 + x^2} = 3$

or, $\sqrt{4y^2 + x^2} = 2$ or, $x^2 + 4y^2 = 4$

... (i)

20. (b) Given, $\sec\theta + \tan\theta = \sqrt{3}$

$\because$ $\sec^2\theta - \tan^2\theta = 1$

$\therefore$ $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

or, $(\sec\theta - \tan\theta)\sqrt{3} = 1$ ... (ii)

or, $\sec\theta - \tan\theta = \dfrac{1}{\sqrt{3}}$

(i) and (ii) adding $2\sec\theta = \sqrt{3} + \dfrac{1}{\sqrt{3}}$

$\Rightarrow \sec\theta = \dfrac{1}{2}\left(\dfrac{3+1}{\sqrt{3}}\right) = \dfrac{2}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

$\therefore \sin\theta = \dfrac{1}{2}$

21. (d) $\dfrac{\cos^4\alpha}{\cos^2\beta}+\dfrac{\sin^4\alpha}{\sin^2\beta}=1=\sin^2\alpha+\cos^2\alpha$

$\therefore\ \dfrac{\cos^4\alpha}{\cos^2\beta}-\cos^2\alpha=\sin^2\alpha-\dfrac{\sin^4\alpha}{\sin^2\beta}$

or, $\dfrac{\cos^4\alpha-\cos^2\alpha\cos^2\beta}{\cos^2\beta}=\dfrac{\sin^2\alpha\sin^2\beta-\sin^4\alpha}{\sin^2\beta}$

or, $\dfrac{\cos^2\alpha\left(\cos^2\alpha-\cos^2\beta\right)}{\cos^2\beta}=\dfrac{\sin^2\alpha\left(\sin^2\beta-\sin^2\alpha\right)}{\sin^2\beta}$

or, $\dfrac{\cos^2\alpha}{\cos^2\beta}=\dfrac{\sin^2\alpha}{\sin^2\beta}$ $\quad(\because \cos^2\alpha-\cos^2\beta=\sin^2\beta-\sin^2\alpha)$

or, $\dfrac{\sin^2\beta}{\cos^2\beta}=\dfrac{\sin^2\alpha}{\cos^2\alpha}$ or, $\tan^2\beta=\tan^2\alpha$ or, $\alpha=\beta$

$\therefore\ \dfrac{\cos^4\beta}{\cos^2\alpha}+\dfrac{\sin^4\beta}{\sin^2\alpha}=\dfrac{\cos^4\alpha}{\cos^2\alpha}+\dfrac{\sin^4\alpha}{\sin^2\alpha}=\cos^2\alpha+\sin^2\alpha=1$

Second method (tricky approach) :

$\cos^2\alpha+\sin^2\alpha=1$

$\dfrac{\cos^4\alpha}{\cos^2\beta}+\dfrac{\sin^4\alpha}{\sin^2\beta}=1$ is possible only when $\alpha=\beta$

Now put $\alpha=\beta$ and get requred value

22. (a) $\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\dfrac{\sin\theta-(\cos\theta-1)}{\sin\theta+(\cos\theta-1)}\times\dfrac{\sin\theta-(\cos\theta-1)}{\sin\theta-(\cos\theta-1)}$

$$=\frac{\left(\sin\theta-(\cos\theta-1)\right)^2}{\sin^2\theta-(\cos\theta-1)^2}$$

$$=\frac{\sin^2\theta+(\cos\theta-1)^2-2\sin\theta(\cos\theta-1)}{\sin^2\theta-\left(\cos^2\theta+1-2\cos\theta\right)}$$

$$=\frac{\sin^2\theta+\cos^2\theta+1-2\cos\theta-2\sin\theta\cos\theta+2\sin\theta}{\sin^2\theta-\cos^2\theta-1+2\cos\theta}$$

$$=\frac{1+1-2\cos\theta-2\sin\theta\cos\theta+2\sin\theta}{-\cos^2\theta-\cos^2\theta+2\cos\theta}$$

$(\because \sin^2\theta-1=-\cos^2\theta)$

$$=\frac{2(1-\cos\theta-\sin\theta\cos\theta+\sin\theta)}{-2\left(\cos^2\theta-\cos\theta\right)}$$

$$=\frac{2(1-\cos\theta)(1+\sin\theta)}{2\cos\theta(1-\cos\theta)}=\frac{1+\sin\theta}{\cos\theta}$$

(Quick approach) putting $\theta = 60°$

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\frac{\sqrt{3}}{2} - \frac{1}{2} + 1}{\frac{\sqrt{3}}{2} + \frac{1}{2} - 1} = \frac{\frac{\sqrt{3}}{2} + \frac{1}{2}}{\frac{\sqrt{3}}{2} - \frac{1}{2}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2\sqrt{3}$$

Now put $\theta = 60°$ in each option, only option (a) gives $2\sqrt{3}$.

23. (a) $\sin(2x - 20°) = \cos(2y + 20°)$

$\Rightarrow \sin(2x - 20°) = \sin(90° - 2y - 20°)$

$\therefore 2x - 20° = 70° - 2y$

or, $2(x + y) = 90°$

or, $x + y = 45°$

$\therefore \sec(x + y) = \sec 45° = \sqrt{2}$

24. (c) $4\sec^2\theta + 9\,\mathrm{cosec}^2\theta$

$= 4(1 + \tan^2\theta) + 9(1 + \cot^2\theta)$

$= 4\tan^2\theta + 9\cot^2\theta + 13$

$= (2\tan\theta - 3\cot\theta)^2 + 2\cdot 2\tan\theta\cdot 3\cot\theta + 13$

$= (2\tan\theta - 3\cot\theta)^2 + 12 \times 1 + 13$ $\quad (\because \tan\theta\cdot\cot\theta = 1)$

But $(2\tan\theta - 3\cot\theta)^2 \geq 0$

$\therefore$ Required expression $\geq 12 + 13 = 25$ which is the minimum value.

Note : Do not work

$(2\tan\theta + 3\cot\theta)^2 - 2\cdot 2\tan\theta\cdot 3\cot\theta$ as $2\tan\theta + 3\cot\theta \neq 0$)

25. (a) $\tan(x + y)\tan(x - y) = 1$

$\Rightarrow \tan(x + y) = \dfrac{1}{\tan(x - y)} = \cot(x - y)$

$\Rightarrow \tan(x + y) = \tan(90° - (x - y))$

$\Rightarrow x + y = 90° - (x - y)$

$\Rightarrow 2x = 90° \quad \Rightarrow \dfrac{2x}{3} = 30°$

$\therefore \tan\dfrac{2x}{3} = \tan 30° = \dfrac{1}{\sqrt{3}}$

26. (b) $x = \mathrm{cosec}\theta - \sin\theta$

$$= \frac{1}{\sin\theta} - \sin\theta = \frac{1 - \sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$$

similary $y = \dfrac{\sin^2\theta}{\cos\theta}$

$$\therefore x^2y^2(x^2 + y^2 + 3) = \frac{\cos^4\theta}{\sin^2\theta}\frac{\sin^4\theta}{\cos^2\theta}\left(\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right)$$

$$= \cos^2\theta\sin^2\theta\left(\frac{\cos^6\theta + \sin^6\theta}{\sin^2\theta\cos^2\theta} + 3\right)$$

$$= \cos^2\theta\sin^2\theta\left\{\frac{(\cos^2\theta+\sin^2\theta)^3-3\sin^2\theta\cos^2\theta(\cos^2\theta+\sin^2\theta)}{\sin^2\theta\cos^2\theta}+3\right\}$$

$$= \cos^2\theta\sin^2\theta\frac{(1-3\sin^2\theta\cos^2\theta+3\sin^2\theta\cos^2\theta)}{\sin^2\theta\cos^2\theta}=1$$

27. (a) $\sin\theta+\sin^2\theta=1 \Rightarrow \sin\theta=1-\sin^2\theta=\cos^2\theta$ ...(i)

Now, $\cos^{12}\theta+3\cos^{10}\theta+3\cos^8\theta+\cos^6\theta-1$

$= (\cos^4\theta)^3+3(\cos^4\theta)^2\cos^2\theta+3\cos^4\theta(\cos^2\theta)^2+(\cos^2\theta)^3-1$

$= (\cos^4\theta+\cos^2\theta)^3-1$

$= (\sin^2\theta+\sin\theta)^3-1$ $(\because \cos^2\theta=\sin\theta)$

$= 1^3-1$ $(\because \sin\theta+\sin^2\theta=1)$

$= 1-1=0$

28. (a) $\tan(x+y)=\tan(x-y)$

$\Rightarrow \tan(x+y)=\frac{1}{\tan(x-y)}=\cot(x-y)$

$\Rightarrow x+y=\frac{\pi}{2}-(x-y)$

or, $2x=\frac{\pi}{2} \Rightarrow x=\frac{\pi}{4}$ $\therefore \tan x=1$

29. (d) Given $\cot A+\operatorname{cosec}A=3$ ...(i)

We know that $\operatorname{cosec}^2A-\cot^2A=1$

or, $(\operatorname{cosec}A+\cot A)(\operatorname{cosec}A-\cot A)=1$

or, $3(\operatorname{cosec}A-\cot A)=1$

or, $\operatorname{cosec}A-\cot A=\frac{1}{3}$ ...(ii)

(i) and (ii) adding

$2\operatorname{cosec}A=3+\frac{1}{3}=\frac{10}{3}$

or, $\operatorname{cosec}A=\frac{5}{3}=\frac{h}{p}$

$\therefore b=\sqrt{h^2-p^2}=\sqrt{25-9}=4$

$\therefore \cos A=\frac{b}{h}=\frac{4}{5}$

30. (d) Given expression $=1-\frac{\sin^2A}{1+\cos A}+\frac{1+\cos A}{\sin A}-\frac{\sin A}{1-\cos A}$

$$=1-\frac{1-\cos^2A}{1+\cos A}+\frac{(1+\cos A)(1-\cos A)-\sin^2A}{\sin A(1-\cos A)}$$

$$=1-\frac{(1+\cos A)(1-\cos A)}{(1+\cos A)}+\frac{1-\cos^2A-\sin^2A}{\sin A(1-\cos A)}$$

$$= 1 - (1 - \cos A) + \frac{1 - (\cos^2 A + \sin^2 A)}{\sin A(1 - \cos A)}$$

$$= 1 - 1 + \cos A + \frac{1 - 1}{\sin A(1 - \cos A)} = \cos A$$

) $2\sin\alpha + 15\cos^2\alpha = 7$

$2\sin\alpha + 15(1 - \sin^2\alpha) = 7$

$15\sin^2\alpha - 2\sin\alpha - 8 = 0$

...olving, $\sin\alpha = \frac{4}{5}$

$\therefore$ $\cot\alpha = \frac{3}{4}$

(d) $a^2 + 4 = (\tan\theta - \cot\theta)^2 + 4$

$$= \tan^2\theta + \cot^2\theta - 2\tan\theta\cot\theta + 4$$

$$= \tan^2\theta + \cot^2\theta - 2 + 4$$

$$= \tan^2\theta + \cot^2\theta + 2 = (\tan\theta + \cot\theta)^2$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)^2 = \frac{1}{\cos^2\theta \cdot \sin^2\theta}$$

$(b^2 - 1)^2 = ((\cos\theta - \sin\theta)^2 - 1)^2$

$$= (\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta - 1)^2$$

$$= (1 - 2\sin\theta\cos\theta - 1)^2$$

$$= 4\sin^2\theta\cos^2\theta$$

$\therefore$ $(a^2 + 4)(b^2 - 1)^2 = \frac{1}{\cos^2\theta\sin^2\theta} 4\sin^2\theta\cos^2\theta = 4$

33. (b) $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$

dividing by $a^2 + b^2$

$$\frac{a^2 - b^2}{a^2 + b^2}\sin\theta + \frac{2ab}{a^2 + b^2}\cos\theta = 1 \qquad \text{... (i)}$$

we will solve it by trial

$$\because \left(\frac{a^2 - b^2}{a^2 + b^2}\right)^2 + \left(\frac{2ab}{a^2 + b^2}\right)^2 = \frac{(a^2 - b^2) + 4a^2b^2}{(a^2 + b^2)^2} = \frac{(a^2 + b^2)^2}{(a^2 + b^2)^2} = 1$$

$\therefore$ From (i) $\frac{a^2 - b^2}{a^2 + b^2} = \sin\theta$ and $\frac{2ab}{a^2 + b^2} = \cos\theta$ can be considered.

so that $\sin^2\theta + \cos^2\theta = 1$

Hence $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a^2 - b^2}{2ab}$

34. (c) $\sin^2 21^\circ + \sin^2 69^\circ = \sin^2 21^\circ + \cos^2 21^\circ = 1$

35. (a) $\sin^2 5^\circ + \sin^2 25^\circ + \sin^2 45^\circ + \sin^2 65^\circ + \sin^2 85^\circ$

$= \sin^2 5° + \sin^2 25° + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 25° + \cos^2 5°$ ($\because \cos(90° - \theta) = \sin\theta$)

$= (\sin^2 5° + \cos^2 5°) + (\sin^2 25° + \cos^2 25°) + \frac{1}{2}$

$= 1 + 1 + \frac{1}{2} = \frac{5}{2} = 2.5$

36. (a) $3\sin^2\alpha + 7\cos^2\alpha = 4$

or, $3(1 - \cos^2\alpha) + 7\cos^2\alpha = 4$

or, $4\cos^2\alpha = 1$

or, $\cos^2\alpha = \left(\frac{1}{2}\right)^2 \Rightarrow \alpha = 60°$

$\therefore$ $\tan\alpha = \tan 60° = \sqrt{3}$

37. (a) $x = \cos^4\alpha + \sin^2\alpha$

$= \cos^4\alpha + 1 - \cos^2\alpha = 1 + \cos^4\alpha - \cos^2\alpha = 1 + \cos^2\alpha(\cos^2\alpha - 1)$

$= 1 - \cos^2\alpha \sin^2\alpha = 1 - \left(\frac{2\sin\alpha\cos\alpha}{2}\right)^2 = 1 - \frac{1}{4}\sin^2 2\alpha$

When $\sin^2 2\alpha = 0, x = 1$

When $\sin^2 2\alpha = 1, x = 1 - \frac{1}{4} = \frac{3}{4}$

Second method

$x = \cos^4\alpha + \sin^2\alpha = \cos^4\alpha + 1 - \cos^2\alpha$

$= \cos^4\alpha - 2\cdot\cos^2\alpha\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1$

$= \left(\cos^2\alpha - \frac{1}{2}\right)^2 + \frac{3}{4}$

When $\cos^2\alpha = \frac{1}{2}$ then $x = \left(\frac{1}{2} - \frac{1}{2}\right)^2 + \frac{3}{4} = \frac{3}{4}$

When $\cos^2\alpha = 1$ then $x = \left(1 - \frac{1}{2}\right)^2 + \frac{3}{4} = \frac{1}{4} + \frac{3}{4} = 1$

$\therefore$ Minimum value $= \frac{3}{4}$, maximum value. $= 1$

38. (a) $\sin^2\alpha = \cos^3\alpha \Rightarrow \frac{1}{\cos\alpha} = \frac{\cos^2\alpha}{\sin^2\alpha} \Rightarrow \sec\alpha = \cot^2\alpha$ ... (i)

Now, $\cot^6\alpha - \cot^2\alpha = \sec^3\alpha - \sec\alpha$ (from (i))

$= \sec\alpha(\sec^2\alpha - 1) = \sec\alpha\cdot\tan^2\alpha$

$= \frac{1}{\cos\alpha}\cdot\frac{\sin^2\alpha}{\cos^2\alpha} = \frac{\sin^2\alpha}{\cos^3\alpha}$

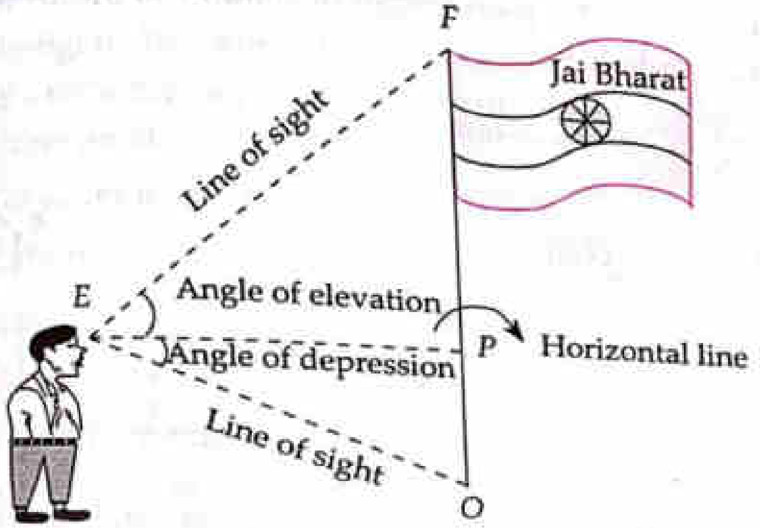
$= \frac{\sin^2\alpha}{\sin^2\alpha} = 1$ ($\because \cos^3\alpha = \sin^2\alpha$)

★★★

12 Height and Distance

Important terminology :

In the figure given below a person is eying at a flag "Jai Bharat". Foot of the flag staff is 'O' and its top is F as shown in figure. E is the eye of the man. If a horizontal line is drawn from E to flag staff which meets flag staff at P, then



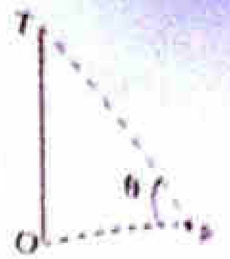
1.1 **Line of Sight :** To see the top 'F' of flagstaff, the man raises up his head and see along the line (ray EF). To see the foot 'O' he looks down towards ground and see along the line EO. These lines EF and EO are called line of sight. Hence line joining the object to the observer eye is called the line of sight.

1.2 **Horizontal line :** If one looks up at some object without raising up or lowering down his head, the line of sight is called horizontal line. In the above figure OP is the horizontal line.

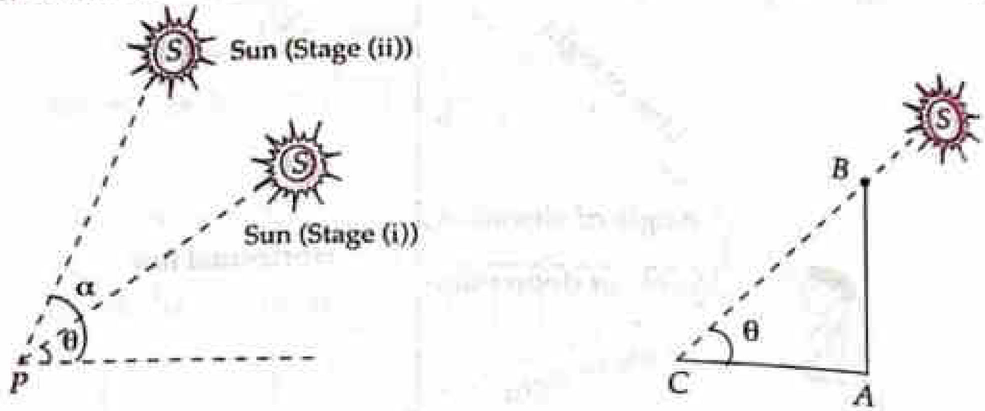
1.3 **Angle of elevation :** If we raise our head (or eye) to see any object then angle between line of sight and horizontal line is called angle of elevation. In the above figure angle of elevation of top 'F' of flag at observer eye is $\angle PEF$.

1.4 **Angle of depression :** If we lower down our head (or eye) to see an object then angle between line of sight and horizontal line is called angle of depression. In the above figure angle of depression of foot O at the observer eye is $\angle PEO$.

2. **Angle subtended by a line at a point :**
In the adjacent figure suppose OT is a tower, where O is the foot and T is the top of a tower. Suppose P is a point any where in the space (including ground). Join $O-P$ and $T-P$, then $\angle OPT = \theta$ is the angle subtened by tower OT at point P.



3. **Sun's altitude and shadow :**
If the Sun is seen from a point P then angle between horizontal line and line of sight is called Sun's altitude. Thus angle of elevation of the Sun from a point P is called Sun's altitude. In the figure given below, the Sun's altitude in first case is θ while in the case (ii), it is α.
Now, If B is the top of tower AB and line joining the sun's and top 'B' meets the ground at C then length of shadow of the tower is AC, when the Sun's altitude is AB.



4. **Fundamental concept to solve the question on height and distance :**
Go through questions carefully and draw an outline of the question. Consider unknown quantities as x, y, h, α, θ etc. Concentrate on right angled triangles in the figure and make equal number of equations as given unknown the solve the equations.

Solved Example

1. The angle of elevation of the top of a tower standing on a horizontal plane from two points on a line passing through the foot of the tower at a distance x and y respectively are complementary angles. Find the height of the tower. *[SSC Tier-I 2014]*

Solution : In the given figure OT is a tower. A and B are two points on a line passing through the foot of the tower and respectively at distances of x meter and y meter from the foot of tower.

If angle of elevation of top of the tower from point A is θ, then the elevation of top from point B is $(90° - \theta)$.

In ΔOAT, $\tan\theta = \frac{h}{x}$

$\tan(90° - \theta) = \frac{h}{y}$

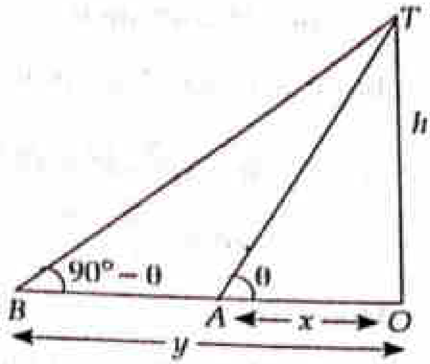
Multiplying,

$$\tan\theta \tan(90-\theta) = \frac{h}{x}\cdot\frac{h}{y}$$

$$\tan\theta\cdot\cot\theta = \frac{h^2}{xy}$$

$$1 = \frac{h^2}{xy}$$

$$\therefore\ h = \sqrt{xy}$$



2. If angle of elevation of top B of a tower AB from a point P on the ground 15 meter away from the foot is 60°, find the height of the tower. Also find the angle of depression of top D of a $10\sqrt{3}$ meter flag staff CD which is at a distance of 5 meter from the tower.

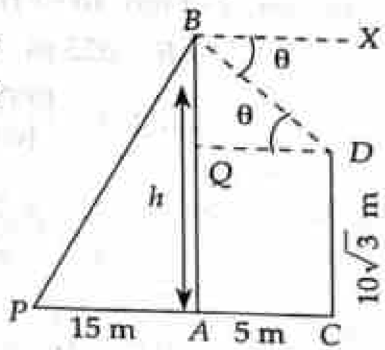
Solution : In the figure AB is a tower. AP is 15 meter and $\angle APB = 60°$ is the angle of elevation of top of tower from point P.

If height of tower is 'h'

then in right angled ΔPAB,

$$\tan 60° = \frac{h}{15}$$

or, $h = 15\tan 60° = 15\sqrt{3}$ meter.



Second part : In the figure CD is a flagstaff where $CD = 10\sqrt{3}$ meter. It is at a distance of 5 meter from the tower i.e., $AC = 5$ meter.

Draw, DQ ⊥r to AB then

$$BQ = AB - AQ$$
$$= 15\sqrt{3} - 10\sqrt{3} = 5\sqrt{3} \text{ meter and } DQ = AC = 5 \text{ m.}$$

If angle of depression by point B on point D is θ then $\angle XBD = \theta$

(see figure)

$\therefore$ $\angle BDQ = \theta$ (alternate angle)

$\therefore$ In right angled ΔBQD,

$$\tan\theta = \frac{BQ}{QD} = \frac{5\sqrt{3}}{5} = \sqrt{3} = \tan 60°$$

$\therefore$ $\theta = 60°$

3. Length of the tight thread of a kite from a point on the ground is 85 m. If thread subtends an angle θ with the ground such that $\tan\theta = \frac{8}{15}$ then find the height at which kite is flying.

Solution : In the given figure OK is the tight thread of the kite and $AK = H$ is the height of the kite.

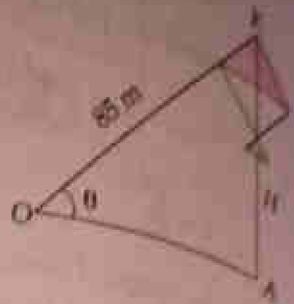
$$\therefore\ \sin\theta = \frac{H}{85}$$

or, $H = 85 \sin \theta$ ...(i)

but it is given that $\tan \theta = \frac{8}{15} = \frac{p}{b}$

$\therefore \quad h = \sqrt{p^2 + b^2} = \sqrt{8^2 + 15^2} = 17$

$\therefore \quad \sin \theta = \frac{p}{h} = \frac{8}{17}$



Hence from (i) $H = 85 \cdot \frac{8}{17} = 5 \cdot 8 = 40$ meter

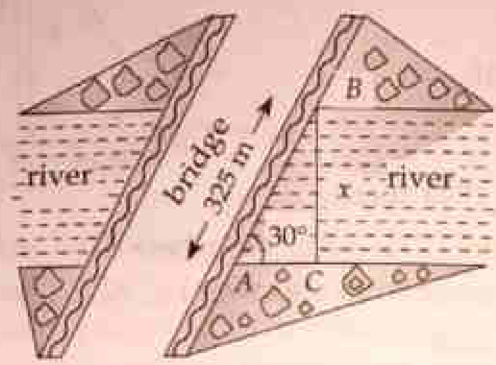
i.e., kite is flying at a height of 40 meter.

4. To cross a river a person covers a straight forward distance of 325 m along a bridge over the river. If bridge substends 30° angle with edge of the river, find the width of the river.

Solution : Let length of river be $BC = x$ m

$AB = 325$ m

$$\sin 30° = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{x}{325}$$



or, $\frac{1}{2} = \frac{x}{325}$

$\therefore \quad x = 162.5$

Hence width of the river is 162.5 m

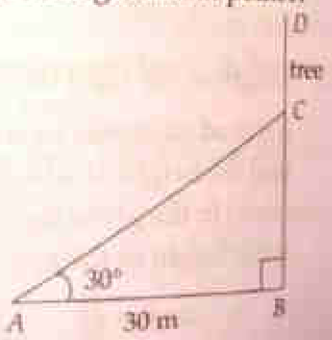
5. In a storm, a tree got bent by the wind whose top meets the ground at an angle of 30°, at a distance of 30 meters from the root. what is the height of the tree.

Solution : Let height of tree $= BD$

Suppose tree bends from point C and touches the ground at point A

where, $AC = CD$

In right angled ΔABC,



$$\tan 30° = \frac{BC}{AB} \Rightarrow BC = AB \tan 30°$$

$$= \frac{30}{\sqrt{3}} \text{ meter} \quad \text{...(i)}$$

and $\cos 30° = \frac{AB}{AC}$

$\Rightarrow AC = \dfrac{AB}{\cos 30^\circ} = 30 \times \dfrac{2}{\sqrt{3}}$ meter ... (ii)

$\therefore$ Total height of the tree $= BC + AC$

$$= \frac{30}{\sqrt{3}} + \frac{60}{\sqrt{3}} = \frac{90}{\sqrt{3}} \quad \text{(from (i) and (ii))},$$

$$= \frac{90}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{90\sqrt{3}}{3} = 30\sqrt{3} \text{ meter.}$$

Shortcut : For such a situation height of the tree $= m\left(\dfrac{1+\sin\theta}{\cos\theta}\right)$.

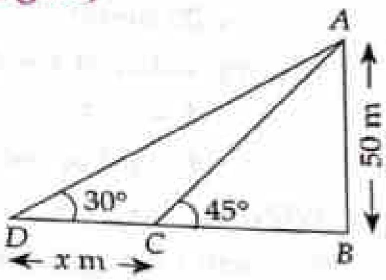
here $m = 30$ meter, $\theta = 30^\circ$

$\therefore$ height of the tree $= 30\left(\dfrac{1+\sin 30^\circ}{\cos 30^\circ}\right) = 30\left(\dfrac{1+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$

$$= 30\left(\frac{3}{\sqrt{3}}\right) = 30\sqrt{3} \text{ meter}$$

6. The height of a tower is 50 meter. When the Sun's altitude increases from 30° to 45°, the length of the shadow of the tower is decreased by x meter. Find the approximate value of x in meter.

Solution : Let height of the tower $= AB$ (see the figure)



In right angled ΔABC,

$$\tan 45^\circ = \frac{AB}{BC}$$

$\Rightarrow 1 = \dfrac{50}{BC}$

$\Rightarrow BC = 50$ meter ... (i)

In right angled ΔABD,

$$\tan 30^\circ = \frac{AB}{BD}$$

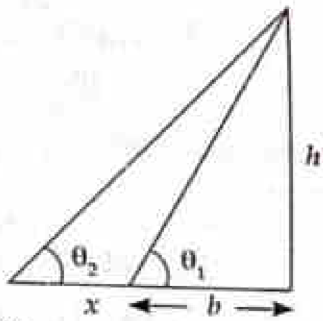
$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{50}{BD}$

$\Rightarrow BD = 50\sqrt{3}$

$\Rightarrow BD = 50(1.732)$

$\Rightarrow BD = 86.6$ meter ... (ii)

$CD = x = BD - BC = 86.6 - 50 = 36.6$ meter



Shortcut : In the given figure if θ_1, θ_2, x are known then

$$h = \frac{x}{\cot\theta_2 - \cot\theta_1} \text{ and } b = \frac{x\cot\theta_1}{\cot\theta_2 - \cot\theta_1}$$

In the given problem, $h = 50$ m; $\theta_2 = 30^\circ$, $\theta_1 = 45^\circ$

$\therefore 50 = \dfrac{x}{\cot 30^\circ - \cot 60^\circ}$

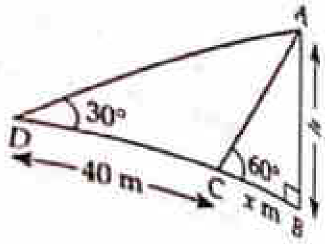
$\Rightarrow x = 50\left(\sqrt{3}-1\right)$ meter $= 50(\sqrt{3}-1)$ meter.

7. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 meter away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and width of the river [take, $\sqrt{3} = 1.732$]

Solution : Let height of tree $AB = h$ meter

Length of river $BC = x$ meter

In right angled ΔABC,



$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \frac{\sqrt{3}}{1} = \frac{h}{x}$$

$$\Rightarrow h = \sqrt{3}x \qquad \cdots (i)$$

In right angled ΔABD,

$$\tan 30° = \frac{AB}{BD} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+40} \text{ meter}$$

$$\Rightarrow \sqrt{3}h = (x+40) \text{ meter} \quad \Rightarrow \quad \sqrt{3}(\sqrt{3}x) - x = 40 \text{meter} \quad \text{[from (i)]}$$

$$\Rightarrow 3x - x = 40 \text{ meter}$$

$$\Rightarrow 2x = 40 \text{ meter}$$

$$\therefore \quad x = 20 \text{ meter}$$

putting value of x in (i),

$$h = \sqrt{3}x = (1.732)\cdot(20) = 34.64 \text{meter} \qquad \cdots (ii)$$

$\therefore$ height of the tree $h = 34.64$ meter

Width of the river $x = 20$ meter [from (ii)]

Shortcut : As mentioned in shortcut of question no. 6

$$h = \frac{40}{\cot 30° - \cot 60°}$$

$$= \frac{40}{\sqrt{3} - \frac{1}{\sqrt{3}}} = \frac{40}{\left(\frac{3-1}{\sqrt{3}}\right)} = 20\sqrt{3} \text{ meter}$$

$$x = \frac{40 \cot 60°}{\cot 30° - \cot 60°}$$

$$= (20\sqrt{3})\cot 60° = 20\sqrt{3} \times \frac{1}{\sqrt{3}} = 20 \text{ meter}$$

8. The angle of elevation θ of the top of vertical tower from a point on the ground is tan θ where $\tan\theta = \frac{5}{12}$. One walking 192 meters towards the tower in the same straight line, the elevation is α where $\tan\alpha = \frac{3}{4}$. Find the height of the tower.

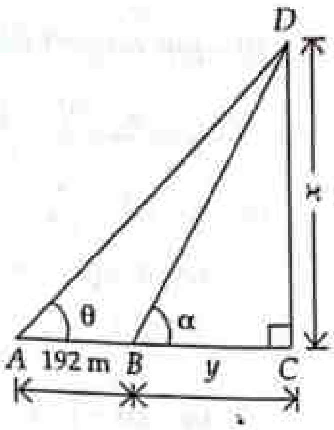
Solution : Let height of the tower $CD = x$ meter

and distance $BC = y$ meter

In right angled ΔBCD,

$$\frac{CD}{BC} = \tan\alpha$$

$$\Rightarrow \quad \frac{x}{y} = \frac{3}{4} \qquad \ldots \text{(i)}$$



Again in right angled ΔACD,

$$\frac{CD}{AC} = \tan\theta \qquad \Rightarrow \qquad \frac{x}{192+y} = \frac{5}{12}$$

Dividing (i) by (ii) we get,

$$\frac{x}{y} \times \frac{192+y}{x} = \frac{3}{4} \times \frac{12}{5}$$

$$\Rightarrow \quad \frac{192+y}{y} = \frac{9}{5}$$

$$\Rightarrow \quad 9y = 5(192+y) \qquad \Rightarrow \quad 9y = 960 + 5y$$

$$\Rightarrow \quad 9y - 5y = 960 \qquad \Rightarrow \quad 4y = 960$$

$$\Rightarrow \quad y = 240$$

putting value of y in (i)

$$\frac{x}{240} = \frac{3}{4} \quad \Rightarrow \quad 4x = 720 \quad \Rightarrow \quad x = 180$$

Hence height of the tower = 180 meter

Shortcut : As mentioned in shortcut of Q.N. 6 (height = x)

$$x = \frac{192}{\cot\theta - \cot\alpha}$$

$$= \frac{192}{\left(\frac{12}{5} - \frac{4}{3}\right)} = \frac{192}{\left(\frac{36-20}{15}\right)} = \frac{192}{16} \times 15 = 12 \times 15 = 180 \text{ meter}$$

$$y = \left(\frac{192}{\cot\theta - \cot\alpha}\right)\cot\alpha = 180 \times \frac{4}{3} = 240 \text{meter}$$

8. The angle of depression of a point on the ground from the top of a tree is 60°. Lowering down 20 m from the top, the angle of depression changes to 30°. Find the distance between the point and the foot of the tree. Also find the height of the tree.

Solution : In figure, AB is the tree and C is a point on the ground.

According to question,

$$\angle LAC = 60° = \angle ACB$$

$$\angle MDC = 30° = \angle DCB$$

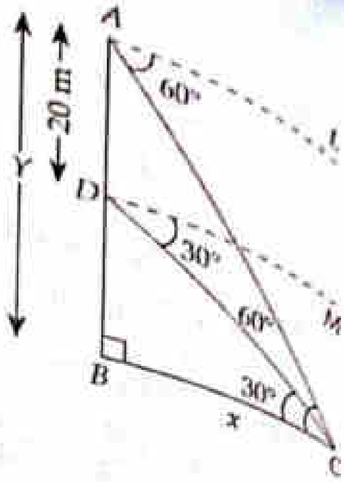
Let $BC = x$ m and $AB = y$ m

In right angled ΔABC,

$$\frac{y}{x} = \tan 60° = \sqrt{3}$$

$$\therefore \quad y = \sqrt{3} \cdot x \qquad \ldots \text{(i)}$$

In right angled ΔDBC, $\frac{BD}{x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$,



$$\therefore \quad \frac{y-20}{x} = \frac{1}{\sqrt{3}}$$

$$\text{or,} \quad y - 20 = \frac{x}{\sqrt{3}} \quad \ldots \text{(ii)}$$

from equation (i) and (ii),

$$20 = \sqrt{3} \cdot x - \frac{x}{\sqrt{3}}$$

$$\text{or,} \quad 20 = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)x$$

$$\text{or,} \quad 20 = \frac{3-1}{\sqrt{3}}x$$

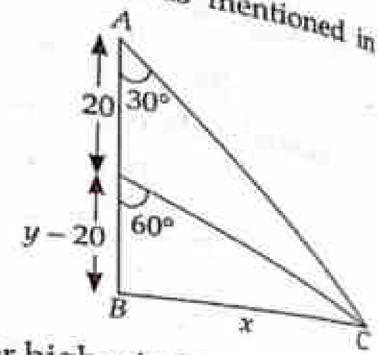
$$\text{or,} \quad x = 10\sqrt{3} = 10 \times 1.732 = 17.32$$

$\therefore$ distance between point and foot of tree = 17.32 m

$$\therefore \quad y = \sqrt{3} \cdot x = \sqrt{3} \times 10\sqrt{3} = 10 \times 3 = 30$$

$\therefore$ height of the tree = 30 m

Shortcut : Imagine figure in horizontal direction as mentioned in shortcut of Q.N.–6.



$$x = \frac{20}{\cot 30^\circ - \cot 60^\circ} = \frac{20}{\sqrt{3} - \frac{1}{\sqrt{3}}} = 10\sqrt{3}$$

$$\text{and } y - 20 = (10\sqrt{3}) \cot 60^\circ$$

$$= 10\sqrt{3} \times \frac{1}{\sqrt{3}} = 10$$

$$\therefore \quad y = 20 + 10 = 30$$

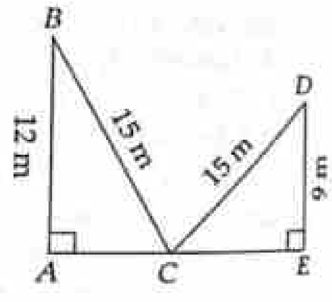
10. The upper end of a ladder touches a 12 meter high window on one side of a street. The ladder is rotated to opposite side of the street keeping its foot fixed and touches a 9 meter high window. If length of the ladder is 15 m, then find the width of the road.

Solution : Let ladder = $BC = CD = 15$ meter

B and D are windows on opposite

Given, $DE = 9$ meter

$AB = 12$ meter



In right angled ΔBAC, $AC^2 + AB^2 = BC^2$

$$\text{or,} \quad AC^2 + 12^2 = 15^2$$

$$AC + 144 = 225$$

$$AC^2 = 225 - 144 = 81$$

$$AC = \sqrt{81} = 9\text{meter}$$

Similarly $CE = \sqrt{15^2 - 9^2} = \sqrt{225 - 81} = \sqrt{144} = 12$meter

$\therefore$ Width of the road $AE = AC + CE = 9 + 12 = 21$ meter

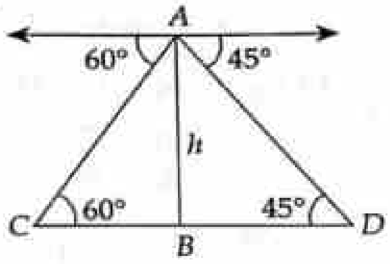
As observed from the top of a light house the angle of depression of two ships in opposite direction are respectively 60° and 45°. If distance between the two ships in $200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$ meter then find the height of the light house.

Solution : Let height of lamp post $AB = h$

C and D are positions of both ships

Given, distance between ships $CD = 200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$

In right angled ΔABC, $\frac{AB}{CB} = \tan 60°$



$\Rightarrow \quad \frac{h}{CB} = \sqrt{3}$

$\Rightarrow \quad CB = \frac{h}{\sqrt{3}} \qquad \ldots \text{(i)}$

In right angled ΔABD, $\frac{AB}{BD} = \tan 45°$

$\Rightarrow \quad \frac{h}{BD} = 1$

$\Rightarrow \quad BD = h \qquad \ldots \text{(ii)}$

Now, $CD = CB + BD$

$\Rightarrow \quad 200\frac{(\sqrt{3}+1)}{\sqrt{3}} = \frac{h}{\sqrt{3}} + h \qquad$ [from (i) & (ii)]

$\Rightarrow \quad h\left(\frac{1}{\sqrt{3}}+1\right) = 200\frac{(\sqrt{3}+1)}{\sqrt{3}}$

$\Rightarrow \quad h\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right) = 200\frac{(\sqrt{3}+1)}{\sqrt{3}}$

$\Rightarrow \quad h = 200\frac{(\sqrt{3}+1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}+1} = 200$

$\therefore$ Height of light house = 200 meter

12. From the top and bottom of a 40 meter high tower, the angle of elevation of top of a light house are respectively 30° and 60°. Find the height of the light house. Also find the distance between top of light house and foot of tower.

Solution : Let tower $BC = 40$ meter

Light house $= AD$;

$AE = h$ meter $\qquad$ (see the figure)

and $BE = CD = x$ meter

In right angled ΔBEA,

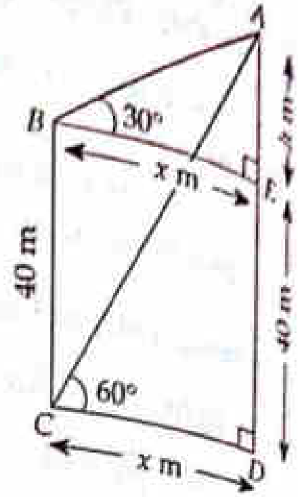
$\frac{h}{x} = \tan 30°$

$\Rightarrow \quad \frac{h}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad x = h\sqrt{3}$ ... (i)

$\Rightarrow \quad BE = CD = h\sqrt{3}$ meter

In right angled ΔADC, $\frac{AD}{CD} = \tan 60°$



$\Rightarrow \quad \frac{h+40}{x} = \sqrt{3}$

$\Rightarrow \quad h + 40 = \sqrt{3}x$

$\Rightarrow \quad h + 40 = \sqrt{3} \times h\sqrt{3}$ [By (i)]

$\Rightarrow \quad 40 = 3h - h$

$\Rightarrow \quad 2h = 40 \qquad \Rightarrow h = 20$ meter

$\therefore$ height of light house = 20 + 40 = 60 meter

In right angled ΔADC, $\frac{AD}{AC} = \sin 60°$

$\frac{60}{AC} = \frac{\sqrt{3}}{2} \qquad \Rightarrow \sqrt{3}AC = 60 \times 2$

$\Rightarrow \quad AC = \frac{60 \times 2}{\sqrt{3}}$

$\Rightarrow \quad AC = 60 \times \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow \quad AC = \frac{60 \times 2 \times \sqrt{3}}{3} \qquad \Rightarrow AC = 40\sqrt{3}$ meter

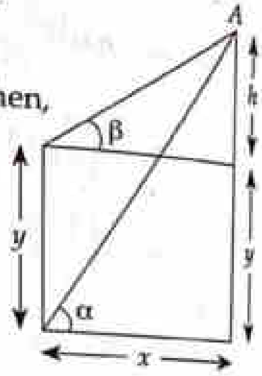
$\therefore$ Required distance $AC = 40\sqrt{3}$ meter

Shortcut : I the given figure if y, α, β are known then,

$$h = \frac{y}{\tan\alpha\cot\beta - 1}$$

$$= \frac{y\cot\alpha}{\cot\beta - \cot\alpha}$$

$$x = \frac{y}{\tan\alpha - \tan\beta}$$



13. A boy standing on a horizontal plane find that angle of elevation of a bird 100 meter away from him at 30°. A girl standing at a house 20 meter above the plane find that elevation of the bird is 45°. If boy and girl are in the opposite direction find the distance between the bird and the girl.

Solution : Let A, E and C be respectively position of bird, girl and boy.

In ΔABC,

$\sin 30° = \frac{AB}{AC}$

$\Rightarrow \quad \frac{1}{2} = \frac{AB}{100}$

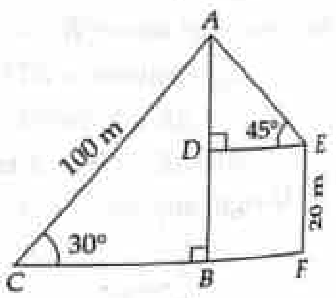
$\Rightarrow \quad 2AB = 100$

$\Rightarrow \quad AB = 50$ meter ... (i)

Now, $AD = AB - BD$

$= 50 - EF$ [From (i)]

$= 50 - 20 = 30$ meter ... (ii)



In ΔADE,

$$\sin 45° = \frac{AD}{AE}$$

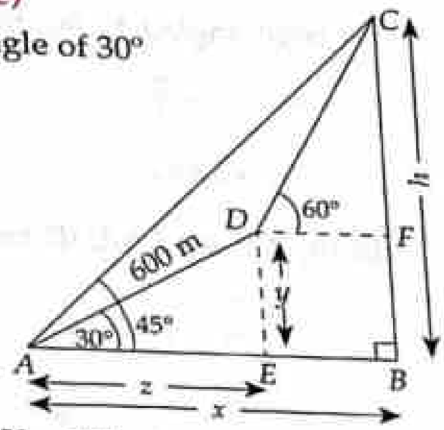
$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{30}{AE}$ **[from (ii)]**

$\Rightarrow \quad AE = 30\sqrt{2} = 30(1.41) = 42.3$ meter $\quad (\because \sqrt{2} = 1.41)$

4. At the foot of a mountain the elevation of its summit is 45°. After ascending 600 meter towards the mountain, upon an incline of 30°, the elevation changes to 60°. Find the height of the mountain.

Solution : Let height of the mountain be $BC = h$ meter

Let A be the foot of the mountain, C is its summit then according to question $\angle CAB = 45°$ **(see the figure)**



AD is inclanation of the mountain at an angle of 30°

$\therefore \quad AD = 600$ meter and $\angle DAE = 30°$

Draw $DF \perp BC$ and $DE \perp AB$

Similarly, $\quad \angle CDF = 60°$ **(given)**

In ΔDFC,

$$\angle DCF = 180° - (90° + 60°) = 30°$$

In ΔABC

$$\angle ACB = 180° - (90° + 45°) = 45°$$

$\therefore \quad \angle ACD = \angle ACB - \angle DCF = 45° - 30° = 15°$

and $\angle CAD = 45° - 30° = 15°$

In ΔACD,

$AD = DC$ ***[sides equal to opposite angles are equal]***

$\therefore \quad DC = 600$ meter $\quad (\because AD = 600$ meter$)$

In right angled ΔAED, $\sin 30° = \frac{DE}{AD}$

$\Rightarrow \quad \frac{1}{2} = \frac{DE}{600}$

$\Rightarrow \quad 2DE = 600$

$\Rightarrow \quad DE = 300$ meter

In right angled, ΔDFC, $\sin 60° = \frac{FC}{DC}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{FC}{600}$

$\Rightarrow \quad 2FC = 600\sqrt{3} \quad \Rightarrow \quad FC = 300\sqrt{3}$

$\therefore \quad$ height of the mountain $BC = BF + FC = DE + FC \quad (\because BF = DE)$

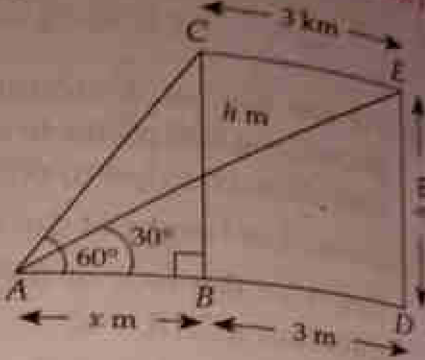
$$= 300 + 300\sqrt{3} = 300(1 + \sqrt{3}) \text{ meter}$$

[**Shortcut :** when angles are 45° and 30° and $AD = x$, height of mountain $= \frac{x}{2}(1 + \sqrt{3})$]

15. The angle of elevation of an aeroplane from a point on the ground is 60°. After flying for 15 seconds, the elevation changes to 30°. If the aeroplane is flying at a speed of 720 kmph, then find the constant height at which aeroplane is flying. [take $\sqrt{3} = 1.732$]

Solution : Let constant height of plane = h meter

and $AB = x$ meter



speed of plane = 720 kmph

$$= 720 \times \frac{5}{18} = 200 \text{ m/s}$$

distance covered by plane in 15 sec.

$$= 200 \times 15 = 3000 \text{ meter}$$

In right angled ΔABC, $\tan 60° = \frac{BC}{AB}$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3}x = h \qquad \Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \text{...(i)}$$

In right angled ΔADE, $\tan 30° = \frac{DE}{AD}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+3000} \qquad \Rightarrow \quad \sqrt{3}h = x + 3000$$

$$\Rightarrow \quad \sqrt{3}h = \frac{h}{\sqrt{3}} + 3000 \qquad \text{[from (i)]}$$

$$\Rightarrow \quad \sqrt{3}h = \frac{h + 3000\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \quad 3h = h + 3000\sqrt{3}$$

$$\Rightarrow \quad 3h - h = 3000\sqrt{3}$$

$$\Rightarrow \quad 2h = 3000\sqrt{3}$$

$$\therefore \quad h = \frac{3000}{2} \times 1.732 = 2598 \text{ m.}$$

16. From a point on the ground the angle of elevation of top of a tower is α. On moving 'a' meters towards the tower, the elevation changes to β. Prove that height of the tower is $\frac{a \tan\alpha \tan\beta}{\tan\beta - \tan\alpha}$.

Solution : Let tower $AB = h$

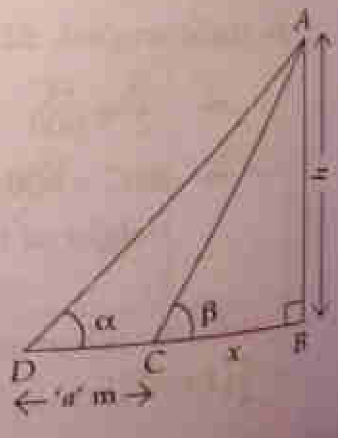
and $CB = x$

In right angled ΔABC,

$$\tan\beta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\beta = \frac{h}{x}$$

$$\Rightarrow \quad x\tan\beta = h$$



$\Rightarrow x = \tan\beta$

In right angled ΔABD,

$$\tan\alpha = \frac{AB}{BD}$$

$$\Rightarrow \tan\alpha = \frac{h}{x+a}$$

$$\Rightarrow h = (x+a)\tan\alpha$$

$$\Rightarrow h = x\tan\alpha + a\tan\alpha$$

$$\Rightarrow h = \frac{h}{\tan\beta}\cdot\tan\alpha + a\tan\alpha \qquad \text{[from (i)]}$$

$$\Rightarrow h = \frac{h\cdot\tan\alpha + a\cdot\tan\alpha\cdot\tan\beta}{\tan\beta}$$

$$\Rightarrow h\tan\beta = h\tan\alpha + a\tan\alpha\cdot\tan\beta$$

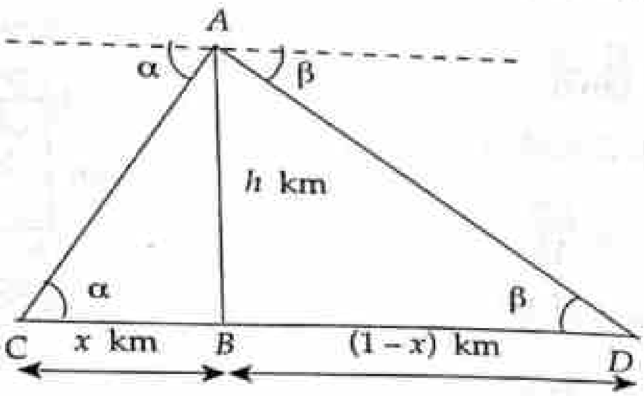
$$\Rightarrow h\tan\beta - h\tan\alpha = a\tan\alpha\cdot\tan\beta$$

$$\Rightarrow h(\tan\beta - \tan\alpha) = a\tan\alpha\cdot\tan\beta$$

$$\therefore \text{ height of tower } h = \frac{a\tan\alpha\cdot\tan\beta}{\tan\beta - \tan\alpha}$$

An aeroplane is flying above a horizontal plane. The angle of depression of two consecutive mile stones at plane in opposite directions are respectively α and β. Prove that height of the aeroplane is $\frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$

Solution : Let height of aeroplane $= AB = h$ km and $BC = x$ km



In figure C and D are positions of two consecutive stones

$\therefore$ $CD = 1$ km and $BD = (1-x)$ km

In right angled ΔABC,

$$\tan\alpha = \frac{AB}{BC} \quad \Rightarrow \quad \tan\alpha = \frac{h}{x}$$

$$\Rightarrow x\tan\alpha = h \quad \Rightarrow \quad x = \frac{h}{\tan\alpha} \qquad \cdots \text{(i)}$$

In right angled ΔABD,

$$\tan\beta = \frac{AB}{BD}$$

$\Rightarrow \quad \tan\beta = \dfrac{h}{1-x}$

$\Rightarrow \quad h = \tan\beta - x\tan\beta$

$\Rightarrow \quad h = \tan\beta - \dfrac{h}{\tan\alpha}\tan\beta$ [from (i)]

$\Rightarrow \quad h = \dfrac{\tan\alpha\tan\beta - h\tan\beta}{\tan\alpha}$

$\Rightarrow \quad h\tan\alpha = \tan\alpha\tan\beta - h\tan\beta$

$\Rightarrow \quad h\tan\alpha + h\tan\beta = \tan\alpha\tan\beta$

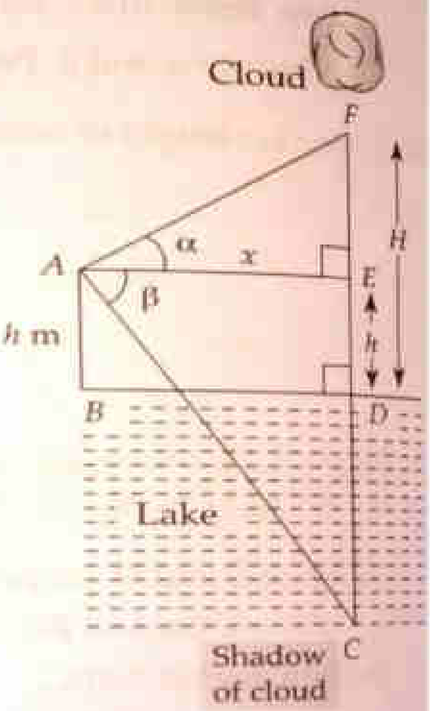
$\Rightarrow \quad h(\tan\alpha + \tan\beta) = \tan\alpha\tan\beta$

$\therefore \quad$ height $(h) = \dfrac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$ km

18. The angle of elevation of a stationary cloud from a point h meter above a lake is 15° and angle of depression of its reflection in the lake is β. Prove that height of the cloud above the lake is $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$.

Solution : Let height of cloud from surface of lake $DF = H$ meter and perpendicular distance of cloud from point $A = AE = x$ meter

$\therefore \quad EF = H - h$ and $EC = H + h$



In right angled ΔAEF, $\tan\alpha = \dfrac{EF}{AE}$

$$\tan\alpha = \frac{H-h}{x}$$

$$\Rightarrow \quad x = \frac{H-h}{\tan\alpha} \qquad \ldots \text{(i)}$$

In right angled ΔAEC,

$$\tan\beta = \frac{EC}{AE}$$

$$\tan\beta = \frac{H+h}{x}$$

$$x = \frac{H+h}{\tan\beta} \qquad \ldots \text{(ii)}$$

Comparing (i) and (ii)

$$\frac{H-h}{\tan\alpha} = \frac{H+h}{\tan\beta}$$

$$H\tan\beta - h\tan\beta = H\tan\alpha + h\tan\alpha$$

$$H\tan\beta - H\tan\alpha = h\tan\alpha + h\tan\beta$$

$$H(\tan\beta - \tan\alpha) = h(\tan\alpha + \tan\beta)$$

$H = \dfrac{h(\tan\alpha + \tan\beta)}{\tan\beta - \tan\alpha}$; **Proved**

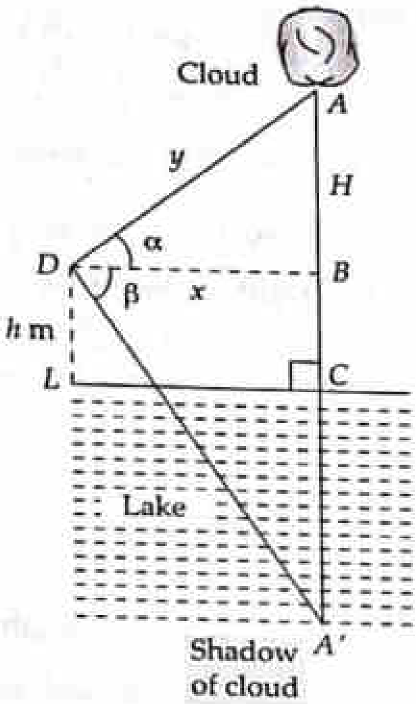
2. The angle of elevation of a cloud from a point h meter above the surface of the lake is α, the angle of depression of its reflection in the lake is β. Prove that height of the cloud from observation points is $\frac{2h \sec \alpha}{\tan \beta - \tan \alpha}$.

Solution : Let distance of cloud (A), from observation point D is

$DA = y$

$DB = x$,

$AC = H$ and $DL = h$



In right angled ΔABD,

$$\tan \alpha = \frac{AB}{DB}$$

$$\tan \alpha = \frac{H-h}{x}$$

$$H - h = x \tan \alpha$$

$$H = h + x \tan \alpha \qquad \ldots \text{(i)}$$

In right angled $\Delta A'BD$,

$$\tan \beta = \frac{A'B}{DB}$$

$$\Rightarrow \tan \beta = \frac{H+h}{x}$$

$$\Rightarrow H + h = x \tan \beta \qquad \ldots \text{(ii)}$$

From equation (i) and (ii),

$$h + x \tan \alpha + h = x \tan \beta$$

$$x \tan \alpha + 2h = x \tan \beta$$

$$2h = x \tan \beta - x \tan \alpha$$

$$2h = x(\tan \beta - \tan \alpha)$$

$$x = \frac{2h}{\tan \beta - \tan \alpha} \qquad \ldots \text{(iii)}$$

In right angled ΔABD,

$$\cos \alpha = \frac{DB}{AD}$$

$$\Rightarrow \cos \alpha = \frac{x}{y}$$

$$\Rightarrow y = \frac{x}{\cos \alpha}$$

$$\Rightarrow y = x \sec \alpha$$

putting value of x in equation (iii)

$$y = \frac{2h \sec \alpha}{\tan \beta - \tan \alpha}$$

3. A spherical balloon of radius r subtends an angle 'α' at an observer's eye. The angle of elevation of centre of the balloon is ϕ. Prove that the height of the centre of the ball is $a \sin \phi \operatorname{cosec} \frac{\theta}{2}$.

Solution : Let height of centre of balloon is h. O is eye of observer and centre of balloon is C. (see the figure)

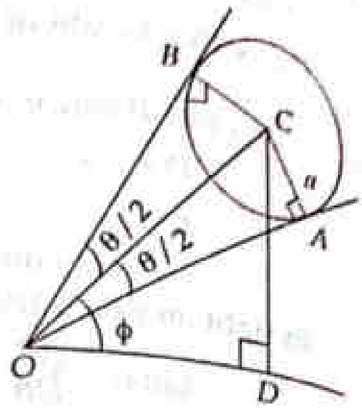
OA and OB are tangents to the ...

$\therefore \quad CA \perp OA$

and $\angle OAC = 90°$

Similarly, $CB \perp OB$

$\therefore \quad \angle OBC = 90°$



In right angled ΔCAO,

$$\text{cosec}\,\frac{\theta}{2} = \frac{OC}{CA}$$

$$\Rightarrow \quad OC = AC\,\text{cosec}\,\frac{\theta}{2}$$

$$\Rightarrow \quad OC = a\,\text{cosec}\,\frac{\theta}{2} \qquad \ldots \text{(i)}$$

In right angled ΔODC,

$$\sin\phi = \frac{CD}{OC}$$

$$\Rightarrow \quad \sin\phi = \frac{h}{OC}$$

$$\Rightarrow \quad h = OC \cdot \sin\phi$$

$$\Rightarrow \quad h = a\,\text{cosec}\,\frac{\theta}{2} \cdot \sin\phi \qquad \text{[from (i)]}$$

$$\therefore \quad \text{Required height} = a \sin\phi\,\text{cosec}\,\frac{\theta}{2}$$

Exercise–12A

1. A 100 m tall radio antenna stands on top of a house. From a point on the ground, the angle of elevation of bottom of the antenna is 45° and angle of elevation of top of the antenna is 60°. What is the height of the house?

   (a) 100 m (b) 50 m (c) $50(\sqrt{3}+1)$ m (d) $50(\sqrt{3}-1)$ m

2. The angle of elevation of the top of a half constructed tower at a distance of 150 m from the base of the tower is 30°. If angle of elevation of top of tower from that point is to be 45°, then by what amount height of the tower be increased ?

   (a) 59.4 m (b) 61.4 m (c) 62.4 m (d) 63.4 m

3. The length of shadow of a s cm tall man is p, cm when Sun's altitude is α. It is of the length q cm when sun's altitude is β. Which of the following is true when $\beta = 3\alpha$.

   (a) $p-q = s\left(\dfrac{\tan\alpha - \tan 3\alpha}{\tan 3\alpha \tan\alpha}\right)$ (b) $p-q = s\left(\dfrac{\tan 3\alpha - \tan\alpha}{3\tan 3\alpha \tan\alpha}\right)$

   (c) $p-q = s\left(\dfrac{\tan 3\alpha - \tan\alpha}{\tan 3\alpha \tan\alpha}\right)$ (d) $p-q = s\left(\dfrac{\tan 2\alpha}{\tan 3\alpha \tan\alpha}\right)$

4. The shadow of a 6 m high tower is 15 m and at the same point of time length of shadow of a tree is 25 m. What is the height of the tree ?

   (a) 21 m (b) 10 m (c) 35 m (d) None of these

5. Suppose angle of elevation of top of a tree from a point E, due east of the tree is 60° and from a point F, due west is 30°. If distance between E and F is 160 feet, what is the height of the tree ?

(a) $40\sqrt{3}$ feet (b) 60 feet (c) $\frac{40}{\sqrt{3}}$ feet (d) 23 feet

6. Two poles of 6 m and 11 m are standing vertically on a plane. If distance between their feet is 12 m, what is the distance between their top ?

(a) 11 m (b) 12 m (c) 13 m (d) 14 m

7. When Sun's altitude is 30°, the shadow of a tower is 15 meter. What is the length of shadow when Sun's altitude is 60° ?

(a) 3 m (b) 4 m (c) 5 m (d) 6 m

8. The angles of elevation of the top of a tower from two points at a distance of 36 meter and 64 meter from the base of the tower and in the same straight line with it are complementary. What is the height of the tower?

(a) 50 meter (b) 48 meter (c) 25 meter (d) 24 meter

9. The angle of elevation of the top of an unfinished tower at a distance of 100 m from its base is 45°. How much higher must the tower be raised so that angle of elevation of its top at the same point may be 60° ?

(a) $50\sqrt{2}$ m (b) 100 m

(c) $100(\sqrt{3}-1)$ m (d) $100(\sqrt{3}+1)$ m

10. The shadow of a tree is 16 meter when elevation of Sun is 60°. What is the height of the tree ?

(a) 8 m (b) 16 m (c) $16\sqrt{3}$ m (d) $\frac{16}{\sqrt{3}}$ m

11. As observed from the top of a light house, the angle of depression of two ships in opposite direction are 30° and 45°. If height of light house is h, then what is the distance between two ships ?

(a) $(\sqrt{3}+1)h$ (b) $(\sqrt{3}-1)h$ (c) $\sqrt{3}h$ (d) $\left(1+\frac{1}{\sqrt{3}}\right)h$

12. An aeroplane is vertically above the another plane flying at a height of 5000 feet from the ground. The angle of elevation of these two planes from a points on the ground are respectively $\frac{\pi}{3}$ and $\frac{\pi}{4}$. What is the vertical distance between these two planes ?

(a) $2500(\sqrt{3}+1)$ feet (b) $5000\sqrt{3}$ feet

(c) $5000(\sqrt{3}-1)$ feet (d) $5000(\sqrt{3}+1)$ feet

... is in north-south direction. From a point on the ... north-east direction and is at a dis- ... dicular distance

14. A man stands at the end point of the shadow of a pole. He measure that length of shadow is $\frac{1}{\sqrt{3}}$ times that of pole. What is the Sun's altitude?

(a) 60° (b) 30° (c) 45° (d) 15°

15. The angle of depression of vertices of a regular hexagon lying in a plane from the top of a 75 m high tower standing at the centre of the hexagon is 60°. What is the length of each side of the hexagon ?

(a) $50\sqrt{3}$ m (b) 75 m (c) $25\sqrt{3}$ m (d) 25 m

16. Two poles, one is double in length of other, are standing opposite to each other at a distance of y meter. If angle of elevation from mid point of the line joining their feet are complementary then what is the height of the shorter pole ?

(a) $\frac{y}{\sqrt{2}}$ (b) $\frac{y}{2\sqrt{2}}$ (c) $\frac{y}{2}$ (d) $\frac{y}{\sqrt{2}+1}$

17. The angle of elevation of top of a house from top and bottom of tree are respectively x and y. If height of the tree is h meter then what is the height of the house ?

(a) $\frac{h\cot x}{\cot x+\cot y}$ (b) $\frac{h\cot y}{\cot x+\cot y}$ (c) $\frac{h\cot x}{\cot x-\cot y}$ (d) $\frac{h\cot y}{\cot x-\cot y}$

18. The angle of elevation of top of a monument from a point on the ground is α and $\tan\alpha = \frac{1}{5}$. One walking 138 meter towards the base of the monument in the same straight line, the angle of elevation of the top of monument is found to be β, where $\sec\beta = \frac{\sqrt{193}}{12}$. Height of monument is

(a) 35 (b) 49 (c) 42 (d) 56

19. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 meters towards the foot of the tower to a point B, the angle of elevation increases to 60°. What is the height of the tower ?

(a) $\sqrt{3}$ m (b) $5\sqrt{3}$ m (c) $10\sqrt{3}$ m (d) $20\sqrt{3}$ m

20. Two vertical pillars stand on either side of a road. One of them is 108 meter height. From the top of this pillar angle of depression of top and bottom of the other pillar are respectively 30° and 60°. What is the height of the second pillar.

(a) 36 m (b) 72 m (c) 108 m (d) 110 m

21. At the foot of a mountain the elevation of its summit is 45°. After ascending 2000 m towards the mountain upon an incline of 30°, the elevation changes to 60°. What is the height of the mountain ?

(a) $2000(\sqrt{3}-1)$ meter (b) $1000(\sqrt{3}-1)$ meter
(c) $2000(\sqrt{3}+1)$ meter (d) $1000(\sqrt{3}+1)$ meter

22. From a point on the ground the angle of elevation of top of a house and top of a chimney surmounted on the house are respectively x° and 45°. If height of the house is h meter then height of the chimney is

(a) $h\cot x + h$ (b) $h\cot x - h$ (c) $h\tan x - h$ (d) $h\tan x + h$

23. From a … On mo… What i…
(a) 75…
24. The ar… directi… two st… plane… (a) 5…
25. The a… feet a… then … (a) 4…
26. A sh… eleva… sphe… is the… (a) …
27. The … 45°. … plan… the …
(a) …
28. Two … nort… and … θ is …
(a) …
(c) …
29. The … a p… bei… val…
(a) …

23. From a point of the ground, angle of elevation of top of a tower is 30°. On moving $50\sqrt{3}$ meter towards the tower the elevation changes to 60°. What is the height of the tower ?

(a) $75\sqrt{3}$ m (b) 75 m (c) $90\sqrt{3}$ m (d) $60\sqrt{3}$ m

24. The angle of depression of two consecutive kilometer stones in opposite direction from an aeroplane lying above the horizontal line joining the two stones are respectively 60° and 45°. What is the height of the aeroplane ?

(a) 500 m (b) $500\sqrt{3}$ m (c) 1500 m (d) $500(3-\sqrt{3})$ m

25. The angle of elevation and depression of top of a statue from a point 22 feet above the lake are respectively α and β. If $\sin\alpha = \frac{5}{13}$ and $\cos\beta = \frac{3}{5}$ then what is the height of the statue from the surface of the lake ?

(a) 44 feet (b) 42 feet (c) 33 feet (d) 55 feet

26. A shperical balloon of of radius 10 feet is in the open air. If angle of elevation of centre of the balloon from a point on the ground is 45° and spherical balloon subtens on angle of 45° on that point then how high is the centre of the balloon from the ground ?

(a) 10 feet (b) 15 feet (c) $20\sqrt{2}$ feet (d) $10\sqrt{2}$ feet

27. The angle of elevation of an aeroplane from a point on the ground is 45°. After flying for 15 second, the elevation changes to 30°. If the aeroplane is flying at a height of 2500 meters, then approximate speed of the aeroplane in kmph is

(a) 440 kmph (b) 400 kmph (c) 360 kmph (d) 480 kmph

28. Two stations due south of a leaning tower which leans towards the north are respectively at a distances x and y from its foot ($y > x$). If α and β be the elevations of the top of the tower from these stations and θ is inclination of the tower to the horizontal then cotθ equals

(a) $\dfrac{x\cot\alpha - y\cot\beta}{x-y}$ (b) $\dfrac{x-y}{x\cot\alpha - y\cot\beta}$

(c) $\dfrac{y\cot\alpha - x\cot\beta}{y-x}$ (d) $\dfrac{y-x}{y\cot\alpha - x\cot\beta}$

29. The angles of elevation of a bird flying in a horizontal straight line from

30. A balloon is slanting down in a straight line passes vertically above two points A and B on a horizontal plane 1000 meters apart. When above A it has an altitude 60° as seen from B and when above B, altitude is 30° as seen from A. What is the distance of the point from point A where the balloon strike the plane.

(a) 500 m (b) $1000\sqrt{3}$ m (c) $\frac{1000}{\sqrt{3}}$ m (d) 1500 m

Answer–12A

1. (c)	2. (d)	3. (c)	4. (b)	5. (a)	6. (c)	7. (c)	8. (b)
9. (c)	10. (c)	11. (a)	12. (c)	13. (a)	14. (a)	15. (c)	16. (b)
17. (c)	18. (c)	19. (c)	20. (b)	21. (b)	22. (b)	23. (b)	24. (d)
25. (d)	26. (d)	27. (a)	28. (c)	29. (a)	30. (d)		

Explanation

1. (c) Let $BC = h$ be height of the house.

In ΔABC, $\tan 45^\circ = \frac{h}{x}$

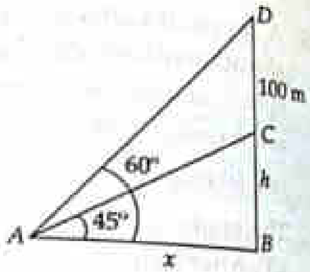
$\Rightarrow x = h$ ... (i)

In ΔABD,

$$\tan 60^\circ = \frac{100+h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{100+h}{h}$$

$$\Rightarrow h = \frac{100}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = 50\,(\sqrt{3}+1) \text{ meter}$$



2. (d) Let $BC = x$ meter be the height of tower and $CD = h$ meter is the height of unfinished tower.

In ΔABC, $\tan 30^\circ = \frac{x}{150}$

$$\Rightarrow x = \frac{150}{\sqrt{3}} \quad \text{... (i)}$$

In ΔABD,

$$\tan 45^\circ = \frac{h+x}{150}$$

$$\Rightarrow 1 = \frac{h+x}{150}$$

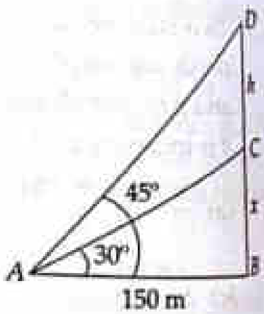
$$\Rightarrow 150 = h + \frac{150}{\sqrt{3}} \quad \text{[from equation (i)]}$$

$$\Rightarrow h = \frac{150(\sqrt{3}-1)}{\sqrt{3}}$$

$$\Rightarrow = 50 \times 1.732\,(1.732 - 1)$$

$$= 50 \times 1.732 \times 0.732$$

$$= 63.39 = 63.4 \text{ m (approximate)}$$



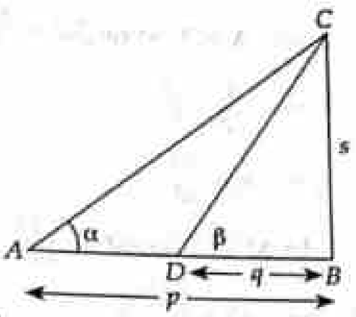
3. (c) In ΔABC, $\tan\alpha = \frac{s}{p}$

$\Rightarrow p = \frac{s}{\tan\alpha}$ ... (i)

In ΔBDC,

$\tan\beta = \frac{s}{q}$

$\Rightarrow q = \frac{s}{\tan 3\alpha}$ ... (ii)



Subtracting equation (ii) from equation (i)

$$p - q = \frac{s}{\tan\alpha} - \frac{s}{\tan 3\alpha} = s\left[\frac{\tan 3\alpha - \tan\alpha}{\tan\alpha \tan 3\alpha}\right]$$

4. (b) For the shadow of 15 meter length, height of tower = 6 meter

$\therefore$ For the shadow of length 1 m, height of tower = $\frac{6}{15}$ meter

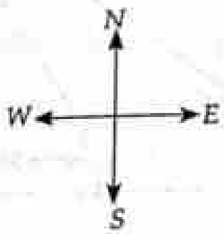
For the shadow of 25 meter length, height of tower $= \frac{6}{15} \times 25$

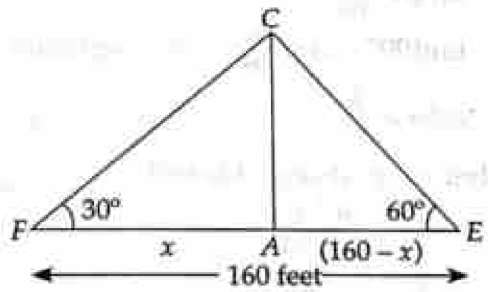
$= 10$ meter

Height of the tree = 10 m.

5. (a) In ΔAFC,

$\tan 30° = \frac{h}{x} \Rightarrow x = \sqrt{3}\,h$ ... (i)





In ΔAEC,

$$\tan 60° = \frac{h}{160 - x}$$

$\Rightarrow \sqrt{3}(160 - x) = h$ (from equation (i))

$\Rightarrow \sqrt{3}(160 - \sqrt{3}h) = h$ $\Rightarrow 160\sqrt{3} - 3h) = h$

$\Rightarrow 4h = 160\sqrt{3}$ $\Rightarrow h = 40\sqrt{3}$ feet

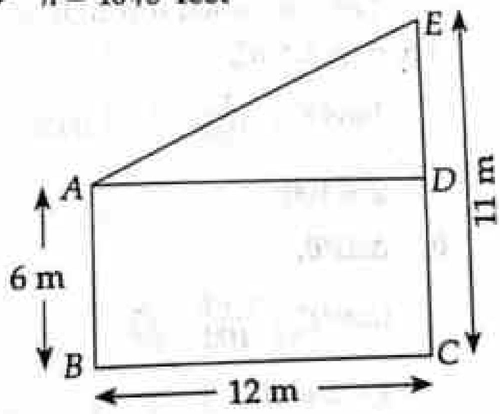
6. (c) $\because$ $AD = BC = 12$ meter and

$ED = 11 - 6 = 5$ meter

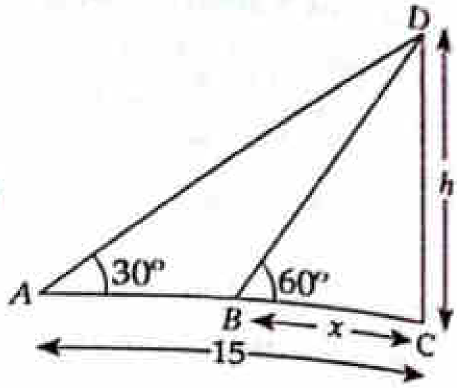
In ΔADE,

$$AE^2 = AD^2 + ED^2$$
$$= 12^2 + 5^2$$
$$= 144 + 25 = 169$$

$\Rightarrow AE = 13$ meter



7. **(c)** ΔACD में, $\tan 30^\circ = \dfrac{CD}{AC}$



$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{h}{15}$

$\Rightarrow \quad h = \dfrac{15}{\sqrt{3}}$ ... (i)

In ΔBCD, $\tan 60^\circ = \dfrac{CD}{BC}$

$\Rightarrow \quad \sqrt{3} = \dfrac{h}{x}$

$\Rightarrow \quad \dfrac{h}{\sqrt{3}} = x \quad \Rightarrow \quad x = \dfrac{15}{3} = 5$ (from equation (i))

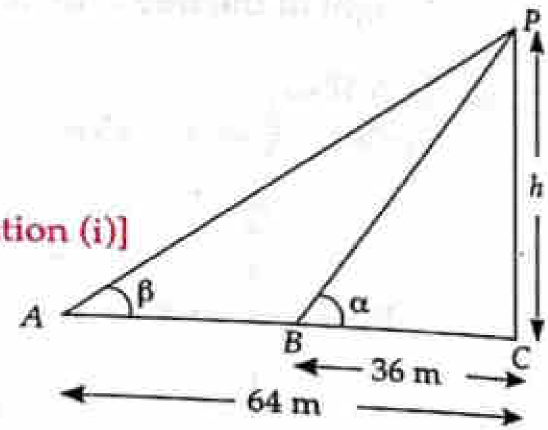
Let of shadow = 5 m

8. **(b)** Given $\alpha + \beta = 90^\circ$ ... (i)

Length height of tower = h

In ΔPBC,

$\tan\alpha = \dfrac{h}{36}$ ... (ii)



In ΔAPC,

$\tan\beta = \dfrac{h}{64}$

$\tan(90^\circ - \alpha) = \dfrac{h}{64}$ [from equation (i)]

$\Rightarrow \quad \cot\alpha = \dfrac{h}{64}$ ... (iii)

Multiplying equation (ii) & (iii)

$\tan\alpha \cdot \cot\alpha = \dfrac{h}{36} \cdot \dfrac{h}{64}$

$\Rightarrow \quad \tan\alpha \cdot \dfrac{1}{\tan\alpha} = \dfrac{h^2}{(6)^2 (8)^2} = 1$

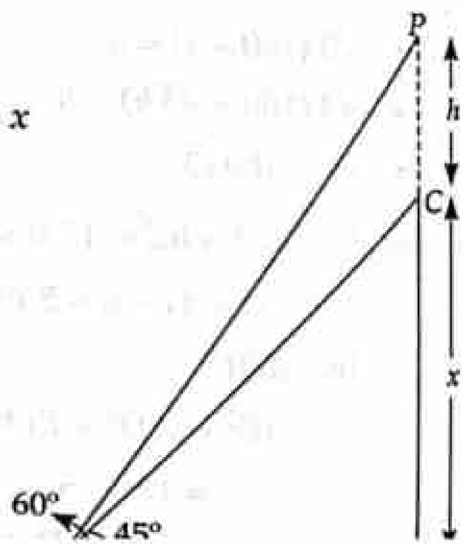
$\Rightarrow \quad h = 6 \times 8 = 48$

$\therefore$ Required height $(h) = 48$ meter

9. **(c)** Let height of unfinished tower $(BC) = x$

Let it is raised through 'h'

Then in ΔABC,



$\tan 45^\circ = \dfrac{x}{100} = 1$

$x = 100$

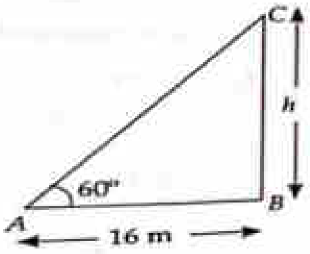
In ΔAPB,

$\tan 60^\circ = \dfrac{x+h}{100} = \sqrt{3}$

$h = 100\sqrt{3} - x = 100\sqrt{3} - 100$

$\therefore$ Required height $(h) = 100(\sqrt{3} - 1)$ meter

10. (c) Let height of the tree $= h$ meter



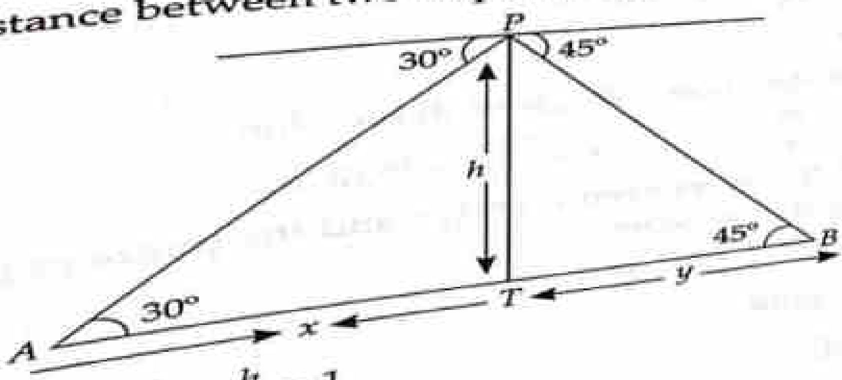
In ΔABC,

$\tan 60^\circ = \frac{h}{16} = \sqrt{3}$

$h = \sqrt{3} \cdot 16$

$\therefore$ Required height $(h) = 16\sqrt{3}$ meter

11. (a) Let distance between two ships $= x + y$



In ΔPTB, $\tan 45^\circ = \frac{h}{y} = 1$ ... (i)

$y = h$

Similarly, In ΔPTA, $\tan 30^\circ = \frac{h}{x} = \frac{1}{\sqrt{3}}$ ... (ii)

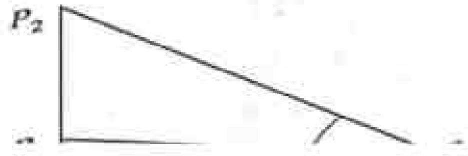
$x = \sqrt{3}h$

$\therefore$ Required distance $= x + y$

$= \sqrt{3}h + h = (\sqrt{3}+1)h$ meter

POP_1 is a right angled isosceles triangle.

$\ldots = 5000$ feet



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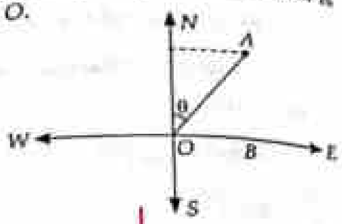
13. (a) In figure O is a point on the national highway NS and school is situated at point A, north-east to point O.

$\therefore \quad \theta = 45°$ and $OA = 600\sqrt{2}$ m

Let perpendicular distance be AB.



In ΔAOB,

$\sin\theta = \dfrac{AB}{OA}$

$\Rightarrow \sin 45° = \dfrac{AB}{600\sqrt{2}} \Leftrightarrow AB = 600$ m

14. (a) Let angle of elevation be θ.

$\tan\theta = \dfrac{h}{h/\sqrt{3}} = \sqrt{3} = \tan 60°$

$\therefore \quad \theta = 60°$

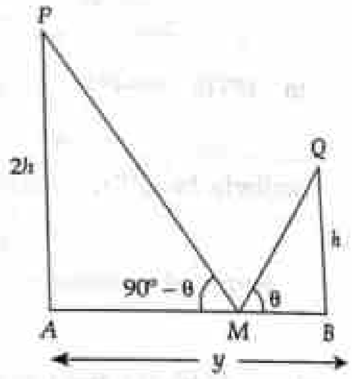
15. (c) Angle of elevation = Angle of depression

$\tan 60° = \dfrac{75}{x} \qquad \therefore \; x = \dfrac{75}{\sqrt{3}} = 25\sqrt{3}$ m

Since distance between a vertex and the centre of the hexagon is equal to length of its sides

$\therefore$ length of side $= 25\sqrt{3}$ m

16. (b) See the figure



$\tan\theta = \dfrac{h}{MB}$ ... (i)

$\tan(90° - \theta) = \dfrac{2h}{AM}$

or, $\cot\theta = \dfrac{2h}{AM}$ ... (ii)

On multiplication,

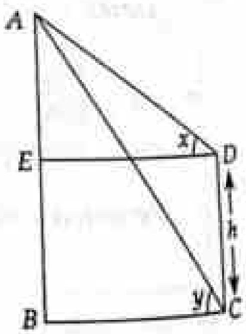
$\tan\theta \cdot \cot\theta = \dfrac{h \cdot 2h}{(MB)(AM)}$

$1 = \dfrac{2h^2}{\left(\dfrac{y}{2}\right)^2} \qquad \left(\because AM = MB = \dfrac{y}{2}\right)$

$\dfrac{y^2}{4} = 2h^2$

$\Rightarrow \quad h = \dfrac{y}{2\sqrt{2}}$

17. (c) As is shortcut of solved example 12.



$AE = \dfrac{h}{\tan y \cot x - 1}$

$= \dfrac{h}{\dfrac{1}{\cot y}\cot x - 1} = \dfrac{h\cot y}{\cot x - \cot y}$

$\therefore \; AB = AE + h$

$= \dfrac{h\cot y}{\cot x - \cot}$

18. (c) $\tan\alpha = \dfrac{1}{5}$ $\Rightarrow$

$\sec\beta = \sqrt{\dfrac{193}{12}}$

$\therefore \; \tan\beta = \sqrt{\sec^2\beta}$

$\Rightarrow \cot\beta = \dfrac{12}{7}$

As in shortcut of

$h = \dfrac{138}{\cot\alpha - c}$

19. (c) As in short

$h = \dfrac{}{\cot\theta}$

$= \dfrac{}{\cot 3}$

$= \dfrac{2}{\sqrt{3}}$

$= 10.$

20. (b) As in sh

$AE = \dfrac{}{\tan 60°}$

$108 - h = \dfrac{}{\sqrt{3}}$

$\Rightarrow \; 108 = h$

$\therefore \; h = 72$

21. (d) Do as

Requi

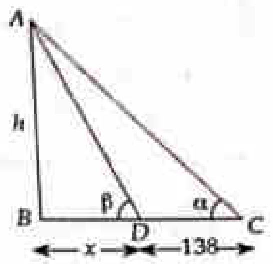
22. (b) In fig

Let $\angle DC$

If AD

$\therefore\ AB = AE + h$

$$= \frac{h\cot y}{\cot x - \cot y} + h = \frac{h\cot x}{\cot x - \cot y}$$



19. (c) $\tan\alpha = \frac{1}{5} \Rightarrow \cot\alpha = 5$

$$\sec\beta = \sqrt{\frac{193}{12}}$$

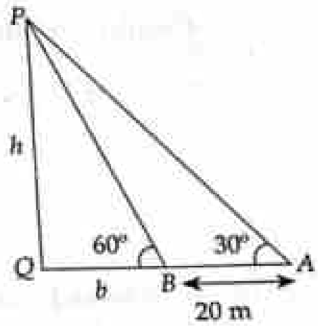
$$\therefore\ \tan\beta = \sqrt{\sec^2\beta - 1} = \sqrt{\frac{193}{144} - 1} = \frac{7}{12}$$

$$\Rightarrow \cot\beta = \frac{12}{7}$$

As in shortcut of solved example 6

$$h = \frac{138}{\cot\alpha - \cot\beta} = \frac{138}{5 - \frac{12}{7}}$$

$$= \frac{138\times 7}{23} = 42 \text{ meter}$$



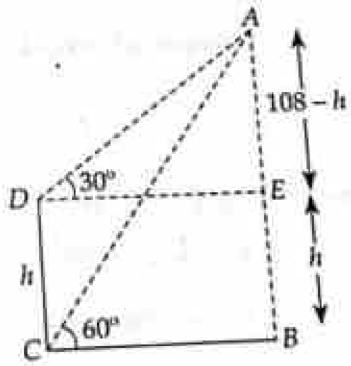
19. (c) As in shortcut of solved example 6.

$$h = \frac{x}{\cot\theta_2 - \cot\theta_1}$$

$$= \frac{20}{\cot 30^\circ - \cot 60^\circ}$$

$$= \frac{20}{\sqrt{3} - \frac{1}{\sqrt{3}}} = \frac{20\sqrt{3}}{3-1}$$

$$= 10\sqrt{3} \text{ meter}$$



20. (b) As in shortcut of solved example 13.

$$AE = \frac{h}{\tan 60^\circ \cot 30^\circ - 1}$$

$$108 - h = \frac{h}{\sqrt{3}\cdot\sqrt{3} - 1} = \frac{h}{2}$$

$$\Rightarrow 108 = h + \frac{h}{2} = \frac{3h}{2}$$

$$\therefore\ h = 72 \text{ meter}$$

21. (d) Do as in solved example 14

Required height $= \frac{x}{2}\left(1+\sqrt{3}\right)$

$$= \left(\sqrt{3}+1\right)1000 \text{ meter}$$

22. (b) In figure, BD = house, AD = chimney

Let $\angle DCB = x^\circ$ and $\angle ACB = 45^\circ$

If $AD = y$ then

In right angled isosceles triangle ABC,

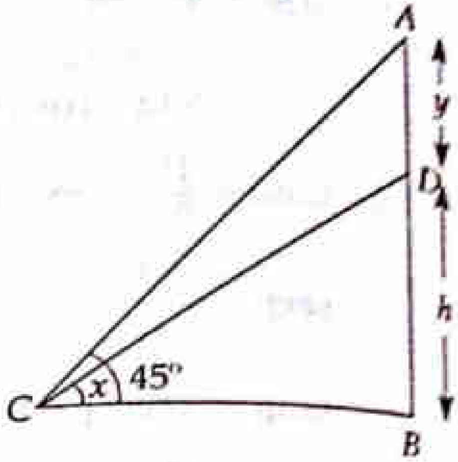
$BC = AB = h + y$

In ΔBCD,

$$\cot x = \frac{BC}{BD} = \frac{h+y}{h}$$

or, $h\cot x = h + y$

or, $y = h\cot x - h$



23. **(b)** As done in solved example 16

$$\text{Required height} = \frac{h\tan\alpha\tan\beta}{\tan\beta - \tan\alpha} = \frac{50\sqrt{3}\cdot\tan 30^\circ\cdot\tan 60^\circ}{\tan 60^\circ - \tan 30^\circ}$$

$$= \frac{50\sqrt{3}\frac{1}{\sqrt{3}}\cdot\sqrt{3}}{\sqrt{3} - \frac{1}{\sqrt{3}}} = 75 \text{ meter.}$$

24. **(d)** As done in solved example 17

$$\text{Required height} = \frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta} \text{ km}$$

$$= \frac{\tan 60^\circ\cdot\tan 45^\circ}{\tan 60^\circ + \tan 45^\circ} = \frac{\sqrt{3}\cdot 1}{\sqrt{3}+1} = \frac{\sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{3-\sqrt{3}}{2} \text{ km} = \frac{3-\sqrt{3}}{2} \times 1000 \text{ meter}$$

$$= 500(3-\sqrt{3}) \text{ meter}$$

25. **(b)** As in solved example 18

$$\text{Required height } = h\left(\frac{\tan\alpha + \tan\beta}{\tan\beta - \tan\alpha}\right) = 22\left(\frac{\frac{5}{12}+\frac{4}{3}}{\frac{4}{3}-\frac{5}{12}}\right) = 22\left(\frac{\frac{5+16}{12}}{\frac{16-5}{12}}\right)$$

$$= 22\left(\frac{21}{11}\right) = 2 \times 21 = 42 \text{ feet}$$

26. **(d)** As in solved example 20

then In $OP'Q'$ $\tan 30^\circ = \frac{2500}{2500+x}$

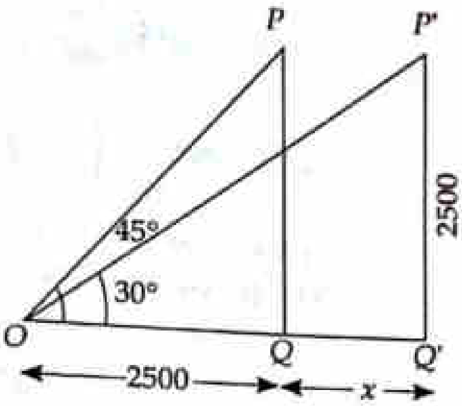
$$x = (2500\sqrt{3} - 2500)$$

$$= 2500 \times 0.732 = 1830 \text{ m}$$

$\because$ Speed $= \frac{x}{t} = \frac{1830}{15}$ m/s

$$= \frac{1830}{15} \times \frac{18}{5} \text{ kmph}$$

$$= 440 \text{ kmph (approx)}$$



28. (c) OT is the leaning tower. A and B are two stations at x and y distances from the foot of tower O.

Let $OC = z$ and $CT = h$

In ΔOCT, $\cot\theta = \frac{z}{h}$ ... (i)

In ΔACT, $\cot\alpha = \frac{z+x}{h} = \frac{z}{h} + \frac{x}{h} = \cot\theta + \frac{x}{h}$ ... (ii)

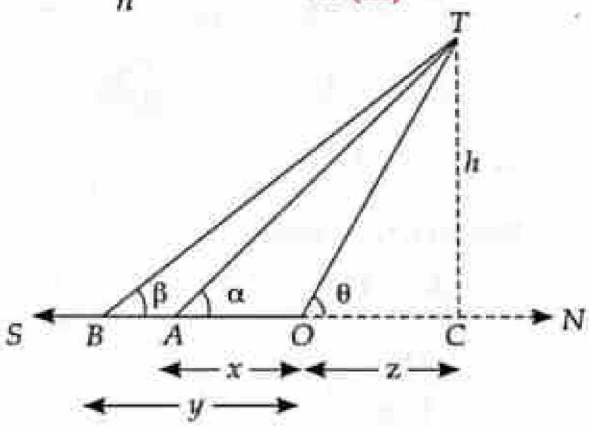
In ΔBCT, $\cot\beta = \frac{z+y}{h} = \frac{z}{h} + \frac{y}{h} = \cot\theta + \frac{y}{h}$ ... (iii)

From (ii) $\cot\alpha - \cot\theta = \frac{x}{h}$

$$\Rightarrow \quad h = \frac{x}{\cot\alpha - \cot\theta}$$

From (iii) $\cot\beta - \cot\theta = \frac{y}{h}$

$$\Rightarrow \quad h = \frac{y}{\cot\beta - \cot\theta}$$



Equating 'h' we get

$$\frac{x}{\cot\alpha - \cot\theta} = \frac{y}{\cot\beta - \cot\theta}$$

$\Rightarrow$ $x\cot\beta - x\cot\theta = y\cot\alpha - y\cot\theta$

$\Rightarrow$ $(y - x)\cot\theta = y\cot\alpha - x\cot\beta$

$\therefore$ $\cot\theta = \frac{y\cot\alpha - x\cot\beta}{y - x}$

29. (a) P, Q, R, S are four positions of bird at equal time interval.

Let $PQ = QR = RS = x$

$\therefore$ $LM = MN = NT = x$

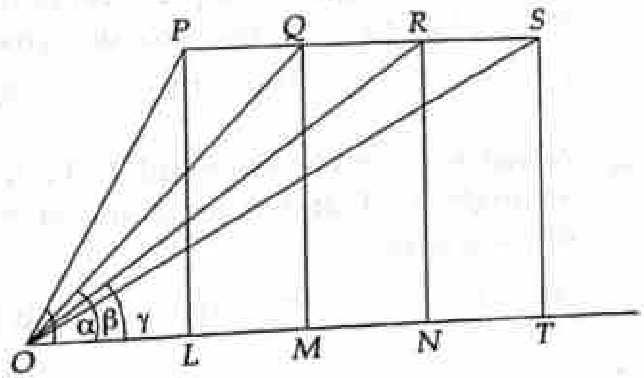
let $PL = h$ and $OL = l$

In ΔOLP, $\cot\alpha = \frac{l}{h}$

In ΔOMQ, $\cot\beta = \frac{l+x}{h}$

In ΔONR, $\cot\gamma = \frac{l+2x}{h}$

In ΔOTS, $\cot\delta = \frac{l+3x}{h}$



$$\therefore \quad \cot^2\alpha - \cot^2\delta = \frac{l^2}{h^2} - \frac{(l+3x)^2}{h^2} = \frac{l^2-(l+3x)^2}{h^2}$$

$$\cot^2\beta - \cot^2\gamma = \left(\frac{l+x}{h}\right)^2 - \left(\frac{l+2x}{h}\right)^2 = \frac{(l+x)^2-(l+2x)^2}{h^2}$$

$$\therefore \quad \frac{\cot^2\alpha-\cot^2\delta}{\cot^2\beta-\cot^2\gamma} = \frac{l^2-(l+3x)^2}{(l+x)^2-(l+2x)^2}$$

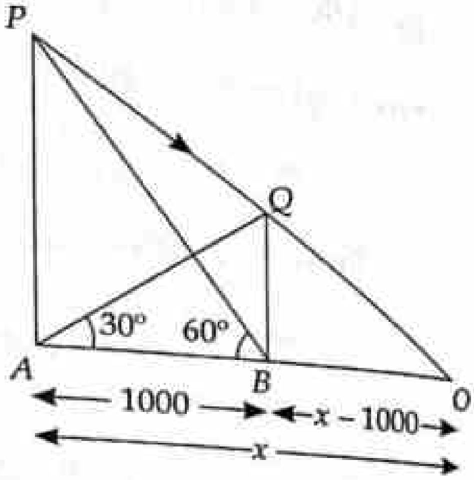
$$= \frac{l^2-l^2-6xl-9x^2}{l^2+x^2+2lx-l^2-4x^2-4xl}$$

$$= \frac{3(-2xl-3x^2)}{(-3x^2-2xl)} = 3$$

30. **(d)** See the figure, AB is a horizontal line 1000 m apart in a plane. PQ is the path of balloon, which is slanting down from P to Q and strikes the horizontal plane at O. Let $AO = x$ then $BO = x - 1000$

In ΔPAB, $\tan 60° = \frac{AP}{1000}$

$\Rightarrow AP = 1000\sqrt{3}$



In ΔQBA, $\tan 30° = \frac{BQ}{1000}$

$\Rightarrow \quad BQ = \frac{1000}{\sqrt{3}}$

But $\Delta OAP \sim \Delta OBQ$

$\therefore \quad \frac{AP}{BQ} = \frac{OA}{OB}$

$\Rightarrow \quad \frac{1000\sqrt{3}}{\left(\frac{1000}{\sqrt{3}}\right)} = \frac{x}{x-1000}$

$\Rightarrow \quad 3 = \frac{x}{x-1000}$

$\Rightarrow \quad 3x - 3000 = x$

$\Rightarrow x = 1500$ m

Exercise–12B

1. P and Q are two points observed from the top of a building $10\sqrt{3}$ m high. If the angles of depression of the points are complementary and $PQ = 20$ m, then the distance of P from the building is
   (a) 30 m (b) 40 m (c) 25 m (d) 45 m

   [SSC Tier-I 2012]

2. A tree is broken by the wind. If the top of the tree struck the ground at an angle of 30° and at a distance of 30 m from the root, then the height of the tree is

3. From the top of a cliff 90 m high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. The height of the tower is :

(a) 45 m (b) 60 m (c) 75 m (d) 30 m

[SSC Tier-I 2012]

4. A pole broken by the storm of wind and its top struck the ground at the angle of 30° and at a distance of 20 m from the foot of the pole. The height of the pole before it was broken was

(a) $20\sqrt{3}$ m (b) $\frac{40\sqrt{3}}{3}$ m (c) $60\sqrt{3}$ m (d) $\frac{100\sqrt{3}}{3}$ m

[SSC Tier-I 2012]

5. The angle of elevation of the top of a tower standing on a horizontal plane from two points on a line passing through the foot of the tower at a distance 9 ft and 16 ft. respectively are complementary angles. The height of the tower is

(a) 9 ft (b) 12 ft (c) 16 ft (d) 144 ft

[SSC Tier-I 2012,

6. When angle of elevation of the Sun increases from 30° to 60°, Shadow of a pole is diminised by 5 meter. The height of the pole is

(a) $\frac{5\sqrt{3}}{2}$ m (b) $\frac{2\sqrt{3}}{5}$ m (c) $\frac{2}{5\sqrt{3}}$ m (d) $\frac{4}{5\sqrt{3}}$ m

[SSC Tier-I 2012]

7. A boy standing in the middle of a field, observes a flying bird in the north at an angle of elevation of 30° and after 2 minutes, he observes the same bird in the south at an angle of elevation of 60°. If the bird flies all along in a straight line at a height of $50\sqrt{3}$ m, then its speed in km/h is

(a) 3 (b) 9 (c) 6 (d) 4.5

8. The angle of elevation from two points at a distance of x and y from the feet of a lower are complementary, the height of the lower is

(a) $\sqrt{xy}$ (b) $\frac{x}{y}$ (c) $\sqrt{\frac{x}{y}}$ (d) $\sqrt{x+y}$

[SSC Tier-I 2012]

Answer–12B

1. (a) 2. (d) 3. (b) 4. (a) 5. (b) 6. (a) 7. (c) 8. (a)

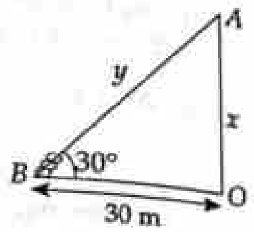
$\Rightarrow \quad 300 = 20x + x^2$

$\Rightarrow \quad x^2 + 20x - 300 = 0 \Rightarrow (x+30)(x-10) = 0$

$\Rightarrow \quad x - 10 = 0$ $\quad (\because x + 30 \neq 0)$

$\Rightarrow \quad x = 10$ m $\qquad$ Required length $= 20 + 10 = 30$ m

2. **(d)** Height of tree $= OA + AB = x + y$



$$\cos 30° = \frac{30}{y}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{30}{y} \Rightarrow y = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

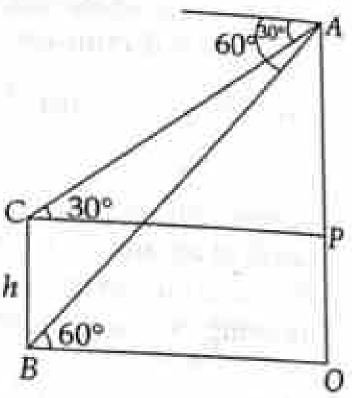
$$\tan 30° = \frac{x}{30}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{30} \quad x \Rightarrow 10\sqrt{3}$$

$$\therefore \quad x + y = 10\sqrt{3} + 20\sqrt{3} = 30\sqrt{3}$$

3. **(b)** Rock $AO = 90$ m

Tower $BC = h$ m (say)



In ΔAOB, $\tan 60° = \dfrac{90}{OB}$

$$\Rightarrow \quad OB = \frac{90}{\tan 60°} = \frac{90}{\sqrt{3}}$$

In ΔACP, $\tan 30° = \dfrac{AP}{PC}$

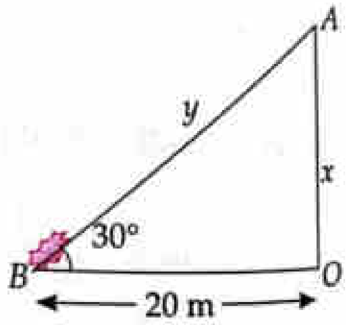
$$\tan 30° = \frac{90° - h}{OB} \qquad (\because OB = PC)$$

$$\therefore \quad \tan 30° \cdot OB = 90° - h$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} \cdot \frac{90}{\sqrt{3}} = 90 - h$$

$$\Rightarrow \quad h = 90 - \frac{90}{3} = 60 \text{ m}$$

4. **(a)** Height of tree $= OA + AB = x + y$ meter



$$\therefore \quad \cos 30° = \frac{20}{y}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{20}{y}$$

$$\Rightarrow \quad y = \frac{40}{\sqrt{3}} \text{ meter}$$

$$\tan 30° = \frac{x}{20}$$

$$\Rightarrow \quad x = 20\tan 30° = 20 \times \frac{1}{\sqrt{3}} = \frac{20}{\sqrt{3}}$$

$\therefore$ Required height $= \frac{20}{\sqrt{3}} + \frac{40}{\sqrt{3}} = \frac{60}{\sqrt{3}} =$

(b) See the figure,

$\tan\theta = \frac{h}{9} \Rightarrow h = 9\tan\theta$

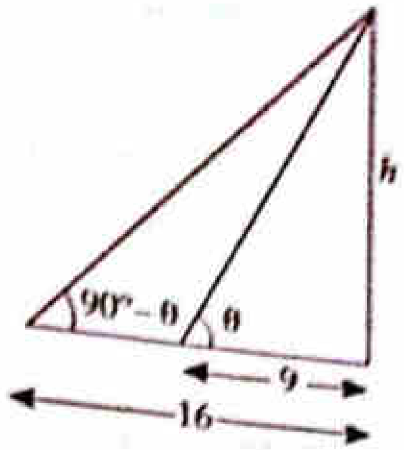
$\tan(90° - \theta) = \frac{h}{16}$

$\Rightarrow h = 16\cot\theta$

Multiplying, $h^2 = (9\tan\theta)(16\cot\theta)$

$$h^2 = 9 \times 16$$

$$h = 3 \times 4 = 12 \text{ ft}$$



(a) $\tan 60° \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$

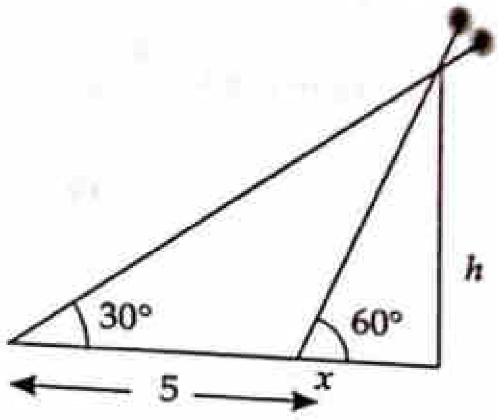
$\tan 30° = \frac{h}{x+5}$

$\Rightarrow (x+5)\tan 30° = h$

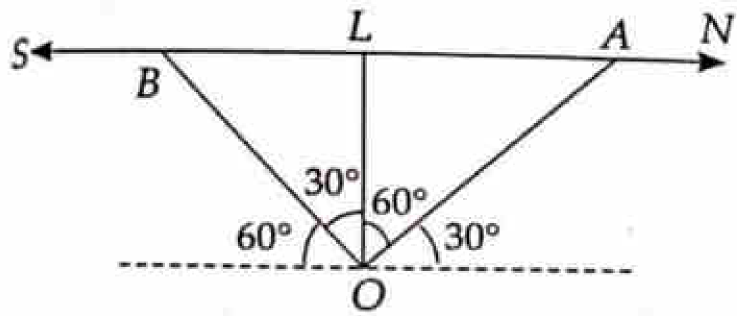
$\Rightarrow \left(\frac{h}{\sqrt{3}} + 5\right)\frac{1}{\sqrt{3}} = h \qquad \left(\because x = \frac{h}{\sqrt{3}}\right)$

$\Rightarrow \frac{h}{3} + \frac{5}{\sqrt{3}} = h \Rightarrow \frac{2h}{3} = \frac{5}{\sqrt{3}}$

$\Rightarrow h = \frac{5\sqrt{3}}{2}$ m



(c) O is the position of boy.



A and B are respectively position of bird at North and South ...

$\therefore$ Distance covered by bird $= AB = AL + BL$

$= 150 + 50 = 200$ m

Speed of bird $= \frac{200}{2}$ m/min

$= 100$ m/min

$= 100 \times \frac{1}{1000} \times 60$ kmph $= 6$ kmph

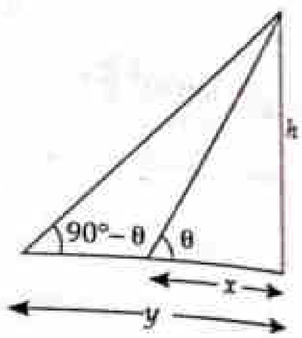
8. **(a)** $\tan\theta = \frac{h}{x}$

$\tan(90° - \theta) = \frac{h}{y}$

$\Rightarrow \cot\theta = \frac{h}{y}$

$\therefore \tan\theta \cdot \cot\theta = \frac{h}{x} \cdot \frac{h}{y}$

$\Rightarrow 1 = \frac{h^2}{xy} \Rightarrow h = \sqrt{xy}$



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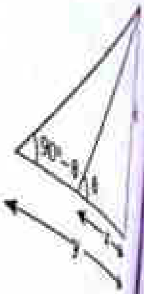
$AB = AL + BL$
$= 150 + 50 = 200$ m

/min

/min

$\frac{1}{1000} \times 60$ kmph $= 6$ kmph



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13 Advanced Trigonometric Identities*

1. **Compound Angle :** Sum or difference of two angles is called compound angle. $A + B$, $A - B$, $A + B + C$ etc are compound angle.

Important formulae :

1.1 $\sin(A+B) = \sin A \cos B + \cos A \sin B$

1.2 $\cos(A+B) = \cos A \cos B - \sin A \sin B$

1.3 $\sin(A-B) = \sin A \cos B - \cos A \sin B$

1.4 $\cos(A-B) = \cos A \cos B + \sin A \sin B$

1.5 $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

1.6 $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

1.7 $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

1.8 $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

1.9.1 $\tan\left(\frac{\pi}{4}+\theta\right) = \dfrac{1+\tan\theta}{1-\tan\theta}$ $\left(\because \tan\frac{\pi}{4} = 1\right)$

1.9.2 $\tan\left(\frac{\pi}{4}-\theta\right) = \dfrac{1-\tan\theta}{1+\tan\theta}$

1.9.3 $\cot\left(\frac{\pi}{4}+\theta\right) = \dfrac{\cot\theta - 1}{\cot\theta + 1}$ $\left(\because \cot\frac{\pi}{4} = 1\right)$

1.9.4 $\cot\left(\frac{\pi}{4}-\theta\right) = \dfrac{\cot\theta + 1}{\cot\theta - 1}$

These results are obtained by putting $A = \frac{\pi}{4}$, $B = \theta$ respectively in 1.5, 1.6, 1.7, 1.8

1.10 (More results)

1.10.1 $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

1.10.2 $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

1.10.2 $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

2. $\sin(A+B)$, $\sin(A-B)$, $\cos(A+B)$ and $\cos(A-B)$ can be transformed to get the following results.

2.1 $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

2.2 $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

2.3 $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

2.4 $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

2.5 $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

2.6 $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

2.7 $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

2.8 $\cos C - \cos D = 2\sin\frac{C+D}{2}.\sin\frac{D-C}{2}$

2.9 $\tan C + \tan D = \frac{\sin(C+D)}{\cos C\cos D}$

2.10 $\tan C - \tan D = \frac{\sin(C-D)}{\cos C\cos D}$

2.11 $\cot C + \cot D = \frac{\sin(D+C)}{\sin C\sin D}$

2.12 $\cot C - \cot D = \frac{\sin(D-C)}{\sin C\sin D}$

3. **Multiple and submultiple angle :** Integral multiple of θ i.e. $2\theta, 3\theta, 4\theta$ etc. are called multiple angle, while $\frac{\theta}{2}, \frac{\theta}{3}, \frac{\theta}{4}$ etc. are called submultiple angle. Formulae for Multiple and submultiple angle can be derived, from compound angle formulae as follows

$\because$ $\sin(A+B) = \sin A\cos B + \cos A\sin B$

Putting, $A = B$

$\sin(A+A) = \sin A\cos A + \cos A\sin A$

or, $\sin 2A = 2\sin A\cos A$

Again replacing $2A$ by A and A by $\frac{A}{2}$

$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$ etc.

Important formulae :

3.1 $\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$

3.2 $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$

Replacing $2A$ by A and A by $\frac{A}{2}$

$\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2} = 2\cos^2\frac{A}{2} - 1 = 1 - 2\sin^2\frac{A}{2} = \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$

3.3 $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

Replacing A by $\dfrac{A}{2}$

$$\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$$

3.4 $\sin 3A = 3\sin A - 4\sin^3 A$

3.5 $\cos 3A = 4\cos^3 A - 3\cos A$

3.6 $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

3.7 Some special condition

3.7.1 $\because\ \cos 2A = 2\cos^2 A - 1$

$$\therefore\ 1+\cos 2A = 2\cos^2 A \text{ or, } 1+\cos A = 2\cos^2\frac{A}{2}$$

3.7.2 $\because\ \cos 2A = 1 - 2\sin^2 A$

$$\therefore\ 1-\cos 2A = 2\sin^2 A \text{ or, } 1-\cos A = 2\sin^2\frac{A}{2}$$

3.7.3 $\because\ \sin 3A = 3\sin A - 4\sin^3 A$

$$\therefore\ \sin^3 A = \frac{3\sin A - \sin 3A}{4}$$

3.7.4 $\because\ \cos 3A = 4\cos^3 A - 3\cos A$

$$\therefore\ \cos^3 A = \frac{3\cos A + \cos 3A}{4}$$

3.7.5 $\cot(A+B) = \dfrac{\cot A\cot B - 1}{\cot B + \cot A}$

$$\Rightarrow\ \cot 2A = \frac{\cot^2 A - 1}{2\cot A}$$

3.7.6 $\because\ \sin^2 A + \cos^2 A \pm 2\sin A\cos A = 1 \pm \sin 2A$

$$\therefore\ 1 \pm \sin 2A = (\sin A \pm \cos A)^2$$

4 Must learn it for shortcut

4.1 $\sin\theta\sin(60^\circ-\theta)\sin(60^\circ+\theta) = \dfrac{3}{4}\sin 3\theta$

4.2 $\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta) = \dfrac{3}{4}\cos 3\theta$

4.3 $\tan\theta\tan(60^\circ-\theta)\tan(60^\circ+\theta) = \tan 3\theta$

4.4 $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$

4.5 $\cos 36^\circ = \dfrac{\sqrt{5}+1}{4}$

4.6 $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

4.7 $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

4.8 $\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$

4.9 $\cot 15^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3}$

4.10 $\tan 22\frac{1}{2}^\circ = \sqrt{2}-1$

4.11 $\cot 22\frac{1}{2}^\circ = \sqrt{2}+1$

5. **Quadrant Rule.**

α equals in rad	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$90-\theta$	$\cos\theta$	$\sin\theta$	$\cot\theta$
$90+\theta$	$\cos\theta$	$-\sin\theta$	$-\cot\theta$
$180-\theta$	$\sin\theta$	$-\cos\theta$	$-\tan\theta$
$180+\theta$	$-\sin\theta$	$-\cos\theta$	$\tan\theta$
$270-\theta$	$-\cos\theta$	$-\sin\theta$	$\cot\theta$
$270+\theta$	$-\cos\theta$	$\sin\theta$	$-\cot\theta$
$360-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$360+\theta$	$\sin\theta$	$\cos\theta$	$\tan\theta$
$2n\pi-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$
$2n\pi+\theta$	$\sin\theta$	$\cos\theta$	$\tan\theta$

To realise above result, we have following two steps—

Step–I. : In different quadrant, sign of *t*-ratio value are as follows.

Quadrant	I	II	III	IV
$\sin\theta$	+ ve	+ ve	– ve	– ve
$\cos\theta$	+ ve	– ve	– ve	+ ve
$\tan\theta$	+ ve	– ve	+ ve	– ve

Step–II.: (i) If angle is $(90^\circ \pm \theta)$ or $(270^\circ + \theta)$; change sin into cos, cos into sin, tan into cot, cot into tan etc.

(ii) If angle is $(180^\circ \pm \theta)$ or $(360^\circ \pm \theta)$ donot make any change i.e sin remains sin, cos remains cos etc.

Some selected *t*-ratio value between 0° and 360°

α	(in rad)	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	U.D.
120°	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135°	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1
150°	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{-1}{\sqrt{3}}$
180°	π	0	-1	0
210°	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
225°	$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1
240°	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
270°	$\frac{3\pi}{2}$	-1	0	U.D.
300°	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315°	$\frac{7\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1
330°	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$
360°	2π	0	1	0

(ii) $\tan 75° = \tan(45° + 30°)$

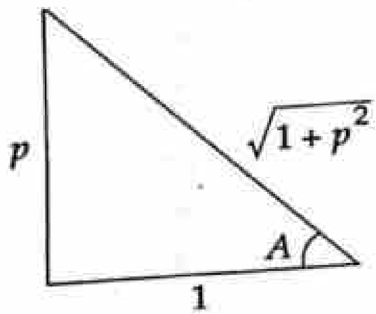
$$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$$

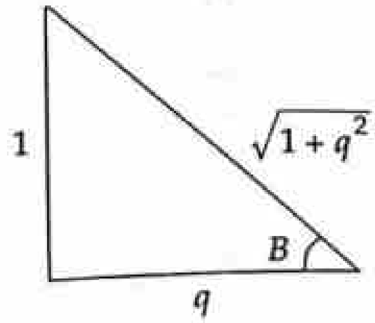
$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \cdot \left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$$

$$= \frac{(\sqrt{3}+1)^2}{3-1} = \frac{3+1+2\sqrt{3}}{2} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

2. If $\tan A = p$ and $\cot B = q$ then express $\sin(A+B)$ in terms of p and q

Solution : $\sin(A+B) = \sin A \cos B + \cos A \sin B$





$$= \frac{p}{\sqrt{1+p^2}} \cdot \frac{q}{\sqrt{1+q^2}} + \frac{1}{\sqrt{1+p^2}} \cdot \frac{1}{\sqrt{1+q^2}} = \frac{pq+1}{\sqrt{(1+p^2)(1+q^2)}}$$

3. If $\sin\theta = 3\sin(\theta + 2\alpha)$ then prove that $\tan(\theta+\alpha) + 2\tan\alpha = 0$

Solution : Given $\sin\theta = 3\sin(\theta + 2\alpha)$

$\sin\{(\theta+\alpha) - \alpha\} = 3\sin\{(\theta+\alpha)+\alpha\}$ ... $\cos\alpha + \cos(\theta+\alpha)\sin\alpha]$

or, $\cot A \cot 2A - 1 = \cot 3A \cot 2A + \cot 3A \cot A$

or $\cot A \cot 2A - \cot 3A \cot 2A - \cot 3A \cot A = 1$, Proved.

5. Prove that $\tan 40^\circ + 2\tan 10^\circ = \tan 50^\circ$

Solution : $\because \quad 40^\circ + 10^\circ = 50^\circ$

$\therefore \quad \tan(40^\circ + 10^\circ) = \tan 50^\circ$

or, $\dfrac{\tan 40^\circ + \tan 10^\circ}{1 - \tan 40^\circ + \tan 10^\circ} = \tan 50^\circ$

or, $\tan 40^\circ + \tan 10^\circ = \tan 50^\circ (1 - \tan 40^\circ \tan 10^\circ)$

or, $\tan 40^\circ + \tan 10^\circ = \tan 50^\circ - \tan 50^\circ \tan 40^\circ \tan 10^\circ$

or, $\tan 40^\circ + \tan 10^\circ = \tan 50^\circ - \cot 40^\circ \tan 40^\circ \tan 10^\circ$

($\because \tan(90^\circ - \theta) = \cot\theta$, here $\theta = 40^\circ$)

or, $\tan 40^\circ + \tan 10^\circ = \tan 50^\circ - 1\cdot\tan 10^\circ$ ($\because \tan\theta\cot\theta = 1$)

or, $\tan 40^\circ + 2\tan 10^\circ = \tan 50^\circ$, Proved.

6. If $A + B = \frac{\pi}{4}$ then prove that $(\cot A - 1)(\cot B - 1) = 2$.

Solution : $A + B = \frac{\pi}{4} \Rightarrow \cot(A + B) = \cot\frac{\pi}{4}$

$\Rightarrow \dfrac{\cot A \cot B - 1}{\cot B + \cot A} = 1$

$\Rightarrow \cot A \cot B - 1 = \cot B + \cot A$

$\Rightarrow \cot A \cot B - \cot B - \cot A = 1$

Adding '1' both sides,

$\cot A \cot B - \cot B - \cot A + 1 = 1 + 1$

or, $\cot B(\cot A - 1) - 1(\cot A - 1) = 2$

or, $(\cot A - 1)(\cot B - 1) = 2$, Proved.

7. Prove that $\dfrac{1}{\tan 3A - \tan A} - \dfrac{1}{\cot 3A - \cot A} = \cot 2A$.

Solution : L.H.S. $= \dfrac{1}{\tan 3A - \tan A} - \dfrac{1}{\cot 3A - \cot A}$

$= \dfrac{1}{\quad} \quad 1$

8. Prove that $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)=1$.

Solution : L.H.S. $=\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)$

$$=\frac{\cot\theta-1}{\cot\theta+1}\cdot\frac{\cot\theta+1}{\cot\theta-1}=1$$

9. Prove that $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{3\pi}{9}\sin\frac{4\pi}{9}=\frac{3}{16}$

Solution : L.H.S. $=\sin\left(\frac{180^\circ}{9}\right)\sin\left(2\cdot\frac{180^\circ}{9}\right)\sin\left(3\cdot\frac{180^\circ}{9}\right)\sin\left(4\cdot\frac{180^\circ}{9}\right)$

$$=\sin 20^\circ\sin 40^\circ\sin 60^\circ\sin 80^\circ$$

$$=\frac{1}{2}(2\sin 20^\circ\sin 40^\circ)\frac{\sqrt{3}}{2}\cdot\sin 80^\circ \qquad \left(\because \sin 60^\circ=\frac{\sqrt{3}}{2}\right)$$

$$=\frac{\sqrt{3}}{4}\{\cos(20^\circ-40^\circ)-\cos(20^\circ+40^\circ)\}\sin 80^\circ$$

$(\because 2\sin A\sin B=\cos(A-B)-\cos(A+B))$

$$=\frac{\sqrt{3}}{4}\{\cos 20^\circ-\cos 60^\circ\}\sin 80^\circ \qquad (\because \cos(-\theta)=\cos\theta)$$

$$=\frac{\sqrt{3}}{4}\left(\sin 80^\circ\cos 20^\circ-\frac{1}{2}\sin 80^\circ\right) \qquad \left(\because \cos 60^\circ=\frac{1}{2}\right)$$

$$=\frac{\sqrt{3}}{4}\left(\frac{2\sin 80^\circ\cos 20^\circ-\sin 80^\circ}{2}\right)$$

$$=\frac{\sqrt{3}}{8}\{\sin(80^\circ+20^\circ)+\sin(80^\circ-20^\circ)-\sin 80^\circ\}$$

$$=\frac{\sqrt{3}}{8}(\sin 100^\circ+\sin 60^\circ-\sin 80^\circ)$$

$$=\frac{\sqrt{3}}{8}\left\{\sin(180^\circ-80^\circ)+\frac{\sqrt{3}}{2}-\sin 80^\circ\right\}$$

$$=\frac{\sqrt{3}}{8}\left(\sin 80^\circ+\frac{\sqrt{3}}{2}-\sin 80^\circ\right)=\frac{\sqrt{3}}{8}\cdot\frac{\sqrt{3}}{2}=\frac{3}{16}$$

10. Prove that $4\cos\theta\cos(60^\circ+\theta)\cos(60^\circ-\theta)=\cos 3\theta$.

Solution : L.H.S. $=4\cos\theta\cos(60^\circ+\theta)\cos(60^\circ-\theta)$

$$=2\cos\theta\{2\cos(60^\circ+\theta)\cos(60^\circ-\theta)\}$$

$$=2\cos\theta[2\cos\{(60^\circ+\theta)-(60^\circ-\theta)\}+\cos\{(60^\circ+\theta)+(60^\circ-\theta)\}]$$

$(\because 2\cos A\cos B=\cos(A-B)+\cos(A+B);$ (here $A=60^\circ+\theta$ and $B=60^\circ-\theta$)

$$=2\cos\theta[\cos 2\theta+\cos 120^\circ]$$

$$=2\cos\theta\cos 2\theta+2\cos\theta\left(\frac{-1}{2}\right) \qquad \left(\because \cos 120^\circ=\frac{-1}{2}\right)$$

$= [\cos(\theta - 2\theta) + \cos(\theta + 2\theta)] - \cos\theta$ (2 cos A cos B formula)

$= \cos\theta + \cos 3\theta - \cos\theta = \cos 3\theta.$ ($\because \cos(-\theta) = \cos\theta$)

9. Prove that the value of $\cos^2\theta + \cos^2(\alpha + \theta) - 2\cos\alpha\cos\theta\cos(\alpha + \theta)$ is independent of θ

Solution : Given expression $= \cos^2\theta + \cos^2(\alpha + \theta) - \{\cos(\alpha - \theta) + \cos(\alpha + \theta)\}\cos(\alpha + \theta)$

$= \cos^2\theta + \cos^2(\alpha + \theta) - \cos(\alpha - \theta)\cos(\alpha + \theta) - \cos^2(\alpha + \theta)$

$= \cos^2\theta - \cos(\alpha - \theta)\cos(\alpha + \theta)$

$= \cos^2\theta - (\cos^2\alpha - \sin^2\theta)$

$= \cos^2\theta + \sin^2\theta - \cos^2\alpha = 1 - \cos^2\alpha = \sin^2\alpha$

Which is independent of θ

12. Prove that $\dfrac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \dfrac{\tan 3\theta + \tan\theta}{1 - \tan 3\theta\tan\theta}$.

Solution : [**Key** : $\theta + 7\theta = 3\theta + 5\theta = 8\theta$]

$$\text{L.H.S.} = \frac{(\sin 7\theta + \sin\theta) + (\sin 5\theta + \sin 3\theta)}{(\cos 7\theta + \cos\theta) + (\cos 5\theta + \cos 3\theta)}$$

$$= \frac{2\sin\frac{7\theta+\theta}{2}\cos\frac{7\theta-\theta}{2} + 2\sin\frac{5\theta+3\theta}{2}\cos\frac{5\theta-3\theta}{2}}{2\cos\frac{7\theta+\theta}{2}.\cos\frac{7\theta-\theta}{2} + 2\cos\frac{5\theta+3\theta}{2}\cdot\cos\frac{5\theta-3\theta}{2}}$$

$$= \frac{2\sin 4\theta\cos 3\theta + 2\sin 4\theta\cos\theta}{2\cos 4\theta\cos 3\theta + 2\cos 4\theta\cos\theta} = \frac{2\sin 4\theta(\cos 3\theta + \cos\theta)}{2\cos 4\theta(\cos 3\theta + \cos\theta)}$$

$$= \tan 4\theta = \tan(3\theta + \theta) = \frac{\tan 3\theta + \tan\theta}{1 - \tan 3\theta\tan\theta}$$

13. Prove that $\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha + \beta + \gamma)$

$$= 4\cos\frac{\alpha+\beta}{2}\cos\frac{\beta+\gamma}{2}\cos\frac{\gamma+\alpha}{2}$$

Solution : L.H.S. $= (\cos\alpha + \cos\beta) + \cos(\alpha + \beta + \gamma) + \cos\gamma$

$$= 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + 2\cos\frac{\alpha+\beta+\gamma+\gamma}{2}\cos\frac{\alpha+\beta+\gamma-\gamma}{2}$$

$$= 2\cos\frac{\alpha+\beta}{2}\left\{\cos\frac{\alpha-\beta}{2} + \cos\frac{\alpha+\beta+2\gamma}{2}\right\}$$

$$\alpha+\beta \left\{ \quad \frac{\alpha-\beta}{2} + \frac{\alpha+\beta+2\gamma}{2} \quad \cos \frac{\frac{\alpha-\beta}{2} - \frac{\alpha+\beta+2\gamma}{2}}{} \right\}$$

14. If cosec A + sec A = cosec B + sec B then prove that $\tan A \tan B = \cot \frac{A+B}{2}$.

Solution : Given that cosec A + sec A = cosec B + sec B

or, cosec A − cosec B = sec B − sec A

or, $\frac{1}{\sin A} - \frac{1}{\sin B} = \frac{1}{\cos B} - \frac{1}{\cos A}$

or, $\frac{\sin B - \sin A}{\sin A \sin B} = \frac{\cos A - \cos B}{\cos B \cos A}$

or, $\frac{2\cos\frac{B+A}{2}\sin\frac{B-A}{2}}{\sin A \sin B} = \frac{2\sin\frac{A+B}{2}\sin\frac{B-A}{2}}{\cos B \cos A}$

or, $\frac{\cos\frac{B+A}{2}}{\sin A \sin B} = \frac{\sin\frac{A+B}{2}}{\cos B \cos A}$

or, $\frac{\cos\frac{A+B}{2}}{\sin\frac{A+B}{2}} = \frac{\sin A \sin B}{\cos A \cos B}$

or, $\cot\frac{A+B}{2} = \tan A \tan B$

or, $\tan A \tan B = \cot\frac{A+B}{2}$.

15. Prove that $\left(\frac{\sin 2A - \sin 2B}{\cos 2A + \cos 2B}\right)^n + \left(\frac{\cos 2A - \cos 2B}{\sin 2A + \sin 2B}\right)^n = 2\tan^n(A-B)$ or 0

According as n is even or n is odd.

Solution : L.H.S. $= \left(\frac{\sin 2A - \sin 2B}{\cos 2A + \cos 2B}\right)^n + \left(\frac{\cos 2A - \cos 2B}{\sin 2A + \sin 2B}\right)^n$

$$= \left[\frac{2\cos\frac{2A+2B}{2}\sin\frac{2A-2B}{2}}{2\cos\frac{2A+2B}{2}\cos\frac{2A-2B}{2}}\right]^n + \left[\frac{2\sin\frac{2A+2B}{2}\sin\frac{2B-2A}{2}}{2\sin\frac{2A+2B}{2}\cos\frac{2A-2B}{2}}\right]^n$$

$$= \left(\frac{\sin(A-B)}{\cos(A-B)}\right)^n + \left(\frac{-\sin(A-B)}{\cos(A-B)}\right)^n$$

$$= (\tan(A-B))^n + (-\tan(A-B))^n$$

$$= \tan^n(A-B) + (-1)^n \tan^n(A-B)$$

When n is an even number $(-1)^n = 1$

$\therefore$ L.H.S $= \tan^n(A-B) + \tan^n(A-B)$

$= 2\tan^n(A-B)$

When n is an odd number $(-1)^n = -1$

$\therefore$ L.H.S $= \tan^n(A-B) - \tan^n(A-B) = 0$.

16. If $x \cot(\theta + 120°) = y \cot(\theta - 30°)$ then prove that $\frac{x+y}{x-y} = 2\cos 2\theta$

Solution : Given that $x \cot(\theta + 120°) = y \cot(\theta - 30°)$

or, $\frac{x}{y} = \frac{\cot(\theta - 30°)}{\cot(\theta + 120°)}$

By componendo–dividendo

$$\frac{x+y}{x-y} = \frac{\cot(\theta - 30°) + \cot(\theta + 120°)}{\cot(\theta - 30°) - \cot(\theta + 120°)}$$

We know that $\cot C + \cot D = \frac{\sin(D+C)}{\sin C \sin D}$ & $\cot C - \cot D = \frac{\sin(D-C)}{\sin C \sin D}$

Thus, we have $$\frac{x+y}{x-y} = \frac{\left(\frac{\sin(\theta + 120° + \theta - 30°)}{\sin(\theta + 120°)\sin(\theta - 30°)}\right)}{\frac{\sin(\theta + 120° - \theta + 30°)}{\sin(\theta + 120°)\sin(\theta - 30°)}}$$

or, $$\frac{x+y}{x-y} = \frac{\sin(90° + 2\theta)}{\sin 150°} = \frac{\cos 2\theta}{\left(\frac{1}{2}\right)}$$

or, $$\frac{x+y}{x-y} = 2\cos 2\theta. \text{ Proved.}$$

17. If $2 \tan A = 3 \tan B$ then prove that $\sin(A + B) = 5 \sin(A - B)$.

Solution : Given $2 \tan A = 3 \tan B$

$\therefore \quad \frac{\tan A}{\tan B} = \frac{3}{2}$

Using componendo-dividendo,

$$\frac{\tan A + \tan B}{\tan A - \tan B} = \frac{3+2}{3-2}$$

or, $$\frac{\left(\frac{\sin(A+B)}{\cos A \cos B}\right)}{\left(\frac{\sin(A-B)}{\cos A \cos B}\right)} = \frac{5}{1}$$

$\left[\left(\because \tan C + \tan D = \frac{\sin(C+D)}{\cos C \cos D}\right) \text{ and } \tan C - \tan D = \frac{\sin(C-D)}{\cos C \cos D}\right)\right]$

or, $\frac{\sin(A+B)}{\sin(A-B)} = 5$

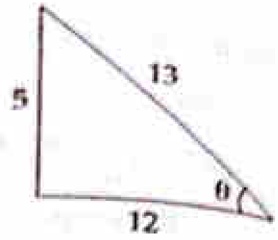
or, $\sin(A + B) = 5 \sin(A - B)$

18. If $\sin\theta = \frac{5}{13}$, $0 < \theta < \frac{\pi}{2}$ then find the value of following.

(i) $\sin 2\theta$ (ii) $\cos 2\theta$ (iii) $\tan 2\theta$ (iv) $\sin 3\theta$

(v) $\cos 3\theta$ (vi) $\sin 4\theta$

Solution : $\because$ $\sin\theta = \frac{5}{13}$



$\therefore$ $\cos\theta = \frac{12}{13}$

and $\tan\theta = \frac{5}{12}$

Now, (i) $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{5}{13}\cdot\frac{12}{13} = \frac{120}{169}$

(ii) $\cos 2\theta = 2\cos^2\theta - 1 = 2\cdot\left(\frac{12}{13}\right)^2 - 1 = \frac{288}{169} - 1 = \frac{119}{169}$

(iii) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2\cdot\frac{5}{12}}{1-\left(\frac{5}{12}\right)^2} = \dfrac{\frac{10}{12}}{\frac{144-25}{144}} = \dfrac{10\times 12}{119} = \dfrac{120}{119}$

(iv) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta = 3\cdot\frac{5}{13} - 4\left(\frac{5}{13}\right)^3$

$$= \frac{15\cdot 13^2 - 500}{13^3} = \frac{2035}{2197}$$

(v) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta = 4\left(\frac{12}{13}\right)^3 - 3\cdot\frac{12}{13}$

$$= \frac{4\cdot 12^3 - 36\cdot 13^2}{13^3} = \frac{828}{2197}$$

(vi) $\sin 4\theta = \sin\left(2(2\theta)\right) = 2\sin 2\theta\cos 2\theta = 2\cdot\frac{120}{169}\cdot\frac{119}{169} = \frac{28560}{28561}$

19. Prove that $\dfrac{1+\tan\theta}{1-\tan\theta} = \dfrac{1+\sin 2\theta}{\cos 2\theta}$

Solution : L.H.S $= \dfrac{1+\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin\theta}{\cos\theta}} = \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\times\dfrac{\cos\theta+\sin\theta}{\cos\theta+\sin\theta}$

$$= \frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta}{\cos^2\theta-\sin^2\theta} = \frac{1+\sin 2\theta}{\cos 2\theta}$$

20. Prove that $\cot^2\frac{\theta}{2} - \tan^2\frac{\theta}{2} = 4\cot\theta\,\mathrm{cosec}\,\theta$

Solution : L.H.S $= \left(\cot\frac{\theta}{2} - \tan\frac{\theta}{2}\right)\left(\cot\frac{\theta}{2} + \tan\frac{\theta}{2}\right)$

$$= \left(\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} - \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)\left(\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} + \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)$$

$$= \left(\frac{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)\left(\frac{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$=\left(\frac{2\cdot\cos\theta}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)\left(\frac{2\cdot 1}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$=\frac{2\cos\theta}{\sin\theta}\cdot\frac{2}{\sin\theta}=4\cot\theta\,\mathrm{cosec}\,\theta=\text{R.H.S.}$$

21. If $\sin\alpha+\sin\beta=a$ and $\cos\alpha+\cos\beta=b$ then proved that

$$\cos(\alpha-\beta)=\frac{a^2+b^2-2}{2}\ \textit{and}\ \tan\frac{\alpha-\beta}{2}=\pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$$

Solution : R.H.S. $=\dfrac{a^2+b^2-2}{2}=\dfrac{(\sin\alpha+\sin\beta)^2+(\cos\alpha+\cos\beta)^2-2}{2}$

$$=\frac{\sin^2\alpha+\sin^2\beta+2\sin\alpha\sin\beta+\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta-2}{2}$$

$$=\frac{\left(\sin^2\alpha+\cos^2\alpha\right)+\left(\sin^2\beta+\cos^2\beta\right)+2(\cos\alpha\cos\beta+\sin\alpha\sin\beta)-2}{2}$$

$$=\frac{1+1+2\cos(\alpha-\beta)-2}{2}=\frac{2\cos(\alpha-\beta)}{2}=\cos(\alpha-\beta)=\text{L.H.S.}$$

Second part : $\because 1-\cos\theta=2\sin^2\frac{\theta}{2}$ and $1+\cos\theta=2\cos^2\frac{\theta}{2}$

$$\therefore\ \frac{1-\cos\theta}{1+\cos\theta}=\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}=\tan^2\frac{\theta}{2}$$ [Note]

Hence, $\tan^2\dfrac{\alpha-\beta}{2}=\dfrac{1-\cos(\alpha-\beta)}{1+\cos(\alpha-\beta)}$

$$=\frac{1-\frac{a^2+b^2-2}{2}}{1+\frac{a^2+b^2-2}{2}}=\frac{2-a^2-b^2+2}{2+a^2+b^2-2}=\frac{4-a^2-b^2}{a^2+b^2}$$

$$\therefore\ \tan\frac{\alpha-\beta}{2}=\pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$$

22. Express $\sin^6x+\cos^6x$ in terms of $\sin 2x$ and hence find the maximum and minimum value of $\sin^6x+\cos^6x$

Solution : $\sin^6x+\cos^6x=(\sin^2x)^3+(\cos^2x)^3$

$$=(\sin^2x+\cos^2x)(\sin^4x-\sin^2x\cos^2x+\cos^4x)$$

$$(\because a^3+b^3=(a+b)(a^2-ab+b^2))$$

$$=1\cdot(\sin^4x+\cos^4x+2\sin^2x\cos^2x-3\sin^2x\cos^2x)$$

$$=(\sin^2x+\cos^2x)^2-\frac{3}{4}(2\sin x\cos x)^2$$

$$=1-\frac{3}{4}\sin^2 2x$$

Now, $-1\le\sin 2x\le 1\Rightarrow 0\le\sin^2 2x\le 1.$

at $\sin^2 2x = 0$, $\sin^6 x + \cos^6 x = 1 - \frac{3}{4} \cdot 0 = 1$

at $\sin^2 2x = 1$, $\sin^6 x + \cos^6 x = 1 - \frac{3}{4} \cdot 1 = \frac{1}{4}$

Hence, Maximum and Minimum value of $\sin^6 x + \cos^6 x$ are respectively 1 and $\frac{1}{4}$.

23. *Prove that* $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

Solution : L.H.S. $= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + \dfrac{8}{\tan 8\alpha}$

$$= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + \frac{8}{\tan 2(4\alpha)}$$

$$= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + \frac{8(1-\tan^2 4\alpha)}{2\tan 4\alpha}$$

$$= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + \frac{4(1-\tan^2 4\alpha)}{\tan 4\alpha}$$

$$= \tan\alpha + 2\tan 2\alpha + \frac{4\tan^2 4\alpha + 4 - 4\tan^2 4\alpha}{\tan 4\alpha}$$

$$= \tan\alpha + 2\tan 2\alpha + \frac{4}{\tan 2(2\alpha)}$$

$$= \tan\alpha + 2\tan 2\alpha + 4 \cdot \frac{1-\tan^2 2\alpha}{2\tan 2\alpha}$$

$$= \tan\alpha + 2\tan 2\alpha + \frac{2(1-\tan^2 2\alpha)}{\tan 2\alpha}$$

$$= \tan\alpha + \frac{2\tan^2 2\alpha + 2 - 2\tan^2 2\alpha}{\tan 2\alpha}$$

$$= \tan\alpha + \frac{2}{\tan 2\alpha} = \tan\alpha + \frac{2(1-\tan^2\alpha)}{2\tan\alpha}$$

$$= \tan\alpha + \frac{1-\tan^2\alpha}{\tan\alpha} = \tan\alpha + \frac{1}{\tan\alpha} - \tan\alpha$$

$$= \frac{1}{\tan\alpha} = \cot\alpha = \text{R.H.S}$$

Second Method :

We know that $\cot\theta - \tan\theta = \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \frac{\cos 2\theta}{\left(\frac{\sin 2\theta}{2}\right)} = \frac{2\cos 2\theta}{\sin 2\theta}$$

or, $\cot\theta - \tan\theta = 2\cot 2\theta$ ... (i)

Now, L.H.S. $= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$

$$= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 4(2\cot 8\alpha)$$

$$= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 4(\cot 4\alpha - \tan 4\alpha)$$

$= \tan\alpha + 2\tan 2\alpha + 4\cot 4\alpha.$

$= \tan\alpha + 2\tan 2\alpha + 2(\cot 2\alpha - \tan 2\alpha)$; ((i) $\theta = 2\alpha$ in (i))

$= \tan\alpha + 2\cot 2\alpha$

$= \tan\alpha + \cot\alpha - \tan\alpha = \cot\alpha =$ R.H.S.

4. Prove that $\sin^3\theta + \sin^3(120^\circ + \theta) + \sin^3(240^\circ + \theta) = \frac{-3}{4}\sin 3\theta$.

Solution : $\because \sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

$$\therefore \sin^3\alpha = \frac{3\sin\alpha - \sin 3\alpha}{4}$$

Now
$$\text{L.H.S} = \frac{3\sin\theta - \sin 3\theta}{4} + \frac{3\sin(120^\circ+\theta) - \sin 3(120^\circ+\theta)}{4} + \frac{3\sin(240^\circ+\theta) - \sin 3(240^\circ+\theta)}{4}$$

$$= \frac{3}{4}\{\sin\theta + \sin(120^\circ+\theta) + \sin(240^\circ+\theta)\} - \frac{1}{4}\{\sin 3\theta + \sin(360^\circ+3\theta) + \sin(720^\circ+3\theta)\}$$

$$= \frac{3}{4}\left\{\sin\theta + 2\sin\frac{120^\circ+\theta+240+\theta}{2}\cos\frac{120^\circ+\theta-240^\circ-\theta}{2}\right\} - \frac{1}{4}\{\sin 3\theta + \sin 3\theta + \sin 3\theta\}$$

$$= \frac{3}{4}\{\sin\theta + 2\sin(180^\circ+\theta)\cos 60^\circ\} - \frac{1}{4}\{3\sin 3\theta\}$$

$$= \frac{3}{4}\left\{\sin\theta - 2\sin\theta\cdot\frac{1}{2}\right\} - \frac{3}{4}\sin 3\theta$$

$$= -\frac{3}{4}\sin 3\theta = \text{RHS}$$

5. Prove that $\cot\theta + \cot(60^\circ + \theta) + \cot(120^\circ + \theta) = 3\cot 3\theta$.

Solution : L.H.S. $= \cot\theta + \cot(60^\circ + \theta) + \cot(120^\circ + \theta)$

$$= \cot\theta + \frac{\cot 60^\circ \cot\theta - 1}{\cot\theta + \cot 60^\circ} + \frac{\cot 120^\circ \cot\theta - 1}{\cot\theta + \cot 120^\circ}$$

$$= \cot\theta + \frac{\frac{1}{\sqrt{3}}\cot\theta - 1}{\cot\theta + \frac{1}{\sqrt{3}}} + \frac{\frac{-1}{\sqrt{3}}\cot\theta - 1}{\cot\theta - \frac{1}{\sqrt{3}}}$$

$$= \cot\theta + \frac{\cot\theta - \sqrt{3}}{\sqrt{3}\cot\theta + 1} - \frac{\cot\theta + \sqrt{3}}{\sqrt{3}\cot\theta - 1}$$

$$= \cot\theta + \frac{(\cot\theta - \sqrt{3})(\sqrt{3}\cot\theta - 1) - (\cot\theta + \sqrt{3})(\sqrt{3}\cot\theta + 1)}{3\cot^2\theta - 1}$$

$$\theta + \frac{(\sqrt{3}\cot^2\theta - \cot\theta - 3\cot\theta + \sqrt{3}) - (\sqrt{3}\cot^2\theta + \cot\theta + 3\cot\theta + \sqrt{3})}{3\cot^2\theta - 1}$$

$$= \cot\theta + \left(\frac{-8\cot\theta}{3\cdot\cot^2\theta - 1}\right) = \frac{3\cot^3\theta - \cot\theta - 8\cot\theta}{3\cot^2\theta - 1}$$

$$= \frac{3\cot^3\theta - 9\cot\theta}{3\cot^2\theta - 1} = 3\left(\frac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}\right) = 3\cot 3\theta.$$

26. Prove that $\sin\theta\cos^3\theta - \cos\theta\sin^3\theta = \frac{1}{4}\sin 4\theta$.

Solution : L.H.S. $= \sin\theta\left(\frac{3\cos\theta + \cos 3\theta}{4}\right) - \cos\theta\left(\frac{3\sin\theta - \sin 3\theta}{4}\right)$

$$= \frac{1}{4}\{\sin\theta\cos 3\theta + \cos\theta\sin 3\theta\} = \frac{1}{4}\sin(\theta + 3\theta) = \frac{\sin 4\theta}{4}$$

27. If α and β are two distinct roots of $a\cos\theta + b\sin\theta = c$, prove that

(i) $\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$ (ii) $\cos\alpha + \cos\beta = \frac{2ac}{a^2+b^2}$

Solution : $\because$ α, β are roots of $a\cos\theta + b\sin\theta = c$

$\therefore$ $a\cos\alpha + b\sin\alpha = c$ ... (i)

and $a\cos\beta + b\sin\beta = c$ ... (ii)

equation (i) – (ii) $a(\cos\alpha - \cos\beta) + b(\sin\alpha - \sin\beta) = 0$

or, $a\cdot 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2} + b\,2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} = 0$

or, $2\sin\frac{\alpha-\beta}{2}\left(-a\sin\frac{\alpha+\beta}{2} + b\cos\frac{\alpha+\beta}{2}\right) = 0$

or, $-a\sin\frac{\alpha+\beta}{2} + b\cos\frac{\alpha+\beta}{2} = 0$

or, $b\cos\frac{\alpha+\beta}{2} = a\sin\frac{\alpha+\beta}{2}$

or, $\tan\frac{\alpha+\beta}{2} = \frac{b}{a}$

Now, $\sin(\alpha+\beta) = \dfrac{2\tan\frac{\alpha+\beta}{2}}{1+\tan^2\frac{\alpha+\beta}{2}} = \dfrac{2\frac{b}{a}}{1+\frac{b^2}{a^2}} = \dfrac{2ab}{a^2+b^2}$

Second part : Given equation is $b\sin\theta = c - a\cos\theta$

squaring both sides $b^2\sin^2\theta = c^2 + a^2\cos^2\theta - 2ac\cos\theta$

or, $b^2(1-\cos^2\theta) = c^2 + a^2\cos^2\theta - 2ac\cos\theta$

or, $(a^2+b^2)\cos^2\theta - 2ac\cos\theta + c^2 - b^2 = 0$

$\because$ two values of θ are α, β

$\therefore$ Roots of above equation are α, β

Hence sum of roots $= \cos\alpha + \cos\beta = \frac{-B}{A} = \frac{2ac}{a^2+b^2}$

28. Find the value of $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}$

Solution : Given $= \sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\left(\pi-\frac{3\pi}{8}\right)+\sin^4\left(\pi-\frac{\pi}{8}\right)$

$\left(\because \frac{5\pi}{8}=\pi-\frac{3\pi}{8}, \frac{7\pi}{8}=\pi-\frac{3}{8}\right)$

$= \sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{\pi}{8}$ $\quad (\because \sin(\pi-\theta)=\sin\theta)$

$$= 2\left\{\left(\sin^2\frac{\pi}{8}\right)^2+\left(\sin^2\frac{3\pi}{8}\right)^2\right\}$$

$$= 2\left\{\left(\frac{1-\cos\frac{\pi}{4}}{2}\right)^2+\left(\frac{1-\cos\frac{3\pi}{4}}{2}\right)^2\right\} \quad \left(\because \sin^2\frac{\theta}{2}=\frac{1-\cos\theta}{2}\right)$$

$$= \frac{2}{2^2}\left\{\left(1-\frac{1}{\sqrt{2}}\right)^2+\left(1+\frac{1}{\sqrt{2}}\right)^2\right\}$$

$$= \frac{1}{2}\left\{2\left(1+\frac{1}{2}\right)\right\} = 1+\frac{1}{2}=\frac{3}{2} \quad \left(\because (a+b)^2+(a-b)^2=2(a^2+b^2)\right)$$

29. Find the value of $\operatorname{cosec} 10^\circ - \sqrt{3}\sec 10^\circ$.

Solution : Given expression $= \operatorname{cosec} 10^\circ - \sqrt{3}\sec 10^\circ$

$$= \frac{1}{\sin 10^\circ}-\frac{\sqrt{3}}{\cos 10^\circ}$$

$$= \frac{\cos 10^\circ-\sqrt{3}\sin 10^\circ}{\sin 10^\circ\cos 10^\circ} = \frac{2\left(\frac{1}{2}\cos 10^\circ-\frac{\sqrt{3}}{2}\sin 10^\circ\right)}{\sin 10^\circ\cos 10^\circ}$$

$$= \frac{2\times 2(\sin 30^\circ\cos 10^\circ-\cos 30^\circ\sin 10^\circ)}{2\sin 10^\circ\cos 10^\circ}$$

$$= \frac{4\sin(30^\circ-10^\circ)}{\sin(2\times 10^\circ)} = \frac{4\sin 20^\circ}{\sin 20^\circ} = 4$$

30. Prove that $\sin 6^\circ\sin 42^\circ\sin 66^\circ\sin 78^\circ = \frac{1}{16}$.

Solution : L.H.S. $= \frac{1}{2}(2\sin 78^\circ\sin 42^\circ)\frac{1}{2}(2\sin 66^\circ\sin 6^\circ)$

$$= \frac{1}{2}\cdot\frac{1}{2}\{\cos(78^\circ-42^\circ)-\cos(78^\circ+42^\circ)\}\{\cos(66^\circ-6^\circ)-\cos(66^\circ+6^\circ)\}$$

$$= \frac{1}{4}(\cos 36^\circ-\cos 120^\circ)(\cos 60^\circ-\cos 72^\circ)$$

$$= \frac{1}{4}\left\{\frac{\sqrt{5}+1}{4}-\left(\frac{-1}{2}\right)\right\}\left\{\frac{1}{2}-\frac{\sqrt{5}-1}{4}\right\}$$

$\left(\because \cos 36^\circ = \frac{\sqrt{5}+1}{4}\right.$ तथा $\left.\cos 72^\circ = \sin 18^\circ = \frac{\sqrt{5}-1}{4}\right)$

$$= \frac{1}{4}\left(\frac{\sqrt{5}+1+2}{4}\right)\left(\frac{2-\sqrt{5}+1}{4}\right)$$

$$= \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}(3+\sqrt{5})(3-\sqrt{5})$$

$$= \frac{1}{16}\cdot\frac{1}{4}(9-5) = \frac{1}{16}\cdot\frac{1}{4}\cdot 4 = \frac{1}{16}$$

31. If $\alpha = \frac{\pi}{13}$, Prove that $\cos\alpha\cos 2\alpha\cos 3\alpha\cos 4\alpha\ \cos 5\alpha\cos 6\alpha = \frac{1}{64}$.

Solution : L.H.S $= \frac{1}{2\sin\alpha}(2\sin\alpha\cos\alpha)\cos 2\alpha\cos 3\alpha\cos 4\alpha\cos 5\alpha\cos 6\alpha$

$$= \frac{(2\sin 2\alpha\cos 2\alpha)}{2\times 2\sin\alpha}\cos 3\alpha\cos 4\alpha\cos 5\alpha\cos 6\alpha$$

$$= \frac{(2\sin 4\alpha\cos 4\alpha)}{2\times 4\sin\alpha}\cos 3\alpha\cos 5\alpha\cos 6\alpha$$

$$= \frac{\sin 8\alpha\cos 3\alpha\cos 5\alpha\cos 6\alpha}{8\sin\alpha}$$

But, $\alpha = \frac{\pi}{13} \Rightarrow 13\alpha = \pi \Rightarrow 8\alpha = \pi - 5\alpha$

$$\therefore\ \text{L.H.S.} = \frac{\sin(\pi-5\alpha)\cos 3\alpha\cos 5\alpha\cos 6\alpha}{8\sin\alpha}$$

$$= \frac{(2\sin 5\alpha\cos 5\alpha)\cos 3\alpha\cos 6\alpha}{2\times 8\sin\alpha}a$$

$$= \frac{\sin 10\alpha\cos 3\alpha\cos 6\alpha}{16\sin\alpha}$$

$$= \frac{\sin(\pi-3\alpha)\cos 3\alpha\cos 6\alpha}{16\sin\alpha} \qquad (\because\ 10\alpha + 3\alpha = \pi)$$

$$= \frac{(2\sin 3\alpha\cos 3\alpha)\cos 6\alpha}{2\times 16\sin\alpha} = \frac{2\sin 6\alpha\cos 6\alpha}{2\times 32\sin\alpha}$$

$$= \frac{\sin 12\alpha}{64\sin\alpha} = \frac{\sin(\pi-\alpha)}{64\sin\alpha} \qquad (\because\ 12\alpha + \alpha = \pi)$$

$$= \frac{\sin\alpha}{64\sin\alpha} = \frac{1}{64} = \text{R.H.S.}$$

32. Prove that $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14} = \frac{1}{64}$.

Solution : L.H.S. $= \sin\frac{\pi}{14}\cdot\sin\frac{3\pi}{14}\cdot\sin\frac{5\pi}{14}\cdot\sin\frac{\pi}{2}\cdot\sin\left(\pi-\frac{5\pi}{14}\right)$

$$\sin\left(\pi-\frac{3\pi}{14}\right)\sin\left(\pi-\frac{\pi}{14}\right)$$

$$= \sin\frac{\pi}{14}\cdot\sin\frac{3\pi}{14}\cdot\sin\frac{5\pi}{14}\cdot 1\cdot\sin\frac{5\pi}{14}\cdot\sin\frac{3\pi}{14}\cdot\sin\frac{\pi}{14}$$

$$= \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2$$

$$= \left\{\cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{14}\right)\right\}^2$$

$$= \left(\cos\frac{7\pi-\pi}{14}\cdot\cos\frac{7\pi-3\pi}{14}\cdot\cos\frac{7\pi-5\pi}{14}\right)^2$$

$$= \left(\cos\frac{3\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{\pi}{7}\right)^2$$

$$= \left[\frac{2\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}\cdot\cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}\right]^2$$

$$= \left[\frac{2}{2}\cdot\frac{\sin\frac{2\pi}{7}\cdot\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}}{2\sin\frac{\pi}{7}}\right]^2$$

$$= \left[\frac{\sin\frac{4\pi}{7}\cdot\cos\left(\pi-\frac{4\pi}{7}\right)}{4\sin\frac{\pi}{7}}\right]^2 \qquad \left(\because \frac{3\pi}{7}+\frac{4\pi}{7}=\pi\right)$$

$$= \left\{\frac{-\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}}{4\sin\frac{\pi}{7}}\right\}^2 \qquad (\because \cos(\pi-\theta)=\cos\theta)$$

$$= \left\{\frac{-2\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}}{2\cdot 4\sin\frac{\pi}{7}}\right\}^2 = \left\{\frac{-\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}\right\}^2$$

$$= \left\{\frac{-\sin\left(\pi+\frac{\pi}{7}\right)}{8\sin\frac{\pi}{7}}\right\}^2$$

$$= \left[\frac{\sin\frac{\pi}{7}}{8\sin\frac{\pi}{7}}\right]^2 = \frac{1}{8^2} = \frac{1}{64}$$

33. For positive integer n if

$f_n(\theta) = \tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\ldots\ldots(1+\sec 2^n\theta)$ then find the value of $f_2\left(\frac{\pi}{16}\right)$ and $f_5\left(\frac{\pi}{128}\right)$

Solution : Given that $f_n(\theta) = \tan(1+\sec\theta)(1+\sec 2\theta)(1+\sec 2^2\theta)\ldots(1+\sec 2^n\theta)$.

here, $\tan\frac{\theta}{2}(1+\sec\theta) = \tan\frac{\theta}{2}\left(1+\frac{1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right)$

$$\left[\because \cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \Rightarrow \sec\theta = \frac{1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right]$$

$$= \tan\frac{\theta}{2}\left(\frac{1-\tan^2\frac{\theta}{2}+1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right)$$

$$\therefore \quad \tan\frac{\theta}{2}(1+\sec\theta) = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \tan\theta$$

Replacing θ by $\theta, 2\theta, 2^2\theta$ $\frac{\theta}{2}$ .... one by one

$\tan\theta(1+\sec 2\theta) = \tan 2\theta$

$\tan 2\theta(1+\sec 2^2\theta) = \tan 2^2\theta$ ...... etc.

Using these identities

$$f_n(\theta) = \left\{\tan\frac{\theta}{2}(1+\sec\theta)\right\}(1+\sec 2\theta)(1+\sec 2^2\theta)\ldots(1+\sec 2^n\theta)$$

$$= \{\tan\theta(1+\sec 2\theta)\}(1+\sec 2^2\theta)\ldots\ldots(1+\sec 2^n\theta)$$

$$= \{\tan 2\theta(1+\sec 2^2\theta)\}(1+\sec 2^3\theta)\ldots\ldots(1+\sec 2^n\theta)$$

Proceeding in the same manner.

$$f_n(\theta) = \tan 2^{n-1}(1+\sec 2^n\theta) = \tan 2^n\theta.$$

$$\therefore \quad f_2\left(\frac{\pi}{16}\right) = \tan\left(2^2\cdot\frac{\pi}{16}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

$$f_5\left(\frac{\pi}{128}\right) = \tan\left(2^5\cdot\frac{\pi}{128}\right) = \tan\left(\frac{32\pi}{128}\right) = \tan\frac{\pi}{4} = 1.$$

34. Find the value of following.

(i) $\cot 82\frac{1}{2}^\circ$ (ii) $\tan 142\frac{1}{2}^\circ$

Solution : (i) We know that $1+\cos\theta = 2\cos^2\frac{\theta}{2}$ and $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$\therefore \quad \frac{1+\cos\theta}{\sin\theta} = \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \cot\frac{\theta}{2}$$

or, $\cot\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta}$

[Note]

putting $\theta = 165^\circ$ $\quad \cot\frac{165^\circ}{2} = \frac{1+\cos 165^\circ}{\sin 165^\circ}$

or $\cot 82\frac{1}{2}^\circ = \frac{1-\cos 15^\circ}{\sin 15^\circ} = \frac{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

$= \frac{2\sqrt{2}-\sqrt{3}-1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{2\sqrt{6}+2\sqrt{2}-3-\sqrt{3}-\sqrt{3}-1}{3-1}$

$= \frac{2\sqrt{6}+2\sqrt{2}-4-2\sqrt{3}}{2} = \sqrt{6}+\sqrt{2}-2-\sqrt{3}$

(ii) $\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}$ and $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\therefore \tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

putting $\theta = 285^\circ$

$\tan\frac{285^\circ}{2} = \frac{1-\cos 285^\circ}{\sin 285^\circ} = \frac{1-\sin 15^\circ}{-\cos 15^\circ}$

$= \frac{1-\frac{\sqrt{3}-1}{2\sqrt{2}}}{-\frac{\sqrt{3}+1}{2\sqrt{2}}} = -\frac{2\sqrt{2}-\sqrt{3}+1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= -\left(\frac{2\sqrt{6}-2\sqrt{2}-3+\sqrt{3}+\sqrt{3}-1}{3-1}\right)$

$= -\left(\frac{2\sqrt{6}-2\sqrt{2}-4+2\sqrt{3}}{2}\right) = -\left(\sqrt{6}-\sqrt{2}-2+\sqrt{3}\right)$

Exercise—13A

1. If p and q are two quantities such that $p^2+q^2=1$, then maximum value of $p+q$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 2

2. If θ is real then $3-\cos\theta+\cos\left(\theta+\frac{\pi}{3}\right)$ lies in the interval

(a) $[-2, 3]$ (b) $[-2, 1]$ (c) $[2, 4]$ (d) $[1, 5]$

3. If $\sin A = \frac{1}{\sqrt{10}}$ and $\sin B = \frac{1}{\sqrt{5}}$, where A and B are acute angle then $A+B$ is

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

4. If $\cos(\theta-\alpha)=a$, $\cos(\theta-\beta)=b$, then the value of $\sin^2(\alpha-\beta)+2ab\cos(\alpha-\beta)$ is

(a) a^2+b^2 (b) a^2-b^2 (c) b^2-a^2 (d) $-a^2-b^2$

5. A positive angle is divided into two parts such that their tangents are respectively $\frac{1}{2}$ and $\frac{1}{3}$. The measure of the angles is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

6. In ΔPQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are roots of equation, $ax^2 + bx + c = 0$, then which of the following is true.
(a) $c = a + b$ (b) $a = b + c$ (c) $b = a + c$ (d) $b = c$

7. The value of $\frac{\sin 55° - \cos 55°}{\sin 10°}$ is
(a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) 1 (d) $\sqrt{2}$

8. The value of $\cos 15° - \sin 15°$ is
(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $-\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

9. Minimum value of $27^{\cos 2x}\, 81^{\sin 2x}$ is
(a) $\frac{1}{243}$ (b) $\frac{1}{27}$ (c) -5 (d) $\frac{1}{5}$

10. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$
(a) 14 (b) 11 (c) 12 (d) 13

11. If $\sin\theta = \sin 15° + \sin 45°$, where $0° < \theta < 90°$, then value of θ is
(a) 45° (b) 54° (c) 60°
(d) 72° (e) 75°

12. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then value of $\cos 2\alpha + \cos 2\beta$ is
(a) $-2\sin(\alpha+\beta)$ (b) $2\cos(\alpha+\beta)$ (c) $2\sin(\alpha-\beta)$
(d) $-2\cos(\alpha+\beta)$ (e) $-2\cos(\alpha-\beta)$

13. If $A + B = 45°$, then $(\tan A - 1)(\tan B - 1)$ is
(a) 1 (b) $\frac{1}{2}$ (c) -1
(d) -2 (e) 2

14. If $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, $\sin\alpha = \frac{4}{5}$ and $\cos(\alpha+\beta) = -\frac{12}{13}$, then the value of $\sin\beta$ is
(a) $\frac{63}{65}$ (b) $\frac{61}{65}$ (c) $\frac{3}{5}$
(d) $\frac{5}{13}$ (e) $\frac{8}{65}$

15. The value of $\tan 40° + \tan 20° + \sqrt{3}\tan 20° \tan 40°$ is
(a) $\sqrt{12}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1
(d) $\frac{\sqrt{3}}{2}$ (e) $\sqrt{3}$

16. The value of $\cos 20° + \cos 100° + \cos 140°$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$
(d) 0 (e) 1

17. If $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$ and $\theta \neq \pm\frac{\pi}{4}$, then the value of $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)$ is

(a) 0 (b) -1 (c) 1
(d) -2 (e) 2

18. If $\sin\theta = 3\sin(\theta + 2\alpha)$, then the value of $\tan(\theta + \alpha) + 2\tan\alpha$ is

(a) 3 (b) 2 (c) -1
(d) 0 (e) 1

19. If $x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3}$ then the value of $xy + yz + zx$ is

(a) -1 (b) 0 (c) 1 (d) 2

20. $\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°}$ equals

(a) $\tan 26°$ (b) $\tan 81°$ (c) $\tan 51°$ (d) $\tan 54°$

21. The value of $\sin 50° - \sin 70° + \sin 10°$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

22. Maximum value of $3\cos\theta + 4\sin\theta$ is

(a) 3 (b) 4 (c) 5 (d) 7

23. If $\tan\alpha = \frac{n}{n+1}$ and $\tan\beta = \frac{1}{2n+1}$, then the value of $\alpha + \beta$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{5}$

24. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then the value of $\frac{\tan x}{\tan y}$ is

(a) 0 (b) ab (c) $\frac{b}{a}$ (d) $\frac{a}{b}$

25. If $\tan\alpha = \frac{5}{6}$, $\tan\beta = \frac{1}{11}$ then the value of $\alpha + \beta$ is

(a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $-\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

26. Maximum value of $\sin\left(x+\frac{\pi}{6}\right) + \cos\left(x+\frac{\pi}{6}\right)$ is

(a) $\sqrt{2}$ (b) $\sqrt{7}$ (c) 2 (d) 8

27. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$, then $\tan 2\alpha =$

(a) $\frac{25}{16}$ (b) $\frac{56}{33}$ (c) $\frac{19}{12}$ (d) $\frac{20}{17}$

28. If $A = \sin^2 x + \cos^4 x$, where x is a real number
(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le \frac{13}{16}$ (c) $\frac{3}{4} \le A \le 1$ (d) $\frac{13}{16} \le A \le 1$

29. If $x = \tan 15°$, $y = \operatorname{cosec} 75°$ and $z = 4 \sin 18°$, then
(a) $x < y < z$ (b) $y < z < x$ (c) $z < x < y$ (d) $x < z < y$

30. If $\sin(x + 3\alpha) = 3 \sin(\alpha - x)$, then
(a) $\tan x = \tan \alpha$ (b) $\tan x = \tan^2 \alpha$
(c) $\tan x = \tan^3 \alpha$ (d) $\tan x = 3 \tan \alpha$

31. Expression $\frac{1}{\cos 290°} + \frac{1}{\sqrt{3} \sin 250°}$ equals
(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

he value of $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is
(b) $\frac{1}{6}$ (c) $\frac{1}{8}$ (d) $\frac{1}{2}$

lue of $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$ is
(a) $\frac{3}{2}$ (b) $-\frac{2}{3}$ (c) -1 (d) 1

34. The value of $\cos\frac{\pi}{15} \cos\frac{2\pi}{15} \cos\frac{4\pi}{15} \cos\frac{8\pi}{15}$ is
(a) $\frac{1}{16}$ (b) $-\frac{1}{16}$ (c) 1 (d) 0

35. $\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}}$ equals
(a) $\sec\frac{x}{2}$ (b) $\sec x$ (c) $\operatorname{cosec} x$ (d) 1

36. The value of $\cos 15° \cos 7\frac{1}{2}° \sin 7\frac{1}{2}°$ is
(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

37. $\frac{\cot x - \tan x}{\cot 2x}$ equals
(a) 1 (b) 2 (c) -1 (d) 4

38. $\tan 67\frac{1°}{2} + \cot 67\frac{1°}{2}$ equals
(a) $2\sqrt{2}$ (b) $2-\sqrt{2}$ (c) $\sqrt{2}$ (d) $3\sqrt{2}$

39. If $\sin 4A - \cos 2A = \cos 4A - \sin 2A$, $\left(0 < A < \frac{\pi}{4}\right)$ then the value of $\tan 4A$ is
(a) 1 (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$
(d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (e) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\frac{1}{8}\left(3-4\cos 2\theta+\cos 4\theta\right)$ equals

(a) $\cos 4\theta$ (b) $\sin 4\theta$ (c) $\sin^4\theta$

(d) $\cos^4\theta$ (e) $\sin^4(\theta/2)$

If $8\cos 2\theta + 8\sec 2\theta = 65,\ 0 < \theta < \frac{\pi}{2}$ then the value of $4\cos 4\theta$ is

(a) $\frac{-23}{8}$ (b) $\frac{-31}{8}$ (c) $\frac{-31}{32}$

(d) $\frac{-33}{32}$ (e) $\frac{-32}{4}$

$\cos A \cos 2A \cos 4A \ldots \cos 2^{n-1}A$ equals

(a) $\frac{\sin 2^n A}{2^n \sin A}$ (b) $\frac{2^n \sin 2^n A}{\sin A}$ (c) $\frac{2^n \sin A}{\sin 2^n A}$ (d) $\frac{\sin A}{2^n \sin 2^n A}$

$\tan 9° - \tan 27° - \tan 63° + \tan 81°$ equals

(a) 2 (b) 3

(c) 4 (d) Non of these

$\frac{\cos\theta}{1+\sin\theta}$ equals

(a) $\tan\left(\frac{\theta}{2}-\frac{\pi}{4}\right)$ (b) $\tan\left(-\frac{\pi}{4}-\frac{\theta}{2}\right)$ (c) $\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ (d) $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

Suppose α, β are such that $\pi < \alpha - \beta < 3\pi$. if $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$ then the value of $\cos\frac{\alpha-\beta}{2}$ is

(a) $-\frac{3}{\sqrt{130}}$ (b) $\frac{3}{\sqrt{130}}$ (c) $\frac{6}{65}$ (d) $-\frac{6}{65}$

The value of $\cos^2(A-B) + \cos^2 B - 2\cos(A-B)\cos A\cos B$ is

(a) $\cos^2 A$ (b) $\sin^2 A$

(c) $\tan^2 A$ (d) $\cot^2 A$

$\cos^2\frac{\pi}{16} + \cos^2\frac{3\pi}{16} + \cos^2\frac{5\pi}{16} + \cos^2\frac{7\pi}{16}$ equals

(a) 0 (b) 1 (c) 2 (d) 3

If $\sin\theta - \cos\theta = \frac{\sqrt{3}-1}{2}$, then θ equals

(a) 30° (b) 45° (c) 60° (d) 90°

Which of the following is true about ...

51. If $1 + \cos x \cos y + \sin x \sin y = 0$, then which of the following is/are true
1. $\cos x + \cos y = 0$
2. $\sin x + \sin y = 0$
3. $\sin x + \cos y = 0$

chose the correct option from the code given below
(a) only 1 (b) only 2 (c) only 3 (d) 1 and 2

Answers—13A

1. (c)	2. (c)	3. (d)	4. (a)	5. (a)	6. (a)	7. (d)	8. (b)
9. (a)	10. (d)	11. (c)	12. (d)	13. (c)	14. (a)	15. (c)	16. (d)
17. (c)	18. (d)	19. (b)	20. (d)	21. (a)	22. (c)	23. (a)	24. (d)
25. (d)	26. (a)	27. (b)	28. (c)	29. (a)	30. (c)	31. (b)	32. (c)
33. (a)	34. (b)	35. (a)	36. (a)	37. (c)	38. (a)	39. (c)	40. (c)
41. (b)	42. (a)	43. (c)	44. (c)	45. (a)	46. (b)	47. (c)	48. (c)
49. (c)	50. (c)	51. (d)					

Explanation

1. (c) $\because \; p^2 + q^2 = 1$, let $p = \sin\theta$, $q = \cos\theta$

$\therefore \; p + q = \sin\theta + \cos\theta$

Its maximum value $= \sqrt{1^2+1^2} = \sqrt{2}$

(Recall that $-\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$)

2. (c) $3 - \cos\theta - \cos(\theta + \frac{\pi}{3}) = 3 - \cos\theta + (\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3})$

$$= 3 - \cos\theta + \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$$

$$= 3 - \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$$

here, $\sqrt{a^2+b^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$

$\therefore$ Maximum value $= 3 + 1 = 4$, Minimum value $= 3 - 1 = 2$

3. (d) $\sin A = \frac{1}{\sqrt{10}} \Rightarrow \tan A = \frac{1}{3}$ and $\sin B = \frac{1}{\sqrt{5}} \Rightarrow \tan B = \frac{1}{2}$

Now, $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{2}+\frac{1}{3}}{1 - \frac{1}{2}\times\frac{1}{3}} = 1 \Rightarrow A + B = 45^\circ$

4. (a) $\cos(\alpha - \beta) = \cos(\alpha - \theta) - (\beta - \theta)$

$$= \cos(\alpha-\theta)\cos(\beta-\theta) + \sin(\alpha-\theta)\sin(\beta-\theta)$$

$$= ab + \sqrt{1-a^2}\sqrt{1-b^2}$$

$(\because \sin(\alpha-\theta) = \sqrt{1-\cos^2(\alpha-\theta)})$

Now,

$\sin^2(\alpha-\beta) + 2ab\cos(\alpha-\beta) = 1-\cos^2(\alpha-\beta) + 2ab\cos(\alpha-\beta)$

$= 1-(ab+\sqrt{1-a^2}\ \sqrt{1-b^2})^2 + 2ab\,(ab+\sqrt{1-a^2}\ \sqrt{1-b^2})$

Simplify it

5. (a) tangent of angle means $\tan\theta$

here, $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$ as in question number (3) $A+B = 45^\circ$

6. (a) $P+Q+R = 180^\circ \Rightarrow P+Q = 90^\circ$ $(\because R = 90^\circ)$

$$\Rightarrow \frac{P}{2}+\frac{Q}{2} = 45^\circ$$

$$\Rightarrow \tan\left(\frac{P}{2}+\frac{Q}{2}\right) = \tan 45^\circ = 1$$

$$\Rightarrow \frac{\tan\frac{P}{2}+\tan\frac{Q}{2}}{1-\tan\frac{P}{2}\tan\frac{Q}{2}} = 1$$

$$\Rightarrow \frac{-\frac{b}{a}}{1-\frac{c}{a}} = 1$$

$$\Rightarrow \frac{-b}{a-c} = 1$$

$$\Rightarrow -b = a-c$$

$$\Rightarrow a+b = c$$

7. (d) $\frac{\sin 55^\circ - \cos 55^\circ}{\sin 10^\circ} = \frac{\sin 55^\circ - \sin 35^\circ}{\sin 10^\circ}$

$$= \frac{2\cos\frac{55^\circ+35^\circ}{2}\sin\frac{55^\circ-35^\circ}{2}}{\sin 10^\circ} = 2\cos 45^\circ = \sqrt{2}$$

8. (b) Recall that $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$ and $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

9. (a) $27^{\cos 2x}81^{\sin 2x} = 3^{3\cos 2x}\,3^{4\sin 2x}$

$$= 3^{3\cos 2x + 4\sin 2x}$$

$\because$ minimum value of $a\cos\theta + b\sin\theta = -\sqrt{a^2+b^2}$

$\therefore$ Minimum value of given expression $= 3^{-\sqrt{3^2+4^2}} = 3^{-5} = \frac{1}{243}$

10. (d) $(\sin x+\cos x)^4 = ((\sin x-\cos x)^2)^2 = (1-2\sin x\cos x)^2$

$\sin^6 x + \cos^6 x = (\sin^2 x+\cos^2 x)(\sin^4 x - \sin^2 x\cos^2 x + \cos^4 x)$

$= 1\cdot\{(\sin^2 x+\cos^2 x)^2 - 3\sin^2 x\cos^2 x\}$

$= 1-3\sin^2 x\cos^2 x$ etc.

Shortcut : Since all the options are free from x, putting $x = 0$

Required value $= 3(0-1)^4 + 6(0+1)^2 + 4(0+1^6) = 13$

11. (c) $\sin\theta = 2\sin\frac{15^\circ+45^\circ}{2}\cos\frac{15^\circ-45^\circ}{2}$

$= 2\sin30^\circ\cos15^\circ = \cos15^\circ = \sin75^\circ$

12. (d) $\cos\alpha + \cos\beta = 0 \Rightarrow \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = 0$

$\sin\alpha + \sin\beta = 0 \Rightarrow \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta = 0$

adding $2 + 2\cos(\alpha-\beta) = 0$

$\Rightarrow \cos(\alpha-\beta) = -1$

Now, $\cos2\alpha + \cos2\beta = 2\cos\left(\frac{2\alpha+2\beta}{2}\right)\cos\left(\frac{2\alpha-2\beta}{2}\right)$

$= 2\cos(\alpha+\beta)\cos(\alpha-\beta) = -2\cos(\alpha+\beta)$

13. (c) put $B = 45^\circ - A$, and simplify

14. (a) $\sin\beta = \sin[(\alpha+\beta)-\alpha]$

$= \sin(\alpha+\beta)\cos\alpha - \cos(\alpha+\beta)\sin\alpha$

$= \frac{5}{13}\cdot\frac{3}{5} + \frac{12}{13}\cdot\frac{4}{5} = \frac{63}{65}$

[$\because \cos(\alpha+\beta) = \frac{-12}{13} \Rightarrow \sin(\alpha+\beta) = \frac{5}{13}$ because $(\alpha+\beta)$ lies in II quadrant so $\sin(\alpha+\beta)$ is positive]

15. (c) $\tan(20^\circ + 40^\circ) = \tan60^\circ$

$\frac{\tan20^\circ + \tan40^\circ}{1 - \tan20^\circ\tan40^\circ} = \sqrt{3}$

$\Rightarrow \tan20^\circ + \tan40^\circ = \sqrt{3} - \sqrt{3}\tan20^\circ\tan40^\circ$

$\Rightarrow \tan20^\circ + \tan40^\circ + \sqrt{3}\tan20^\circ\tan40^\circ = \sqrt{3}$.

16. (d) For $\cos100^\circ + \cos140^\circ$ apply $\cos C + \cos D$.

17. (c) Apply $\cot(A+B)$ and $\cot(A-B)$ and simplify.

18. (d) $\frac{\sin(\theta+2\alpha)}{\sin\theta} = \frac{1}{3}$, Apply componendo-dividendo

19. (b) Let $x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3} = k$

$\therefore x = \frac{-1}{2}y = \frac{-1}{2} = k \Rightarrow x = k, y = -2k, z = -2k$

$\therefore xy + yz + zx = -2k^2 + 4k^2 - 2k^2 = 0.$

20. (d) $\frac{\cos9^\circ + \sin9^\circ}{\cos9^\circ - \sin9^\circ} = \frac{1+\tan9^\circ}{1-\tan9^\circ}$. (divide N^r & D^r by $\cos9^\circ$)

$= \tan(45^\circ + 9^\circ) = \tan54^\circ$

24. (d) Apply Componendo and dividendo

26. (a)

27. (b)

$\therefore$

28. (c

29. (

30.

31.

26. (a) If $x + \frac{\pi}{6} = \theta$

then $\sin(x + \frac{\pi}{6}) + \cos(x + \frac{\pi}{6}) = \sin\theta + \cos\theta$

Its maximum value $= \sqrt{1^2 + 1^2} = \sqrt{2}$.

27. (b) $\cos(\alpha + \beta) = \frac{4}{5} \Rightarrow \tan(\alpha + \beta) = \frac{3}{4}$

$\sin(\alpha - \beta) = \frac{5}{13} \Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$

$\therefore \tan 2\alpha = \tan((\alpha + \beta) + (\alpha - \beta))$

$$= \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4}\frac{5}{12}} = \frac{56}{33}$$

28. (c) $A = \sin^2 x + \cos^4 x$

$= 1 - \cos^2 x + \cos^4 x$

$= 1 - \cos^2 x\,(1 - \cos^2 x)$

$= 1 - \cos^2 x \sin^2 x = 1 - \frac{\sin^2 2x}{4}$

$\because\ 0 \le \sin^2 2x \le 1$

$\therefore$ when $\sin^2 2x = 0,\ A = 1$

when $\sin^2 2x = 1,\ A = 1 - \frac{1}{4} = \frac{3}{4}$

29. (a) $x \tan 15^\circ = 2 - \sqrt{3} = 2 - 1{\cdot}73 = 0{\cdot}27$

$y = \operatorname{cosec} 75^\circ = \frac{1}{\sin 15^\circ} = \frac{2\sqrt{2}}{\sqrt{3} - 1} = \frac{2 \times 1{\cdot}414}{1{\cdot}73 - 1} = \frac{2{\cdot}8}{0{\cdot}73} = 0{\cdot}4$ (Approximate)

$z = 4\sin 8^\circ = 4\left(\frac{\sqrt{5} - 1}{4}\right) = 2{\cdot}23 - 1 = 1{\cdot}23$

$\therefore\ x < y < z$

30. (c) $\sin(x + 3\alpha) = 3\sin(\alpha - x)$

$\Rightarrow \sin x \cos 3\alpha + \cos x \sin 3\alpha = 3(\sin\alpha \cos x - \cos\alpha \sin x)$

$\Rightarrow \sin x(\cos 3\alpha + 3\cos\alpha) = \cos x(3\sin\alpha - \sin 3\alpha)$

$\Rightarrow \sin x\, 4\cos^3\alpha = \cos x\, 4\sin^3 x$

$\Rightarrow \tan x = \tan^3\alpha.$ $\quad (\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta)$

31. (b) $\frac{1}{\cos 290^\circ} + \frac{1}{\sqrt{3}\sin 250^\circ} = \frac{1}{\sin 20^\circ} - \frac{1}{\sqrt{3}\cos 20^\circ}$

$$= \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sqrt{3}\sin 20^\circ \cos 20^\circ}$$

$$= 2\,\frac{\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{\sqrt{3}\left(\frac{2\sin 20^\circ \cos 20^\circ}{2}\right)}$$

$$= \frac{4(\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)}{\sqrt{3}\,\sin 40^\circ}$$

$$= \frac{4\sin(60^\circ - 20^\circ)}{\sqrt{3}\,\sin 40^\circ} = \frac{4}{\sqrt{3}}$$

32. (c) here $\cos\frac{5\pi}{8} = \cos\left(\pi - \frac{3\pi}{8}\right) = -\cos\frac{3\pi}{8}$

and $\cos\frac{7\pi}{8} = \cos\left(\pi - \frac{\pi}{8}\right) = -\cos\frac{\pi}{8}$ etc.

33. (a) Apply $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

34. (b) $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$

$$\frac{\ldots in\frac{\pi}{15}\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}}{2\sin\frac{\pi}{15}}$$

$$= \frac{\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}}{2\sin\frac{\pi}{15}} \text{ etc.}$$

Now take the help of solved example 31.

35. (a) Applying $1 + \cos 2\theta = 2\cos^2\theta$

we have, $\sqrt{2+2\cos 4x} = \sqrt{2(1+\cos 4x)}$

$$= \sqrt{2\cdot 2\cos^2 2x} = 2\cos 2x$$

$$= 2\frac{\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{\sqrt{3}\left(\frac{2\sin 20^\circ \cos 20^\circ}{2}\right)}$$

$$= \frac{4(\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)}{\sqrt{3}\sin 40^\circ}$$

$$= \frac{4\sin(60^\circ - 20^\circ)}{\sqrt{3}\sin 40^\circ} = \frac{4}{\sqrt{3}}$$

32. (c) here $\cos\frac{5\pi}{8} = \cos\left(\pi - \frac{3\pi}{8}\right) = -\cos\frac{3\pi}{8}$

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33. (a) Apply $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

34. (b) $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$

$$= \frac{2\sin\frac{\pi}{15}\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}}{2\sin\frac{\pi}{15}}$$

$$= \frac{\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}}{2\sin\frac{\pi}{15}} \text{ etc.}$$

Now take the help of solved example 31.

35. (a) Applying $1 + \cos 2\theta = 2\cos^2\theta$

we have, $\sqrt{2+2\cos 4x} = \sqrt{2(1+\cos 4x)}$

$$= \sqrt{2 \cdot 2\cos^2 2x} = 2\cos 2x$$

$$\therefore \frac{2}{\sqrt{2+2\sqrt{2+2\cos 4x}}} = \frac{2}{\sqrt{2+\sqrt{2+2\cos 2x}}} \text{ etc.}$$

36. (a) $\tan 67\frac{1}{2}^\circ + \cot 67\frac{1}{2}^\circ = \cot 22\frac{1}{2}^\circ + \tan 22\frac{1}{2}^\circ$

$$= (\sqrt{2}+1) + (\sqrt{2}-1) = 2\sqrt{2}$$

37. (c) $\sin 4A + \sin 2A = \cos 4A + \cos 2A$

$\Rightarrow 2\sin 3A \cos A = 2\cos 3A \cos A$

[use formula of $\sin C + \sin D$ and $\cos C + \cos D$]

$\Rightarrow \tan 3A = 1 \Rightarrow 3A = 45^\circ \Rightarrow A = 15^\circ$

$\therefore \tan 4A = \tan 60^\circ = \sqrt{3}$

44. (c) $\frac{\cos\theta}{1+\sin\theta}=\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{1+\cos\left(\frac{\pi}{2}-\theta\right)}$

$$=\frac{2\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}$$

$$=\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$$

45. (a) $\frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}=\frac{21}{27}$

$$=\frac{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}{2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}=\frac{7}{9}$$

$$\Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right)=\frac{7}{9}$$

$\sin\alpha+\sin\beta=\frac{-21}{65}$ for,

$$2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=\frac{-21}{65}$$

$$2\frac{7}{\sqrt{7^2+9^2}}\cdot\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{-21}{65}$$

$$\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{-21}{65}\times\frac{\sqrt{130}}{14}$$

$$=\frac{-3\times\sqrt{130}}{65\times 2}=\frac{-3}{\sqrt{130}}$$

46. (b) $\cos^2(A-B)+\cos^2B-\cos(A-B)\{2\cos A\cos B\}$

$=\cos^2(A-B)+\cos^2B-\cos(A-B)\{\cos(A-B)+\cos(A+B)\}$

$=\cos^2(A-B)+\cos^2B-\cos^2(A-B)-\cos(A-B)\cos(A+B)$

$=\cos^2B-\cos^2A+\sin^2B=1-\cos^2A=\sin^2A$

47. (c) $\cos^2\frac{7\pi}{16}=\cos^2\left(\frac{\pi}{2}-\frac{\pi}{16}\right)=\sin^2\frac{\pi}{16}$

$=\cos^2\frac{5\pi}{16}$

$=\cos^2\left(\frac{\pi}{2}-\frac{3\pi}{16}\right)=\sin^2\frac{3\pi}{16}$ etc.

$\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

$-\frac{\theta}{2}\Big)$

$sA\cos B]$

$\cos(A-B)+\cos(A+B)]$

$\cos(A-B)\cos(A+B)$

$\sin^2 A$

48. (c) $\sin\theta - \cos\theta = \frac{\sqrt{3}-1}{2}$

Squaring

$(\sin\theta - \cos\theta)^2 = \left(\frac{\sqrt{3}-1}{2}\right)^2$

$\Rightarrow \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = \frac{3+1-2\sqrt{3}}{4}$

$\Rightarrow 1-\sin 2\theta = 1-\frac{\sqrt{3}}{2}$

$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} = \sin 120°$

$\Rightarrow 2\theta = 120°$

$\Rightarrow \theta = 60°$

49. (c) $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

$\Rightarrow \sin^2 x = (1-\cos x)(1+\cos x)$

$\Rightarrow (1-\cos^2 x) = (1-\cos^2 x)$

$\Rightarrow \sin^2 x = \sin^2 x$

but at $x = 180°$, $\sin x = 0$, and $1 + \cos x = 0$, so denominator of given identity is not defirnd when $x = 180°$

$\therefore$ options (c) is correct.

50. (c) $\cos 0° + \cos 1° + \ldots\ldots + \cos 179° + \cos 180°$

$= \cos 0° + \cos 1° + \cos 2° + \ldots + \cos (180° - 2°) + \cos (180° - 1°) + \cos 180°$

$= 1 + \cos 1° + \cos 2° + \ldots\ldots - \cos 2° - \cos 1° - 1 = 0$

51. (d) $\because\ 1 + \cos x \cos y + \sin x \cdot \sin y = 0$

$\Rightarrow 1 + \cos (x-y) = 0$

$\Rightarrow \cos (x-y) = -1 = \cos 180°$

$\Rightarrow x - y = 180° \Rightarrow x = 180° + y$

1. $\cos x + \cos y = \cos (180° + y) + \cos y = -\cos y + \cos y = 0$

$\therefore$ Statement (1) is correct.

2. $\sin x + \sin y = \sin (180° + y) + \sin y = -\sin y + \sin y = 0$

$\therefore$ Statement (2) is correct.

3. $\sin x + \cos y = \sin(180° + y) + \cos y = -\sin y + \cos y \neq 0$

$\therefore$ Statement (3) is not correct.

Hence option (d) is correct.

★★★

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